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Higgs-Boson Mass Limit within the Randall-Sundrum Model

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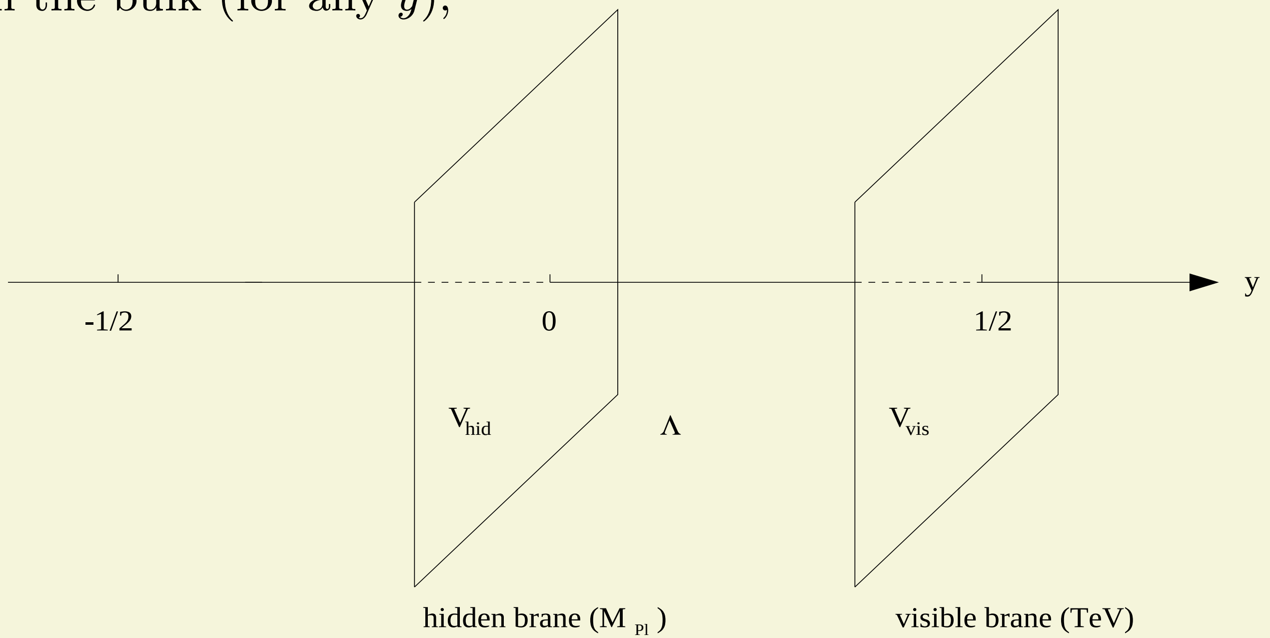
B. G., John F. Gunion, in press (Physics Letters B 653 (2007) 307)

The Randall-Sundrum Model

- x^μ, + 1 extra spatial dimension (y), orbifold (S^1/Z_2): $y \equiv y+1$, $y \equiv -y$
- Standard Model particles on a "visible" brane (at $y = 1/2$),
- Planck mass scale physics on the "hidden" (at $y = 0$),
- Gravity in the bulk (for any y),



The full 5d action:

$$S = -\int d^4x\, dy \sqrt{-g}\left(m^3_{Pl\,5} R + \Lambda\right) + \int d^4x \sqrt{-g_{\rm hid}}(\mathcal{L}_{\rm hid} - V_{\rm hid}) + \int d^4x \sqrt{-g_{\rm vis}}(\mathcal{L}_{\rm vis} - V_{\rm vis})$$

- Neglecting $\mathcal{L}_{hid}$ and $\mathcal{L}_{vis}$ we solve the Einstein's equations.

  The RS metric

$$g_{MN}(x,y) = \left(\begin{array}{c|c} e^{-2m_0 b_0 |y|}\eta_{\mu\nu} & 0 \\ \hline 0 & -b_0^2 \end{array} \right)$$

  is a solution of the Einstein's equations if:

$$V_{\rm hid} = -V_{\rm vis} = 12 m_0\, m_{Pl\,5}^3 \quad \text{and} \quad \Lambda = -12 m_0^2\, m_{Pl\,5}^3 \quad \Longrightarrow \quad \Lambda = -\frac{V_{\rm hid}^2}{12 m_{Pl\,5}^3}$$

  The curvature (the Ricci scalar) for the RS metric: $R_{RS} = 20 m_0^2$.

- An expansion around the background metric:
  - $\eta_{\mu\nu} \to \eta_{\mu\nu} + \epsilon h_{\mu\nu}(x,y)$, for $\epsilon^2 \equiv m_{Pl\,5}^{-3}$
  - $b_0 \to b_0 + b(x)$,

$$h_{\mu\nu}(x,y) = \sum_n h_{\mu\nu}^n(x) \frac{\chi^n(y)}{\sqrt{b_0}} \quad \Longrightarrow \quad \mathcal{L}_{\rm int} = -\frac{1}{\Lambda_W} \sum_{n\neq 0} h_{\mu\nu}^n T^{\mu\nu} - \frac{\phi}{\Lambda_\phi} T_\mu^\mu$$

for

$$\Lambda_W \simeq \sqrt{2} m_{Pl} \Omega_0, \quad \Lambda_\phi = \sqrt{3}\Lambda_W, \quad \Omega_0 = e^{-m_0 b_0/2} \quad \text{and} \quad \phi(x) \equiv \sqrt{6} m_{Pl} e^{-m_0(b_0 + b(x))/2}$$

Advantages:

- "Solution" of the hierarchy problem:

$$\textbf{All mass parameters } (m_{Pl\,5},\ v,\ \cdots) \textbf{ of the 5d theory } \mathcal{O}(m_{Pl\,5})$$

$$\Downarrow$$

$$\textbf{Effective 4d mass scale } v_0 = \Omega_0 v = e^{-m_0 b_0/2} v \sim 1 \textbf{ TeV if } m_0 b_0/2 \sim 35.$$

Drawbacks:

- (No stabilization $\Leftrightarrow$ massless radion) $\Longrightarrow$ "Goldberger-Wise-like"models
- Fine tuning of the cosmological constants

The Lee-Quigg-Thacker bound for the Higgs boson mass

$$W_L^+ W_L^- \to W_L^+ W_L^-$$

$$\mathcal{T}(s,\cos\theta) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta), \qquad a_J(s) = \frac{1}{32\pi(2J+1)} \int_{-1}^{1} \mathcal{T}(s,\cos\theta) P_J(\cos\theta) d\cos\theta$$

