Naïve approach to the little hierarchy problem and its consequences

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- The little hierarchy problem
- Dark Matter
- Neutrino physics
- Summary

B.G., J. Wudka, arXiv:0902.0628 (to appear in PRL), work in progress

The little hierarchy problem

- $$m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 600 \text{ GeV}$$

- For $\Lambda \gtrsim 600$ GeV there must be a cancellation between the tree-level Higgs mass^2 $m_h^{(B)\,2}$ and the 1-loop leading correction $\delta^{(SM)} m_h^2$:

$$m_h^{(B)\,2} \sim \delta^{(SM)} m_h^2 > m_h^2$$
$$\Downarrow$$

the perturbative expansion is breaking down.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

♠ Suppression of corrections growing with Λ^2 at the 1-loop level:

- The Veltman condition, no Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

- SUSY:

$$\delta^{(SUSY)}m_h^2 \sim m_{\tilde{t}}^2 \, \frac{3g_t^2}{8\pi^2} \ln\left(\frac{\Lambda^2}{m_{\tilde{t}}^2}\right)$$

then for $\Lambda \sim 10^{16-18}$ GeV one gets $m_{\tilde{t}}^2 \lesssim 1$ TeV^2 in order to have $\delta^{(SUSY)}m_h^2 \sim m_h^2$.

♠ Increase of the allowed value of m_h:

- The inert Higgs model by Barbieri, Hall, Rychkov, arXiv:hep-ph/0603188, (see also Ma) $\Rightarrow$ $m_h \sim 400 - 600$ GeV, (m_h^2 terms in T parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).

- The little hierarchy

$$\delta^{(SM)} m_h^2 \lesssim m_h^2 \qquad \Longrightarrow \qquad \Lambda \lesssim 600 \text{ GeV}$$

- "The "LEP paradox"", Barbieri & Strumia, hep-ph/0007265

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \sum_i c_i \mathcal{O}_i$$

EWPT (LEP)

$$\Downarrow$$

$$\Lambda \gtrsim 5 \text{ TeV}$$

$$\Downarrow$$

There is no physics beyond the SM without some fine tunning

Motivation: to lift the cutoff to few TeV range in the most economic way

- N_φ extra gauge singlets φ_i with $\langle\varphi_i\rangle = 0$ (no $H \leftrightarrow \varphi_i$ mixing from $\varphi_i^2|H|^2$).

- Symmetries $\mathbb{Z}_2^{(i)}$: $\varphi_i \to -\varphi_i$ (to eliminate $|H|^2\varphi_i$ couplings).

- Gauge singlet neutrinos: $\nu_{R\,j}$ for $j = 1, 2, 3$.

$$V(H,\varphi_i) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\varphi^2\sum_{i=1}^{N_\varphi}\varphi_i^2 + \frac{\lambda_\varphi}{24}\sum_{i=1}^{N_\varphi}\varphi_i^4 + \lambda_x|H|^2\sum_{i=1}^{N_\varphi}\varphi_i^2$$

with $O(N_\varphi)$ symmetry imposed

$$\langle H\rangle = \frac{v}{\sqrt{2}}, \qquad \langle\varphi_i\rangle = 0 \qquad \text{for} \qquad \mu_\varphi^2 > 0$$

then

$$m_h^2 = 2\mu_H^2 \qquad \text{and} \qquad m^2 = 2\mu_\varphi^2 + \lambda_x v^2$$

- Positivity (stability): $\lambda_H, \lambda_\varphi, \lambda_x > 0$

- Unitarity in the limit $s \gg m_h^2, m^2$: $\lambda_H \leq \frac{4\pi}{3}$ (the SM requirement) and $\lambda_\varphi \leq 8\pi$, $\lambda_x < 4\pi$

$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \ln\left(1+\frac{\Lambda^2}{m^2}\right)\right]$$
$$|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2| = D_t m_h^2$$
$$\Downarrow$$
$$\lambda_x = \lambda_x(m, m_h, D_t, \Lambda, N_\varphi)$$