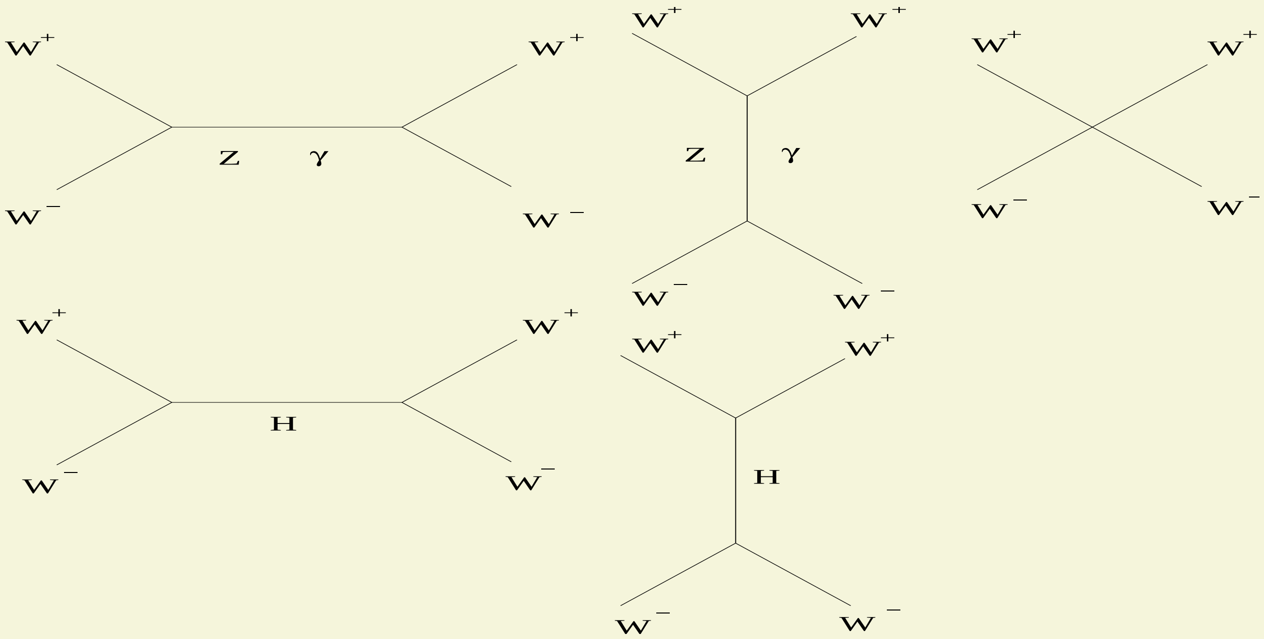
- **"high-energy divergences": Cornwall, Levin and Tiktopoulos, Phys. Rev. D10, 1145, 1974:**
  Proper high-energy behavior of tree-level amplitudes, i.e.

  $$\mathcal{T}^{(n)}|_{E\to\infty} = \mathcal{O}(E^{4-n} \ln^k E), \qquad \text{for} \qquad k > 0$$

  is a necessary condition for perturbative renormalizability.

- **"small partial-wave amplitudes":**

  Unitarity of the S**-matrix implies Re**$(a_J) \leq \frac{1}{2}$

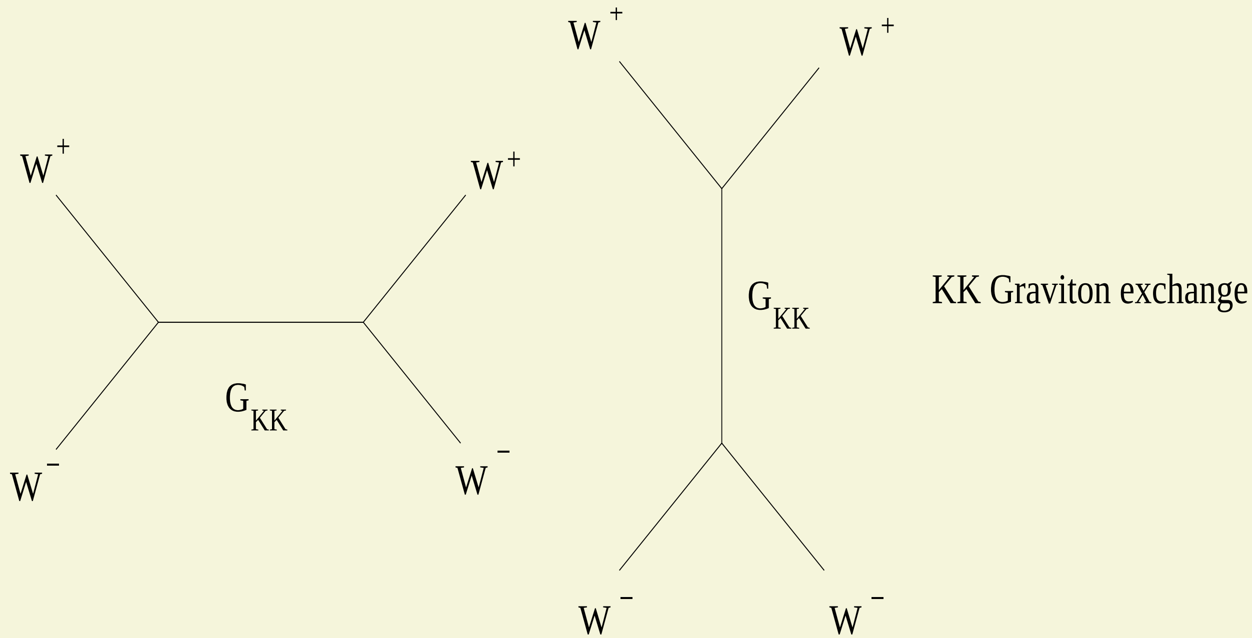


$$a_J = A_J \left(\frac{s}{m_W^2}\right)^2 + B_J \left(\frac{s}{m_W^2}\right) + C_J \quad \text{for} \quad \epsilon_\mu^{W_L}(k) = \frac{k_\mu}{m_W} + \mathcal{O}\left(\frac{m_W}{E}\right)$$

- $A_J, B_J \neq 0$ for $J = 0, 1$ and 2
- A-terms (for $J = 0, 1$ and 2) vanish by the virtue of the gauge invariance and $B_2 = 0$
- for $J = 0$ and 1, the B-term is canceled by the Higgs-boson exchange
- eventually a_J turns out to be m_H-dependent constant in the high-energy asymptotic region, that implies the Lee-Quigg-Thacker unitarity bound for the Higgs boson mass:

$$\mathbf{Re}(a_J) \leq \frac{1}{2} \quad \Rightarrow \quad m_H \lesssim 870 \text{ GeV}$$

Tree-level unitarity in $W_L^+ W_L^- \to G_{KK}, \phi \to W_L^+ W_L^-$



- The massive graviton propagator

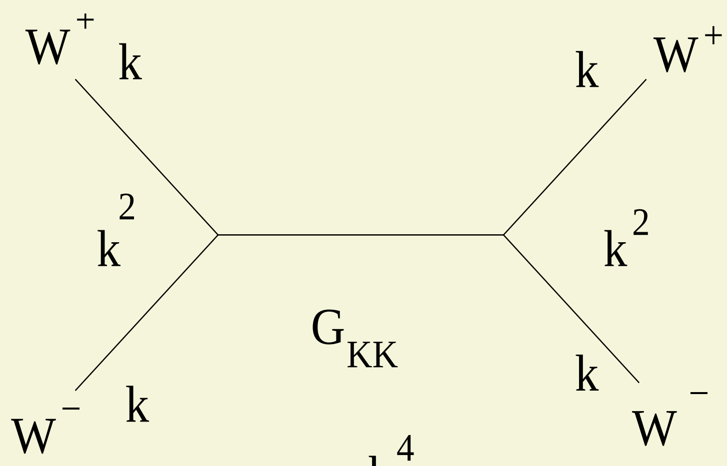
$$D^{\mu\nu,\alpha\beta}(k) = \frac{1}{2}\left(\bar{\eta}^{\mu\alpha}\bar{\eta}^{\nu\beta} + \bar{\eta}^{\mu\beta}\bar{\eta}^{\nu\alpha} - \frac{2}{3}\bar{\eta}^{\mu\nu}\bar{\eta}^{\alpha\beta}\right)\frac{i}{k^2 - m_G^2 + i\epsilon}\,,$$

  where $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{k^\mu k^\nu}{m_G^2}$ for $\eta^{\mu\nu}$ being the Minkowski metric.

- The graviton couples to the energy-momentum tensor $T_{\mu\nu}$, so the amplitude reads

$$T_{\mu\nu} D^{\mu\nu,\alpha\beta} T_{\alpha\beta}$$

- $\epsilon_\mu^{W_L}(k) = \frac{k_\mu}{m_W} + \mathcal{O}\left(\frac{m_W}{E}\right)$



KK Graviton exchange

$$\frac{k^4}{k^2}$$

$$\Downarrow$$

$$a_J \propto k^{10}$$

$$k_\mu T^{\mu\nu} = 0$$

$$\langle 0|T^{\mu\nu}|W_L^+ W_L^-\rangle =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6}[(1-2\beta_W)d^0_{0,0} + 2(\beta_W - 2)d^2_{0,0}]s & 0 & -\frac{1}{\sqrt{6}}(s+4m_W^2)d^2_{1,0} \\ 0 & 0 & -\frac{1}{2}sd^0_{0,0} & 0 \\ 0 & -\frac{1}{\sqrt{6}}(s+4m_W^2)d^2_{1,0} & 0 & \frac{1}{6}[(1+\beta_W)d^0_{0,0} + 2(\beta_W - 2)d^2_{0,0}]s\,, \end{pmatrix}$$

in the CM frame. The scattering angle is measured relatively to the direction of motion W^-, $d^J_{\mu\mu'}(\cos\theta) = d^J_{\mu\mu'}$ stands for the Wigner d function and $\beta_W \equiv 1 - 4m_W^2/s$.

$$\Downarrow$$

$$a_J \propto k^2$$

Note that the 4d RS effective theory contains dim 5 operators: $\propto \frac{1}{\Lambda_W} h^n_{\mu\nu} T^{\mu\nu}$, having a cutoff $\mathcal{O}(1\text{TeV})$, therefore the amplitude should satisfy the unitarity conditions up to the cutoff $\mathcal{O}(1\text{TeV})$.

Determination of the cutoff

Graviton-matter interactions in D-dim:

$$\mathcal{L} = \frac{1}{\Lambda_W^{D/2-1}}\, h_{MN} T^{MN}$$

- The goal: to determine the cutoff at which the interactions become strong.
- The Naive Dimensional Analysis (NDA) condition for the cutoff ($\Lambda_{\rm NDA}$):

The factor generated by extra internal graviton line = 1.

$$\Downarrow$$

$$\left(\frac{\Lambda_{\rm NDA}^n}{\Lambda_W^{D/2-1}}\right)^2 \frac{1}{\Lambda_{\rm NDA}^2} \left(\frac{1}{\Lambda_{\rm NDA}^n}\right)^2 \Lambda_{\rm NDA}^D\, l_D = 1$$

for

$$l_D = (4\pi)^{D/2}\Gamma(D/2) \qquad n = \begin{cases} 1 & \text{for fermions} \\ 2 & \text{for bosons} \end{cases}$$

In D=4: $$\Lambda_{\text{NDA}} = 4\pi\Lambda_W$$

- Let's apply the NDA to estimate the cutoff in 4D effective RS model with the tower of KK graviton modes

- $$m_n = x_n \frac{m_0}{m_{Pl}} \frac{\Lambda_W}{\sqrt{2}} \quad \text{with} \quad x_n \simeq \pi n \ \text{ for } \ n \gg 1$$

- In the effective theory one should include an exchange of N gravitons in the loop (in the NDA arguments), such that $m_N = \Lambda_{\text{NDA}}$

$$\Downarrow$$

$$\Lambda_{\text{NDA}} = 2^{7/6}\pi \left(\frac{m_0}{m_{Pl}}\right)^{1/3} \Lambda_W$$

Parameters of the effective RS model:

$$\begin{aligned}
\Lambda_W &\simeq \sqrt{2} m_{Pl} \Omega_0 \,, \\
\Lambda_\phi &= \sqrt{6} m_{Pl} \Omega_0 = \sqrt{3} \Lambda_W \\
m_n &= m_0 x_n \Omega_0 \,,
\end{aligned}$$

where $\Omega_0 m_{Pl} = e^{-m_0 b_0/2} m_{Pl}$ should be of order a TeV to solve the hierarchy problem. The x_n are the zeroes of the Bessel function J_1 ($x_1 \sim 3.8$, $x_n \sim x_1 + \pi(n-1)$). A useful relation following from the above equations is:

$$m_n = x_n \frac{m_0}{m_{Pl}} \frac{\Lambda_\phi}{\sqrt{6}} \quad \text{with} \quad m_1 = 15.5 \text{ GeV} \times \left(\frac{m_0/m_{Pl}}{0.01} \right) \left(\frac{\Lambda_\phi}{1 \text{ TeV}} \right) .$$

- To trust the RS solution of the Einstein equations the curvature m_0 must be small comparing to the 5d scale of quantum gravity $m_{Pl\,5}$: $m_0 < m_{Pl\,5}$.
- From the matching to the General Relativity

$$m_{Pl}^2 = 2 \frac{m_{Pl\,5}^3}{m_0} ,$$

that implies

$$\frac{m_0}{m_{Pl}} = \frac{1}{\sqrt{2}} \left(\frac{m_0}{m_{Pl\,5}} \right)^{3/2} \lesssim 1$$

- We define $\overline{\Lambda}$ (the cutoff) to be the largest $\sqrt{s}$ for which we would expect $W_L W_L \to W_L W_L$ scattering to be unitarity when computed using the RS effective theory. Since

$$\mathcal{L}_{\text{int}} = -\frac{1}{\Lambda_W} \sum_{n \neq 0} h^n_{\mu\nu} T^{\mu\nu} - \frac{\phi}{\Lambda_\phi} T^\mu_\mu$$

  therefore well-motivated choices for the upper cutoff seem to be $\overline{\Lambda} = \Lambda_\phi$ and $\overline{\Lambda} = \Lambda_W$