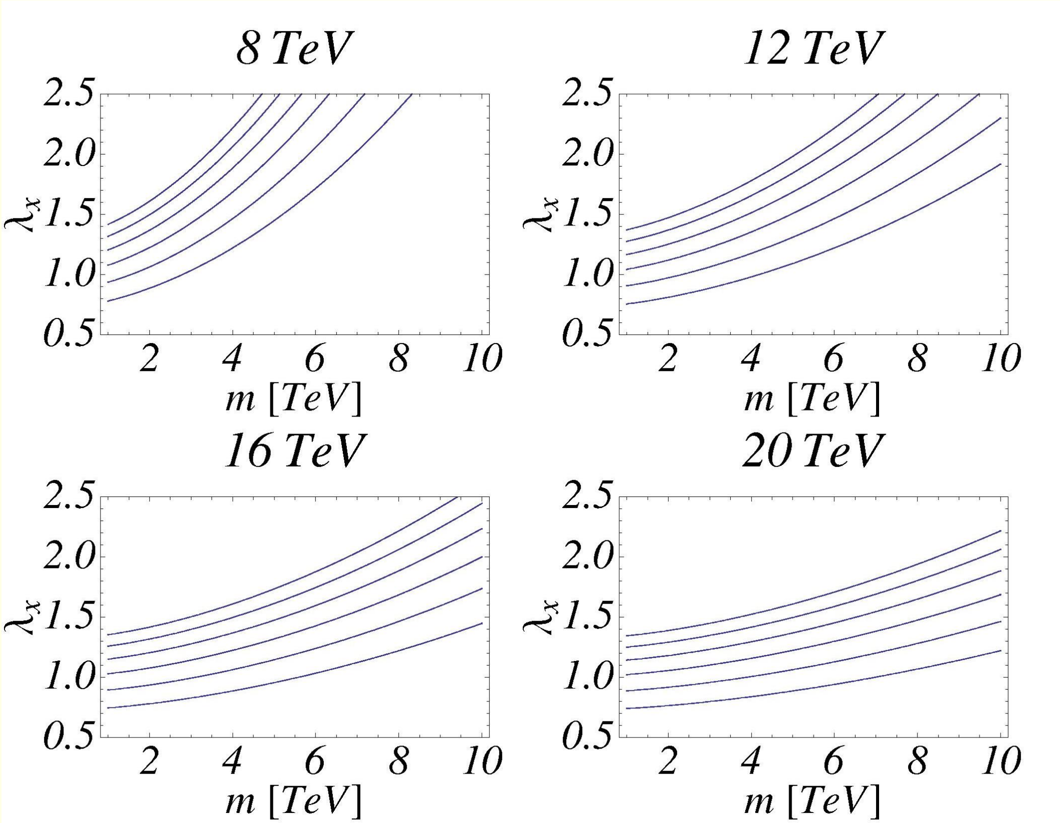

Figure 1: Plots of λ_x as a function of m for $N_\varphi = 3$, $D_t = 0$ and various choices of $\Lambda = 8, 12, 16$ and 20 TeV shown above each panel. The curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve).