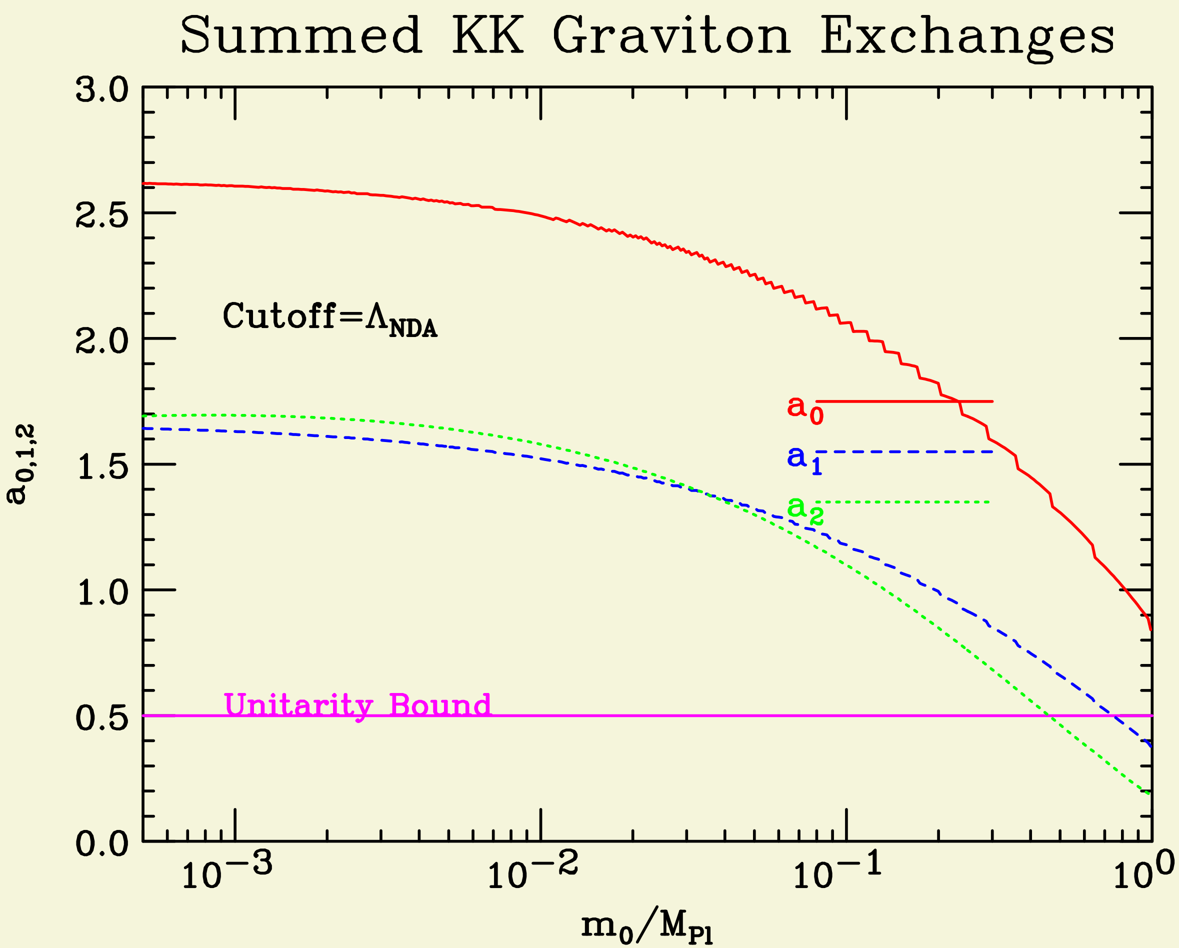
- We include all KK states with $m_n \leq \overline{\Lambda}$:

$$a_J(\sqrt{s}) = \sum_{n, m_n < \overline{\Lambda}} a_J(m_G = m_n, \sqrt{s})$$

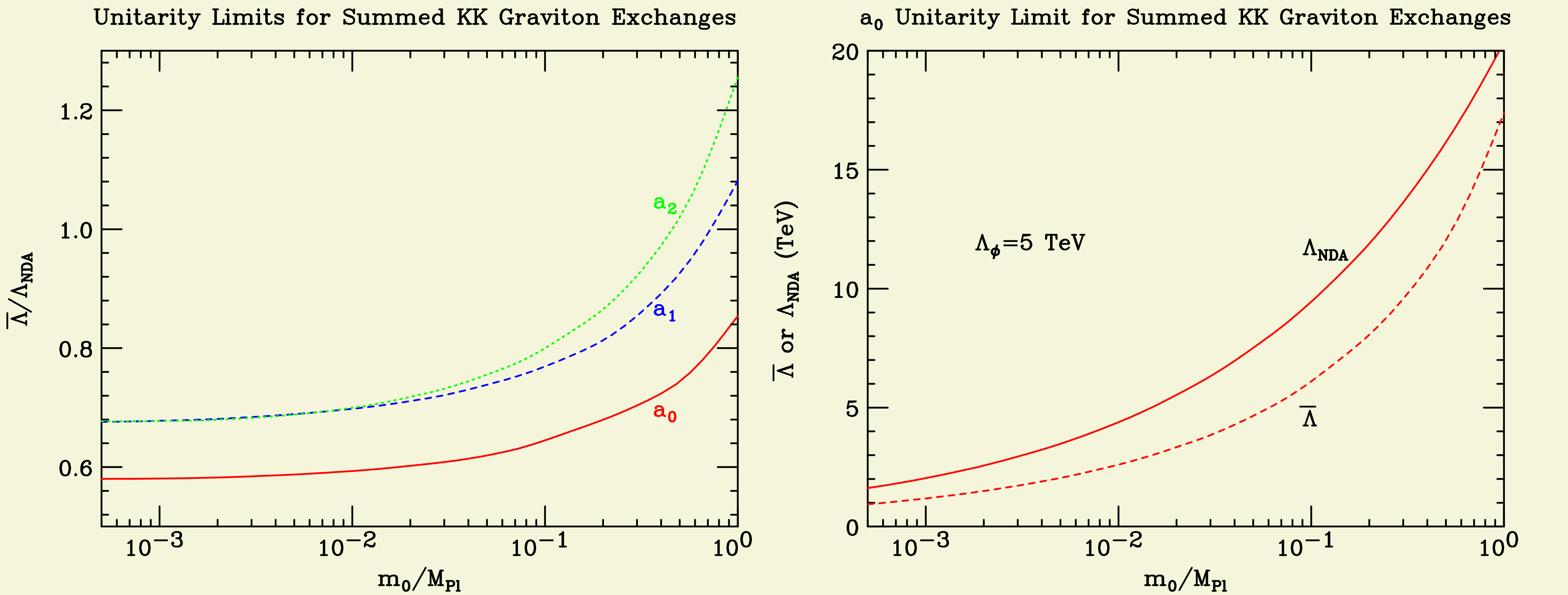
-

$$m_n = x_n \frac{m_0}{m_{Pl}} \frac{\Lambda_\phi}{\sqrt{6}}$$

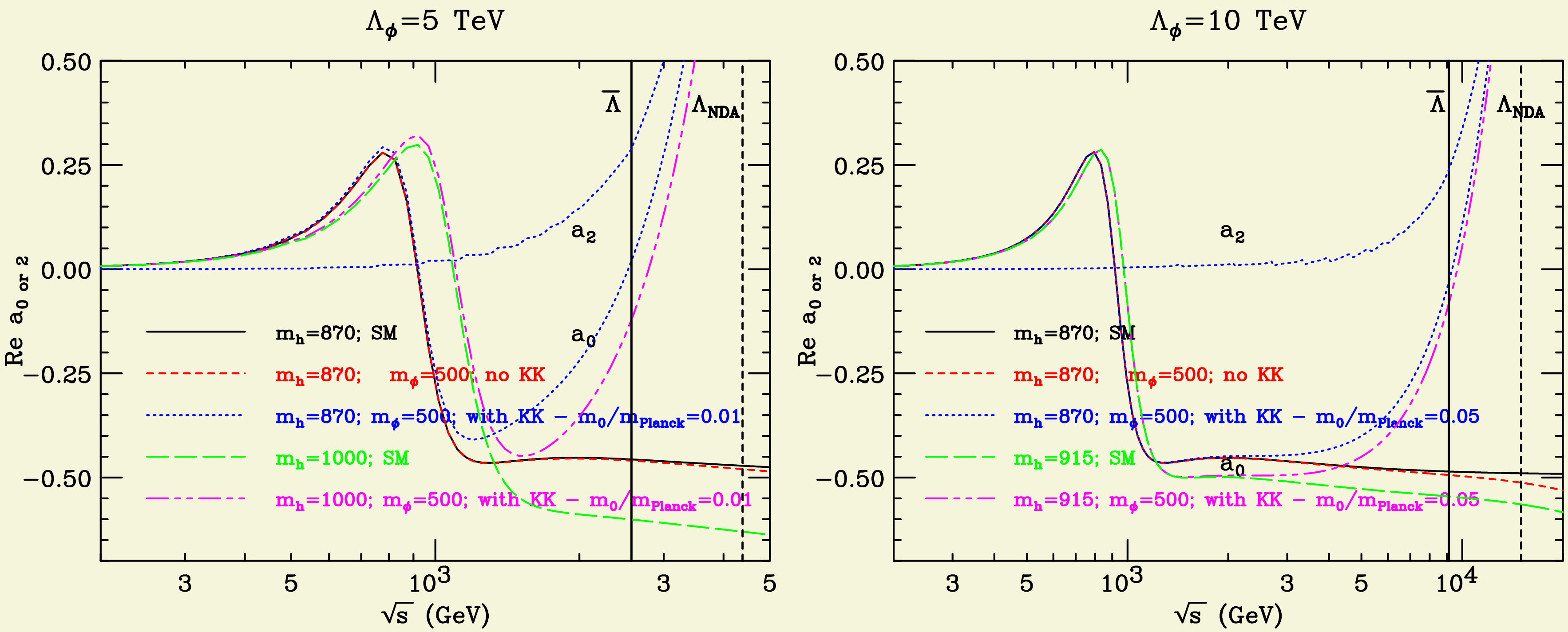
  $m_0/m_{Pl} \ll 1$ implies many KK graviton modes below the cutoff $\overline{\Lambda}$. It will be shown that because of the presence of many KK modes $\overline{\Lambda} \neq \Lambda_W$.



Re$a_{0,1,2}$ as functions of m_0/m_{Pl} as computed at $\sqrt{s} = \Lambda_{\rm NDA}$ and summing over all KK graviton resonances with mass below $\Lambda_{\rm NDA}$, but without including Higgs or radion exchanges. Note that $\Lambda_{\rm NDA} \propto (m_0/m_{Pl})^{1/3}\Lambda_W$ and that $a_J(\sqrt{s} = \Lambda_{\rm NDA})$ do not depend on Λ_W.



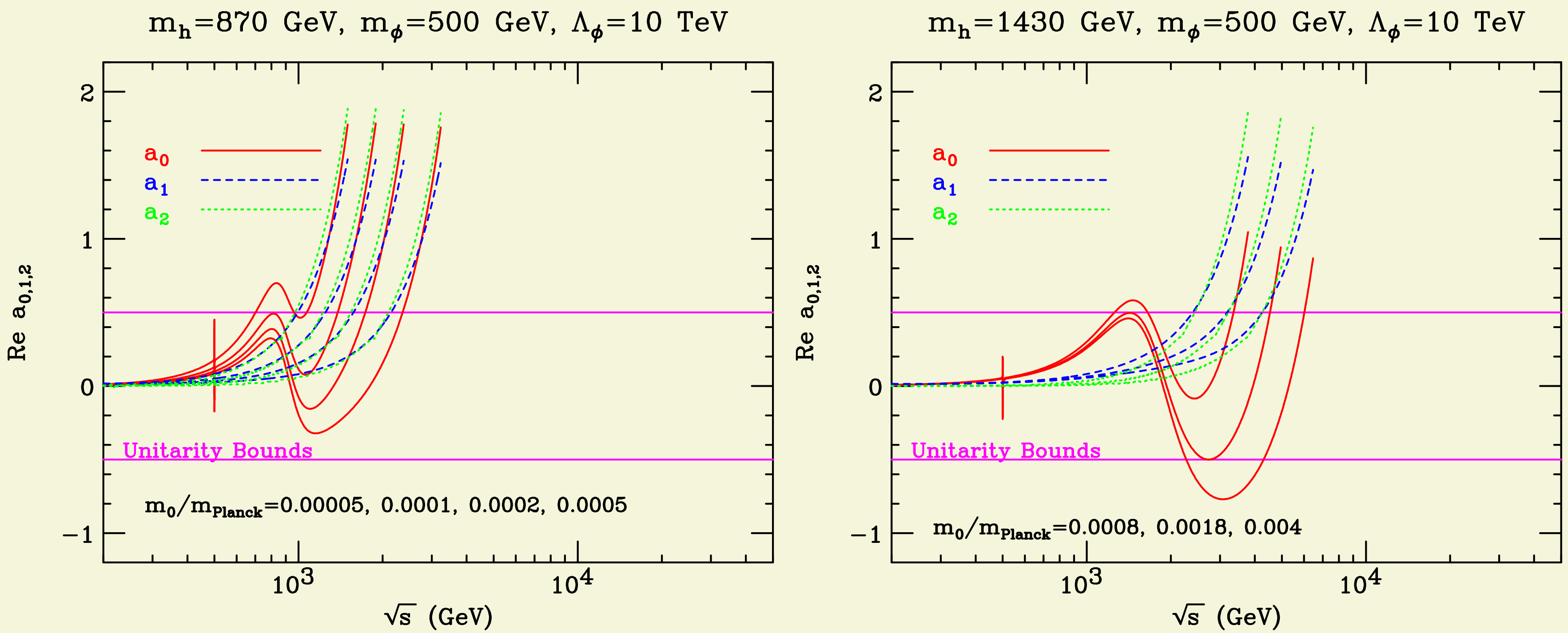
In the left hand plot, $\bar{\Lambda}/\Lambda_{\rm NDA}$ as a function of m_0/m_{Pl}, where $\bar{\Lambda}$ is the largest $\sqrt{s}$ for which $W_L^+W_L^-$ scattering is unitary after including KK graviton exchanges with mass up to $\bar{\Lambda}$, but before including Higgs and radion exchanges. Results are shown for the $J=0$, 1 and 2 partial waves. With increasing $\sqrt{s}$ unitarity is always violated earliest in the $J=0$ partial wave, implying that $J=0$ yields the lowest $\bar{\Lambda}$. The right hand plot shows the individual absolute values of $\bar{\Lambda}(J=0)$ and $\Lambda_{\rm NDA}$ for the case of $\Lambda_\phi = 5$ TeV; $\bar{\Lambda}/\Lambda_{\rm NDA}$ is independent of Λ_ϕ