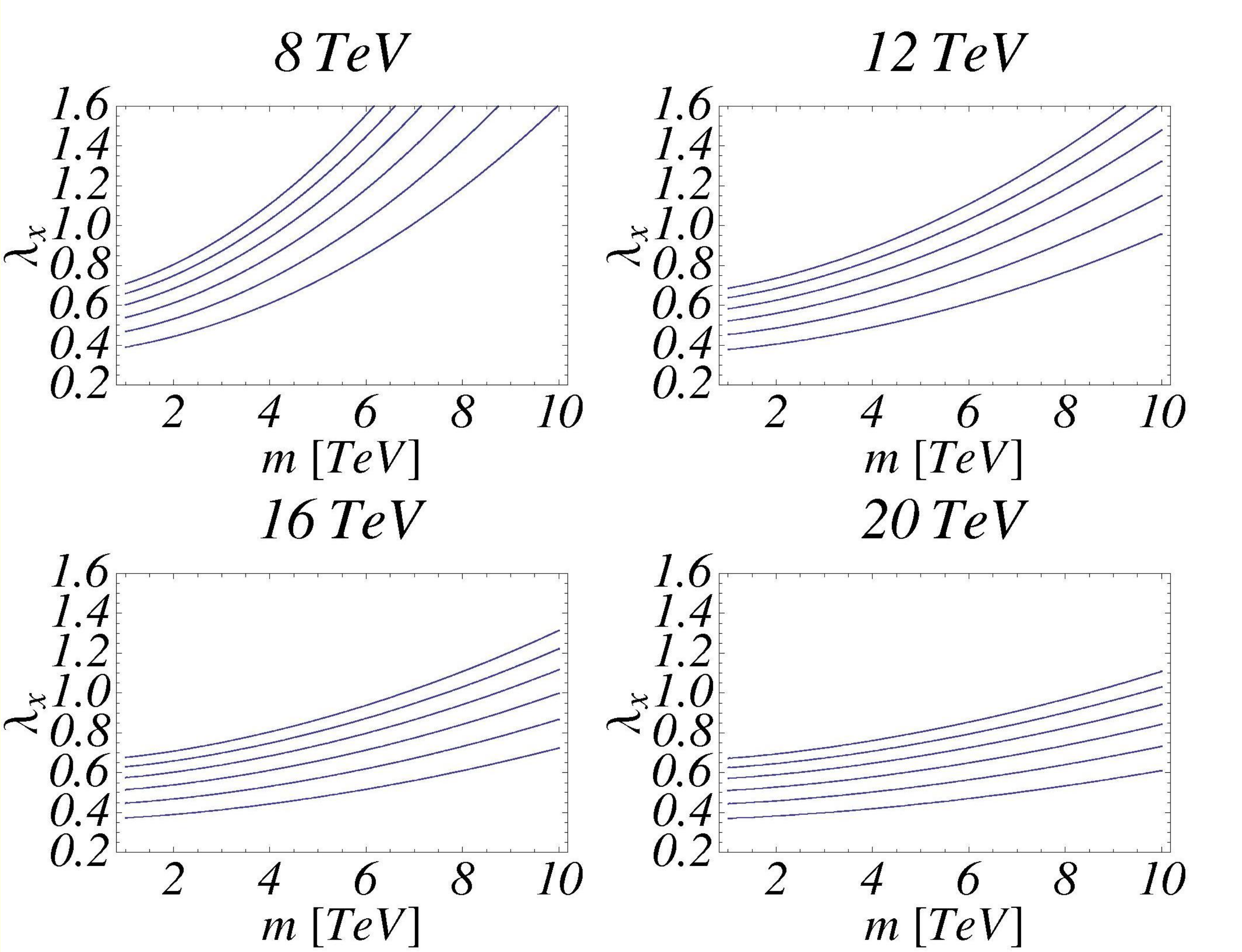

Figure 2: Plots of λ_x as a function of m for $N_\varphi = 6$, $D_t = 0$ and various choices of $\Lambda = 8, 12, 16$ and 20 TeV shown above each panel. The curves correspond to $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve).

Stability of the fine tunning

$$\begin{aligned}\delta^{(SM)} m_h^2 &= \frac{\Lambda^2}{16\pi^2}\left(12 g_t^2 - \frac{9}{2} g_2^2 - \frac{3}{2} g_1^2 - 12\lambda_H\right) \\ \delta^{(\varphi)} m_h^2 &= -N_\varphi \frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \ln\left(1 + \frac{\Lambda^2}{m^2}\right)\right]\end{aligned}$$

In general

$$\delta m_h^2 = \underbrace{\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2}_{\simeq 0} + 2\Lambda^2 \sum_{n=1}^{\infty} f_n(\lambda, \dots) \ln^n\left(\frac{\Lambda}{\mu}\right)$$

where (see I. Jack and D. R. T. Jones, Phys. Lett. **B**234, 321 (1990))

$$(n+1) f_{n+1} = \mu \frac{\partial}{\partial \mu} f_n = \beta_i \frac{\partial}{\partial \lambda_i} f_n \qquad \text{and} \qquad f_n \propto \left(\frac{N_\varphi \lambda_x}{16\pi^2}\right)^{n+1}$$

From 1-loop condition $(n=0)$

$$\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 \simeq 0$$

we have

$$\lambda_x = \frac{1}{N_\varphi}\left\{4.8 - 3\left(\frac{m_h}{v}\right)^2 + 2D_t\left[\frac{2\pi}{\Lambda/\text{ TeV}}\right]^2\right\}\left[1 - \frac{m^2}{\Lambda^2}\ln\left(\frac{m^2}{\Lambda^2}\right)\right] + \mathcal{O}\left(\frac{m^4}{\Lambda^4}\right).$$

Therefore at the 2-loop $(n=1)$

$$D_t \equiv \frac{\delta m_h^2}{m_h^2} \simeq \left(\frac{N_\varphi \lambda_x}{16\pi^2}\right)^2 \frac{\Lambda^2}{m_h^2}\ln\left(\frac{\Lambda}{m_h}\right) \simeq \left(\frac{4}{16\pi^2}\right)^2 \frac{\Lambda^2}{m_h^2}\ln\left(\frac{\Lambda}{m_h}\right)$$

for $D_t \lesssim 1$

$$\Lambda \lesssim 3-5 \text{ TeV} \qquad \text{for} \qquad m_h = 130-230 \text{ GeV}$$