$\mathbf{Re}a_0$ as a function of $\sqrt{s}$ for five cases:

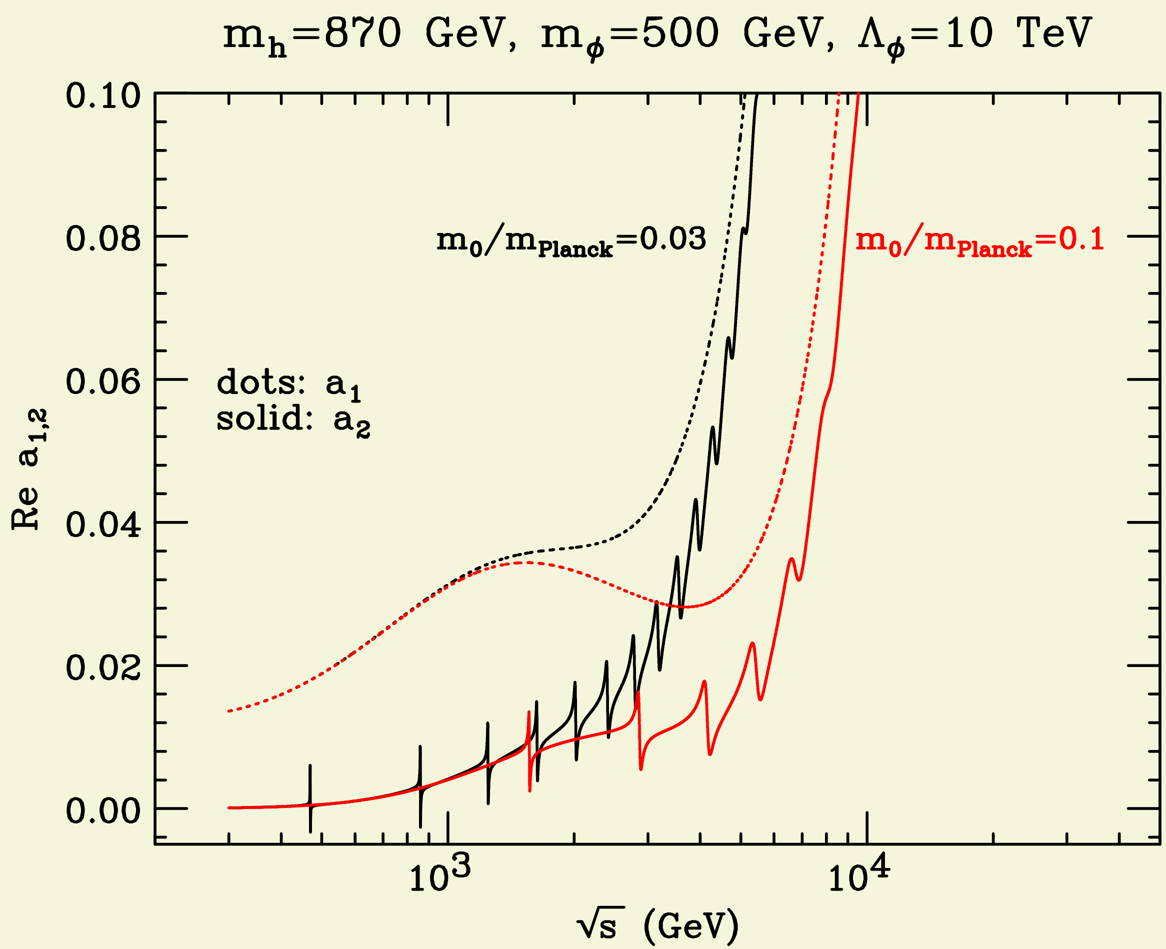
1. solid (black) $m_h = 870$ GeV, SM contributions only ($\gamma = 0$)
2. short dashes (red) $m_h = 870$ GeV, with radion of mass $m_\phi = 500$ GeV included, but no KK gravitons (we do not show the very narrow ϕ resonance)
3. dots (blue) as in 2), but including the sum over KK gravitons taking $m_0/m_{Pl} = 0.01$ ($m_0/m_{Pl} = 0.05$) — $\mathbf{Re}a_2$ is also shown for this case
4. long dashes (green) $m_h = 1000$ GeV (915 GeV), with radion of mass $m_\phi = 500$ GeV, but no KK gravitons
5. as in 4., but including the sum over KK gravitons taking $m_0/m_{Pl} = 0.01$ ($m_0/m_{Pl} = 0.05$). The $\bar{\Lambda}$ and $\Lambda_{\rm NDA}$ values for $m_0/m_{Pl} = 0.01$ ($m_0/m_{Pl} = 0.05$) are indicated by vertical lines.

The curvature dependence:



Re$a_{0,1,2}$ as functions of $\sqrt{s}$ for $m_h = 870$ GeV and $m_h = 1430$ GeV, taking $m_\phi = 500$ GeV and $\Lambda_\phi = 10$ TeV, and for the m_0/m_{Pl} values indicated on the plot.

The graviton excitations can be revealed:



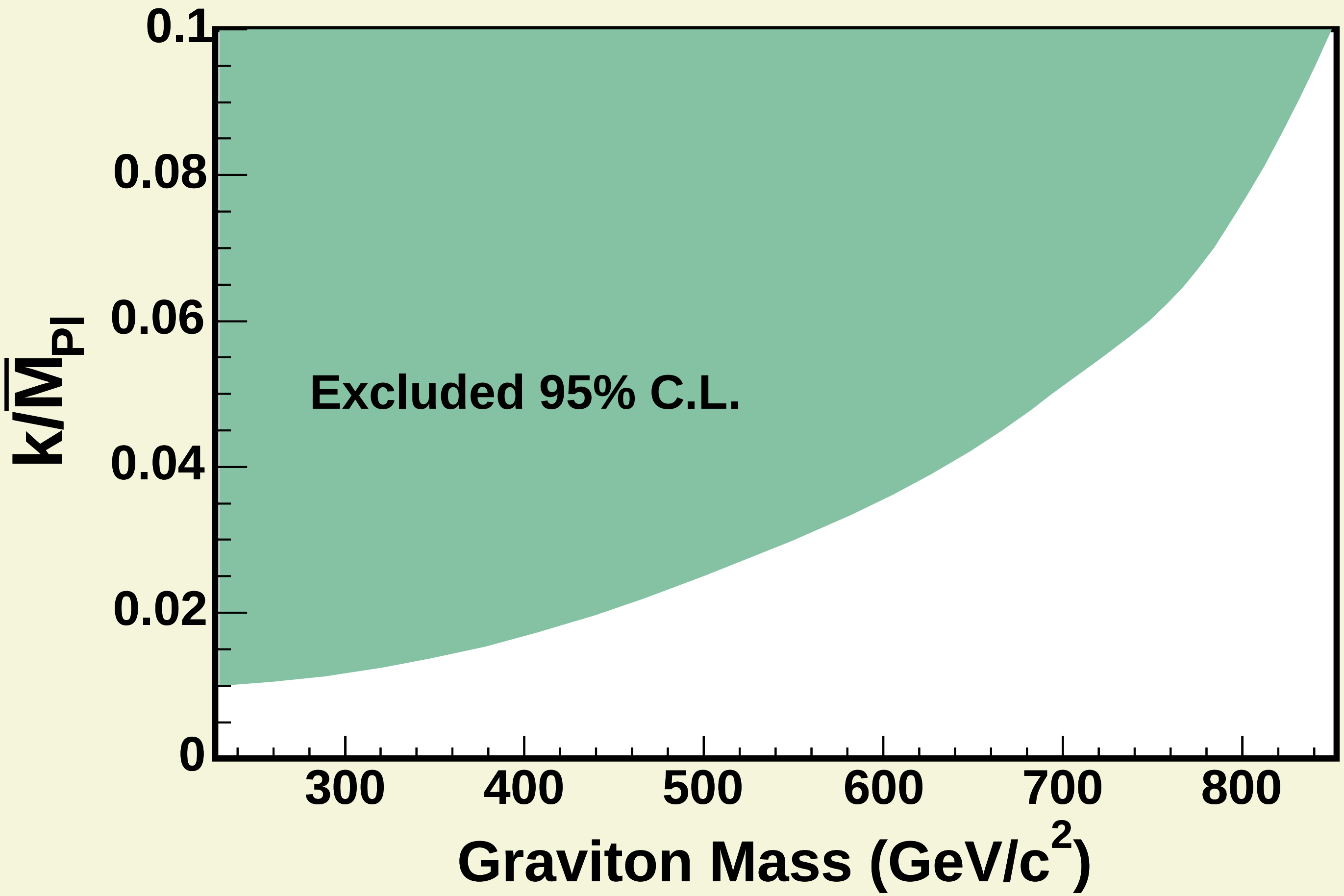
Re$a_{1,2}$ for $m_h = 870$ GeV, $m_\phi = 500$ GeV and $\Lambda_\phi = 10$ TeV as functions of $\sqrt{s}$ for the m_0/m_{Pl} values indicated on the plot.

Λ_ϕ(TeV)	5	10	20	40
Absolute maximum Higgs mass				
m_h^{max}(GeV)	1435	1430	1430	1430
required m_0/m_{Pl}	1.32×10^{-2}	1.8×10^{-3}	2.3×10^{-4}	2.9×10^{-5}
associated m_1(GeV)	103.2	28.2	7.2	1.8
$m_0/m_{Pl} = 0.005$: Tevatron limit: $m_1 >$??				
m_h^{max}(GeV)	1300	930	920	905
associated m_1(GeV)	39	78	156	313
$m_0/m_{Pl} = 0.01$: Tevatron limit: $m_1 > 240$ GeV				
m_h^{max}(GeV)	1405	930	910	895
associated m_1(GeV)	78	156	313	626
$m_0/m_{Pl} = 0.05$: Tevatron limit: $m_1 > 700$ GeV				
m_h^{max}(GeV)	930	915	900	885
associated m_1(GeV)	391	782	1564	3129
$m_0/m_{Pl} = 0.1$: Tevatron limit: $m_1 > 865$ GeV				
m_h^{max}(GeV)	920	910	893	883
associated m_1(GeV)	782	1564	3128	6257

Experimental constraints

- Tevatron KK-graviton search: $\sigma(p\bar{p} \to G) BR(G \to e^+e^-, \mu^+\mu^-, \gamma\gamma) \propto f_{p\bar{p}}(m_G)/\Lambda_W^2$

  $\Longrightarrow$ For a given graviton mass an upper limit for $\left(\frac{m_0}{m_{Pl}}\right)$ can be determined.



The 95% C.L. excluded region in the plane of $k/\bar{M}_{Pl}$ (m_0/m_{Pl}) and the graviton mass (T. Aaltonen *et al.* [CDF Collaboration], arXiv:0707.2294 [hep-ex]).

Can LHC measure the curvature of the 5D space-time?

- $\sigma(pp \to G) BR(G \to e^+ e^-) \propto f_{pp}(m_G)/\Lambda_W^2 \Longrightarrow m_G$ and Λ_W determination at LHC

B. C. Allanach, *et al.*JHEP **0212**, 039 (2002) [arXiv:hep-ph/0211205]:

$$\Downarrow$$

$$\Delta m_G = 10.5 \text{ GeV (for } m_G = 1.5 \text{ TeV)}, \quad \frac{\Delta \Lambda_W}{\Lambda_W} = 1 \div 17\% \text{ (for } \Lambda_W = 1 \div 39 \text{ TeV)}$$