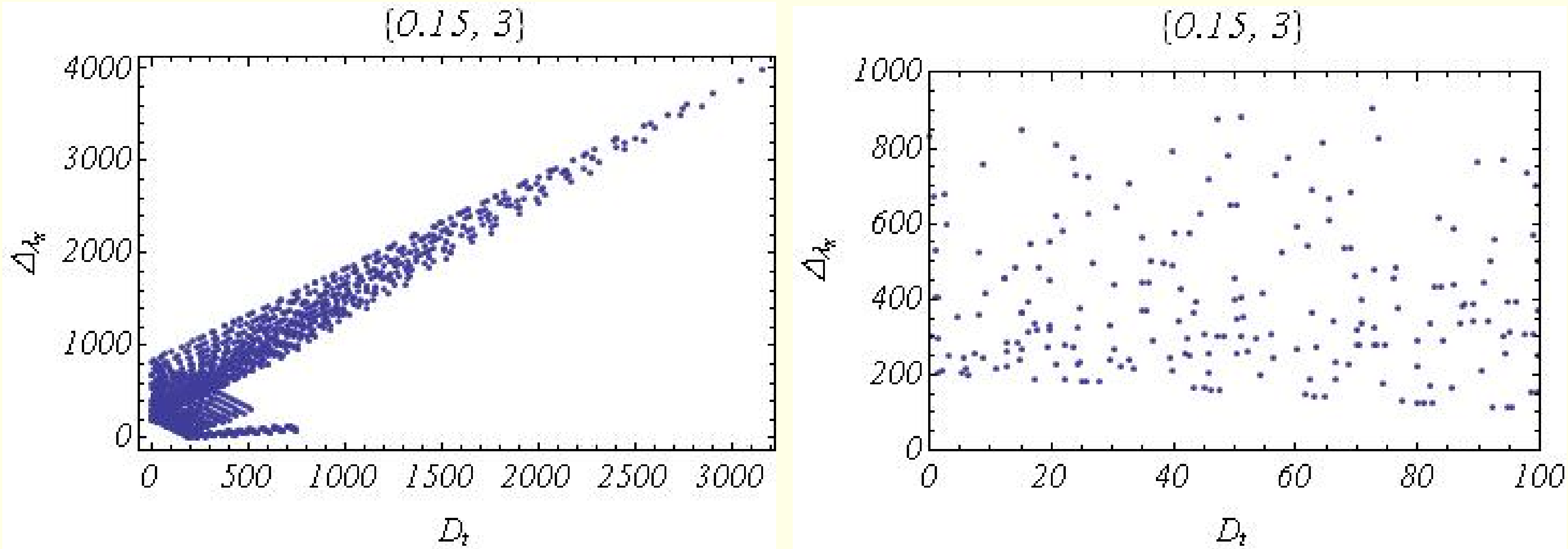

Figure 3: Contour plots of the Barbieri-Giudice parameters Δ_{λ_x} plotted against corresponding value of $D_t \equiv \delta m_h^2/m_h^2$ for $m_h = 150$ GeV, $N_\varphi = 3$ and $0.2 \leq \lambda_x \leq 6$, 1 TeV $\leq m \leq 10$ TeV and 10 TeV $\leq \Lambda \leq 20$ TeV.

$$\Delta_{\lambda_x} \equiv \frac{\lambda_x}{m_h^2}\frac{\partial m_h^2}{\partial \lambda_x} = \frac{|\delta^{(\varphi)} m_h^2|}{m_h^2}$$

$$\frac{\delta m_h^2}{m_h^2} = \Delta_{\lambda_x}\frac{\delta\lambda_x}{\lambda_x}$$

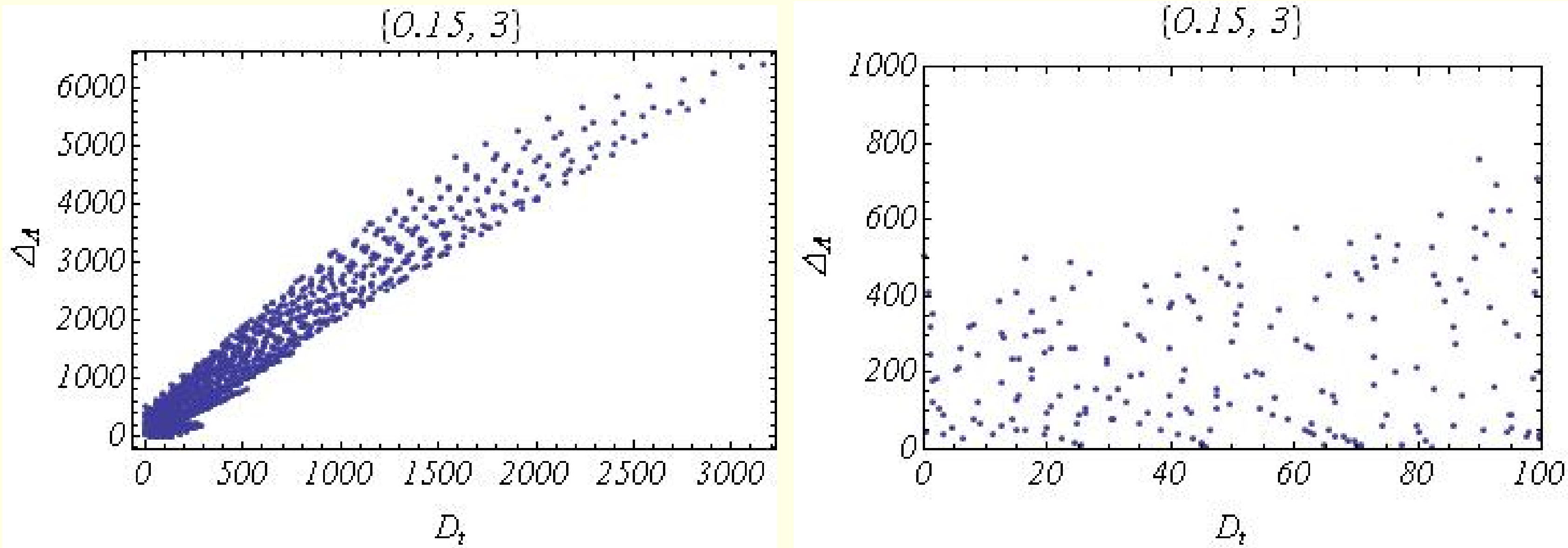

Figure 4: Contour plots of the Barbieri-Giudice parameters Δ_Λ plotted against corresponding value of $D_t \equiv \delta m_h^2/m_h^2$ for $m_h = 150$ GeV, $N_\varphi = 3$ and $0.2 \leq \lambda_x \leq 6$, 1 TeV $\leq m \leq 10$ TeV and 10 TeV $\leq \Lambda \leq 20$ TeV.

$$\Delta_\Lambda \equiv \frac{\Lambda}{m_h^2}\frac{\partial m_h^2}{\partial \Lambda} = \left| 2\frac{\delta^{(SM)} m_h^2}{m_h^2} - \frac{\Lambda^2}{m_h^2}\frac{N_\varphi \lambda_x}{4\pi^2}\frac{\Lambda^2}{m^2+\Lambda^2} \right|$$

$$\frac{\delta m_h^2}{m_h^2} = \Delta_\Lambda \frac{\delta \Lambda}{\Lambda}$$

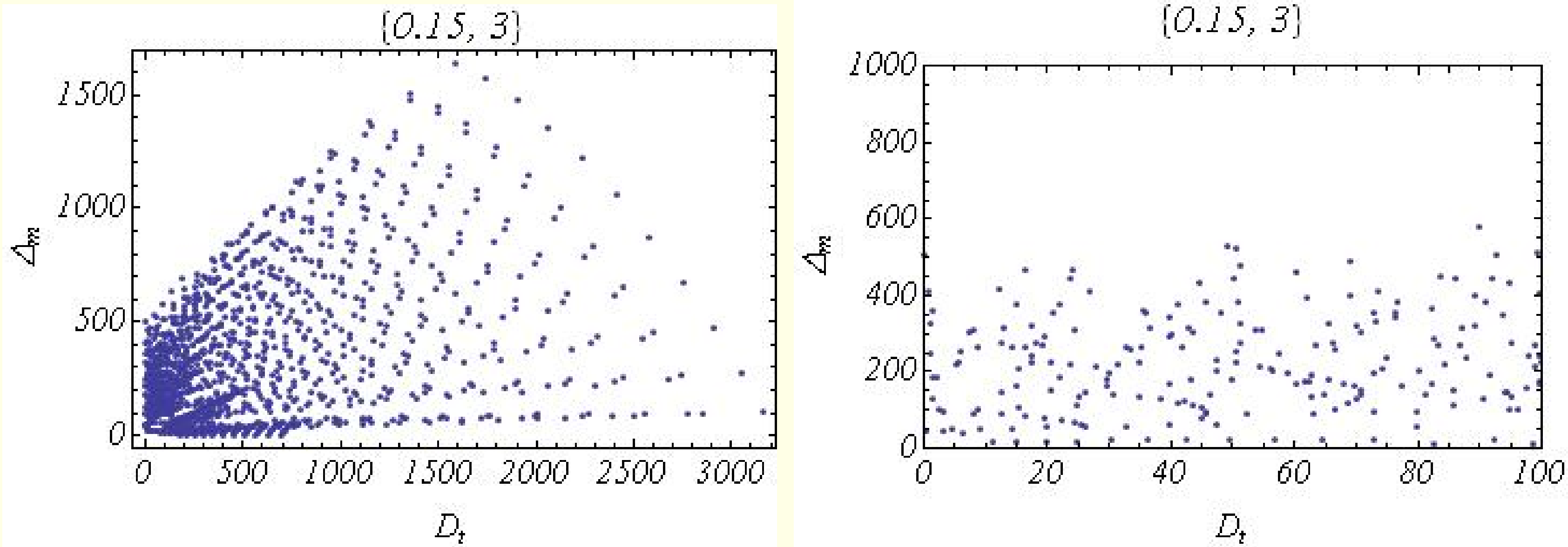

Figure 5: Contour plots of the Barbieri-Giudice parameters Δ_m plotted against corresponding value of $D_t \equiv \delta m_h^2/m_h^2$ for $m_h = 150$ GeV, $N_\varphi = 3$ and $0.2 \leq \lambda_x \leq 6$, 1 TeV $\leq m \leq 10$ TeV and 10 TeV $\leq \Lambda \leq 20$ TeV.