- $$m_n = x_n \frac{m_0}{m_{Pl}} \frac{\Lambda_W}{\sqrt{2}}$$

$$\Downarrow$$

If n is known then the curvature m_0/m_{Pl} can be determined at the LHC

Summary

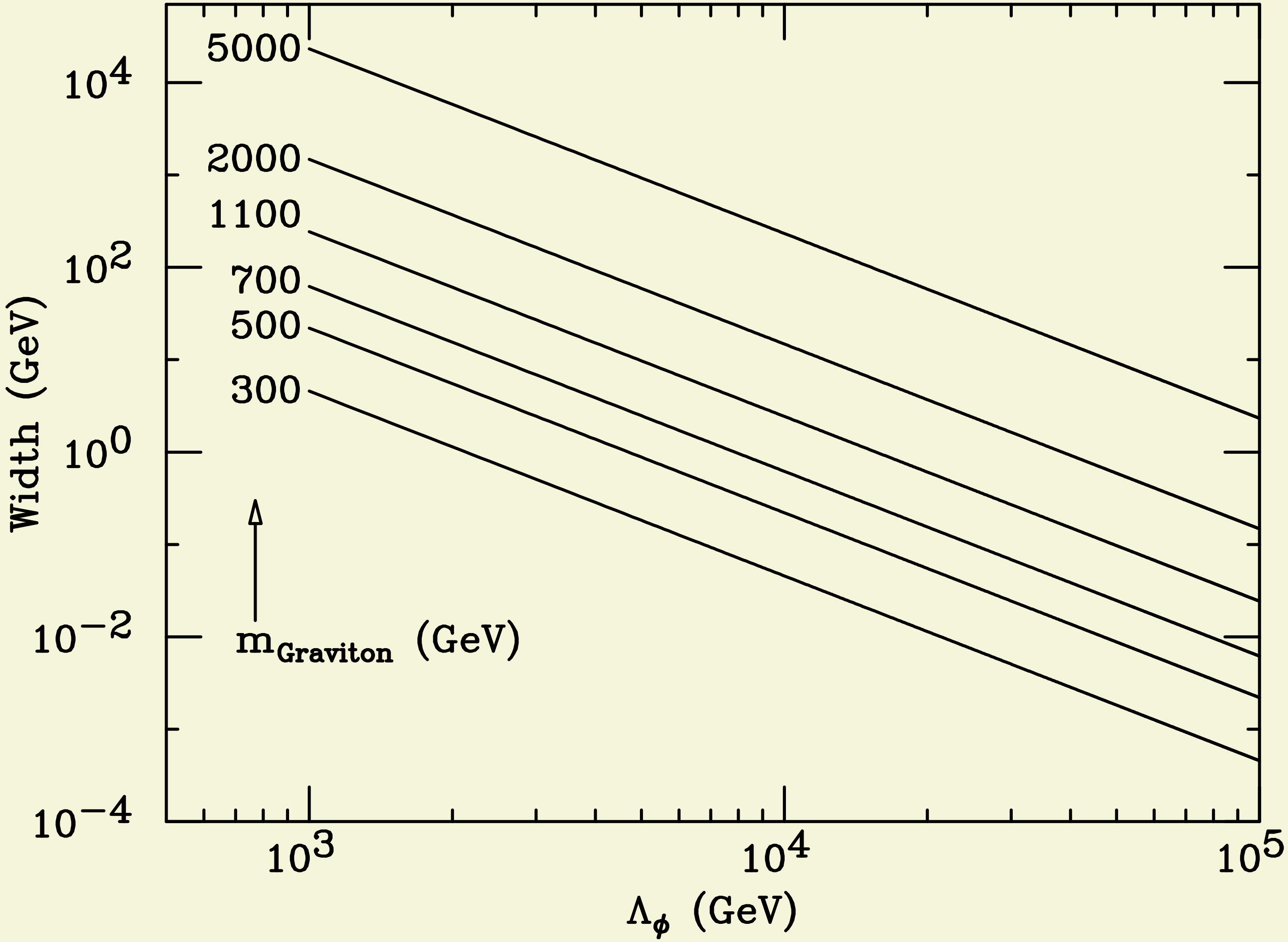
- The graviton and/or radion exchange lead to divergent partial wave amplitudes for $V_L V_L \to V_L V_L$, $a_J \propto \frac{s}{\Lambda_W^2}$, and therefore can substantially modify their high-energy behavior.

- The tree-level unitarity requirement can be adopted to determine the cutoff in the Randall-Sundrum model.

- The results obtained here for the graviton exchange are applicable to models which have massive gravitons which couples as $\frac{1}{\Lambda_W} \Sigma_{n\neq 0} h^n_{\mu\nu} T^{\mu\nu}$.

- In the curvature-Higgs mixing scenario, $\xi \sqrt{g_{\rm vis}} R(g_{\rm vis})\, H^\dagger H$, the presence of the radion-Higgs mixing can substantially spoil the cancellation of $a_{1,0} \propto s$ by the Higgs-boson exchange. Therefore the requirement of proper high-energy behavior severely constraints the allowed region for the mixing parameter ξ.

- When 5 TeV $\leq \Lambda_\phi \leq$ 40 TeV, then the requirement of proper unitarity behavior allows to determine an absolute maximum Higgs-boson mass: $m_h \lesssim 1.43$ TeV.



The KK graviton width as a function of Λ_ϕ for various values of the graviton mass. This plot applies independently of the level n of the excitation.

diagram	$\mathcal{O}(\frac{s^2}{v^4})$	$\mathcal{O}(\frac{s^1}{v^2})$
γ, Z s-channel	$-\frac{s^2}{g^2 v^4} 4\cos\theta$	$-\frac{s}{v^2} \cos\theta$
γ, Z t-channel	$-\frac{s^2}{g^2 v^4}(-3 + 2\cos\theta + \cos^2\theta)$	$-\frac{s}{v^2}\frac{3}{2}(1 - 5\cos\theta)$
$WWWW$ contact	$-\frac{s^2}{g^2 v^4}(3 - 6\cos\theta - \cos^2\theta)$	$-\frac{s}{v^2} 2(-1 + 3\cos\theta)$
G s-channel	0	$-\frac{s}{24\hat{\Lambda}_W^2}(-1 + 3\cos^2\theta)$
G t-channel	0	$-\frac{s}{24\hat{\Lambda}_W^2}\frac{13+10\cos\theta+\cos^2\theta}{-1+\cos\theta}$
$(h - \phi)$ s-channel	0	$-\frac{s}{v^2} R^2$
$(h - \phi)$ t-channel	0	$-\frac{s}{v^2}\frac{-1+\cos\theta}{2} R^2$

The leading contributions to the $W_L^+ W_L^- \to W_L^+ W_L^-$ amplitude. $R^2 \equiv g_{vvh}^2 + g_{vv\phi}^2 = 1 + \gamma^2$ for $\gamma \equiv \frac{v}{\Lambda_\phi}$

$$
\begin{aligned}
a_2 &= -\frac{1}{960\pi\hat{\Lambda}_W^2}\left\{\left[91+30\log\left(\frac{m_G^2}{s}\right)\right]s+\left[241+210\log\left(\frac{m_G^2}{s}\right)\right]m_G^2+32g^2v^2\right\}+\mathcal{O}(s^{-1}) \\
a_1 &= -\frac{1}{1152\pi\hat{\Lambda}_W^2}\left\{\left[73+36\log\left(\frac{m_G^2}{s}\right)\right]s+36\left[1+3\log\left(\frac{m_G^2}{s}\right)\right]m_G^2+37g^2v^2\right\}+ \\
&\quad +\frac{1}{96\pi}\left[\frac{s}{v^2}(1-R^2)-R^2g^2+\frac{12\cos^2\theta_W-1}{2\cos^2\theta_W}g^2\right]+\mathcal{O}(s^{-1}) \\
a_0 &= -\frac{1}{384\pi\hat{\Lambda}_W^2}\left\{\left[11+12\log\left(\frac{m_G^2}{s}\right)\right]s-\left[10-12\log\left(\frac{m_G^2}{s}\right)\right]m_G^2+19g^2v^2\right\}+ \\
&\quad +\frac{1}{32\pi}\left[\frac{s}{v^2}(1-R^2)+R^2g^2-4\frac{\overline{m}_{\rm scal}^2}{v^2}\right]+\mathcal{O}(s^{-1})
\end{aligned}
$$

where $\overline{m}_{\rm scal}^2 = g_{vvh}^2 m_h^2 + g_{vv\phi}^2 m_\phi^2$ and $R^2 \equiv g_{vvh}^2 + g_{vv\phi}^2$ satisfies the following sum rule

$$
R^2 = 1+\gamma^2 \ \text{ for } \gamma \equiv \frac{v}{\Lambda_\phi}
$$

$$
a_0 = \frac{1}{32\pi}\left[-\frac{s}{\Lambda_\phi^2}+g^2R^2-4\frac{\overline{m}_{\rm scal}^2}{v^2}+\ \text{graviton contributions}\ \right],
$$