$$\Delta_m \equiv \frac{m}{m_h^2}\frac{\partial m_h^2}{\partial m} = \left|\frac{m^2}{m_h^2}\frac{N_\varphi \lambda_x}{4\pi^2}\left[\ln\left(1+\frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{m^2+\Lambda^2}\right]\right|$$

$$\frac{\delta m_h^2}{m_h^2} = \Delta_m \frac{\delta m}{m}$$

Dark Matter

V. Silveira and A. Zee, (1985), J. McDonald, (1994), C. P. Burgess, M. Pospelov and T. ter Veldhuis, (2001), H. Davoudiasl, R. Kitano, T. Li and H. Murayama, (2005), J. J. van der Bij, (2006), S. Andreas, T. Hambye and M. H. G. Tytgat, (2008)

It is possible to find parameters Λ, λ_x and m such that
both the hierarchy is ameliorated to the prescribed level and
such that $\sum_i \Omega_{\varphi_i} h^2$ is consistent with $\Omega_{DM} h^2$

$$\varphi_i\varphi_i \to hh, W^+W^-, ZZ, l\bar{l}, q\bar{q}, gg, \gamma\gamma \quad \Rightarrow \quad \langle\sigma_i v\rangle = \langle\sigma_i v\rangle(\lambda_x, m)$$

$$\langle\sigma_i v\rangle \simeq \frac{\lambda_x^2}{8\pi m^2} + \frac{\lambda_x^2 v^2 \Gamma_h(2m)}{8m^5} \simeq \frac{1.73}{8\pi}\frac{\lambda_x^2}{m^2}$$

The Boltzmann equation $\quad \Rightarrow \quad x_f \left(\equiv \frac{m}{T_f}\right) \simeq \ln\left[0.038\frac{m_{Pl} m \langle\sigma v\rangle}{g_\star^{1/2} x_f^{1/2}}\right]$

$$\Omega_{\varphi_i} h^2 \simeq 1.06 \cdot 10^9 \frac{x_f}{g_\star^{1/2} m_{Pl} \langle\sigma v\rangle \text{ GeV}}$$

$$|\delta m_h^2| = D_t m_h^2 \qquad \text{and} \qquad \sum_{i=1}^{N_\varphi} \Omega_{\varphi_i} h^2 = \Omega_{DM} h^2 = 0.106 \pm 0.008 \quad \Rightarrow \quad m = m(\Lambda)$$

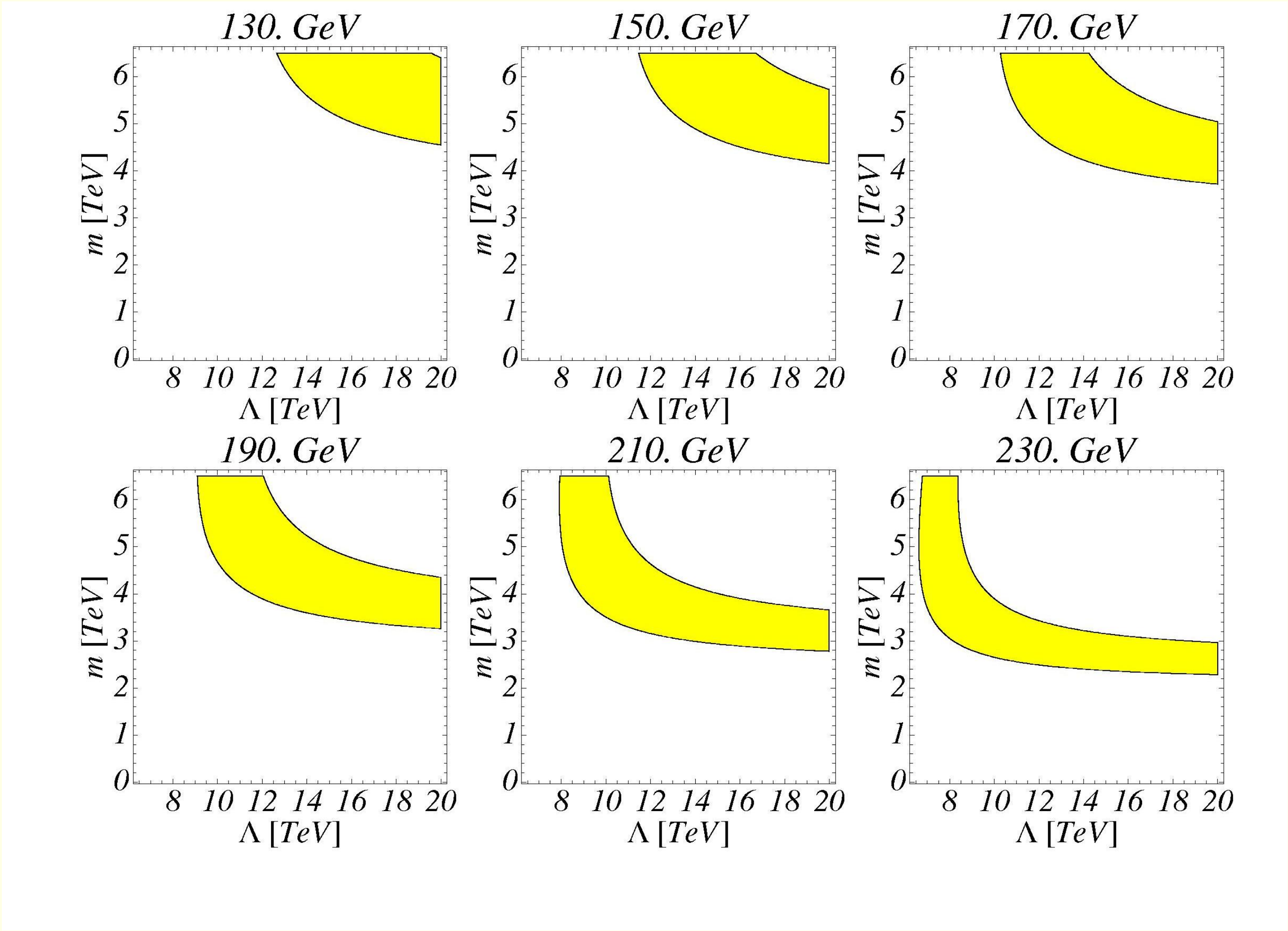

Figure 6: Allowed regions in the space (m, Λ) are plotted for $D_t(m) = 0$, $N_\varphi = 3$ and assuming that each φ_i contributes the same to the total Ω_{DM} at the 3σ level: $\Omega_\varphi h^2 = 0.106 \pm 0.008$ for the Higgs mass shown above each panel; $m_h = 130, 150, 170, 190, 210, 230$ GeV.

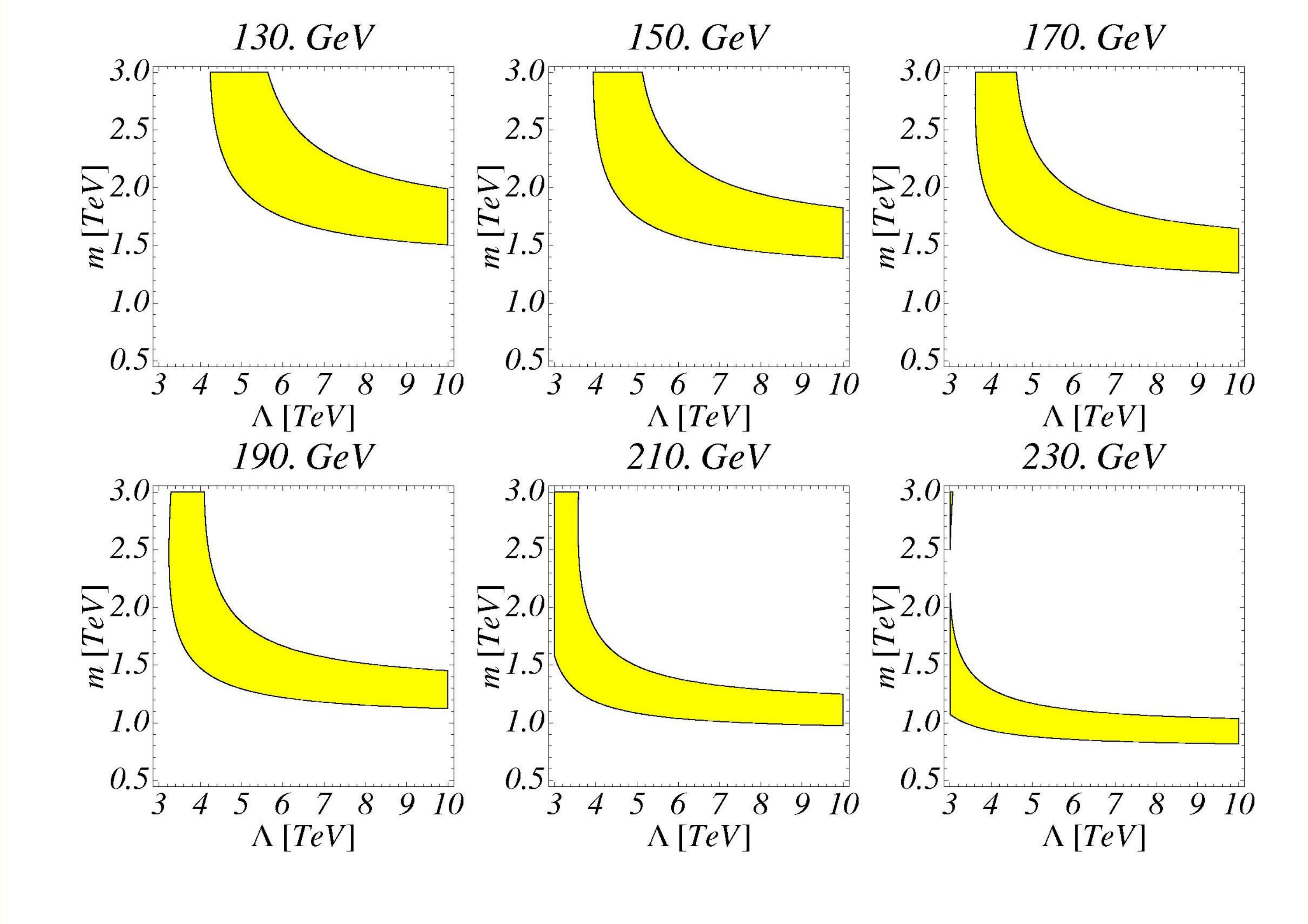

Figure 7: Allowed regions in the space (m, Λ) are plotted for $D_t(m) = 0$, $N_\varphi = 6$ and assuming that each φ_i contributes the same to the total Ω_{DM} at the 3σ level: $\Omega_\varphi h^2 = 0.106 \pm 0.008$ for the Higgs mass shown above each panel; $m_h = 130, 150, 170, 190, 210, 230$ GeV.

Neutrino physics

$$\mathcal{L}_Y = -\bar{L} Y_l H l_R - \bar{L} Y_\nu \tilde{H} \nu_R - \frac{1}{2}\overline{(\nu_R)^c} M \nu_R - \varphi_i \overline{(\nu_R)^c} Y_{\varphi_i} \nu_R + \text{H.c.}$$

$$\mathbb{Z}_2^{(i)}: \qquad H \to H,\ \varphi_i \to -\varphi_i,\ L \to S_L L,\ l_R \to S_{l_R} l_R,\ \nu_R \to S_{\nu_R} \nu_R$$

The symmetry conditions ($S_i S_i^\dagger = S_i^\dagger S_i = \mathbb{1}$):

$$S_L^\dagger Y_l S_{l_R} = Y_l, \quad S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu, \quad S_{\nu_R}^T M S_{\nu_R} = +M, \quad S_{\nu_R}^T Y_\varphi S_{\nu_R} = -Y_\varphi$$

The implications of the symmetry (in the basis in which M is diagonal):

$$S_{\nu_R}^T M S_{\nu_R} = +M \quad \Rightarrow \quad S_{\nu_R} = \pm \mathbb{1}, \qquad S_{\nu_R} = \pm\, \text{diag}(1, 1, -1)$$

$$S_{\nu_R} = \pm\mathbb{1} \quad \Rightarrow \quad Y_\varphi = 0 \ \text{ or } \ S_{\nu_R} = \pm\,\text{diag}(1,1,-1) \quad \Rightarrow \quad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

Basis choice: Y_l real and diagonal.

$$S_L^\dagger Y_l S_{l_R} = Y_l \quad \Rightarrow \quad S_L = S_{l_R} = \text{diag}(s_1, s_2, s_3), \quad |s_i| = 1$$

$S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu \quad \Rightarrow$ 10 Dirac neutrino mass textures

For instance the solution corresponding to $s_{1,2,3} = \pm 1$:

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}$$

$$\mathcal{L}_m = -(\bar{n} M_n n + \bar{N} M_N N)$$

with the see-saw mechanism explaining $M_n \ll M_N$:

$$M_N \sim M \qquad \text{and} \qquad M_n \sim (vY_\nu)\frac{1}{M}(vY_\nu)^T$$

where

$$\nu_L = n_L + M_D \frac{1}{M} N_L \qquad \text{and} \qquad \nu_R = N_R - \frac{1}{M} M_D^T n_R$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix} \quad \Rightarrow \quad M_n$$

To compare our results with the data, we use the following approximate lepton mixing matrix (tri-bimaximal lepton mixing) that corresponds to $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \arcsin(1/\sqrt{3})$:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

Writing the diagonal light neutrino mass matrix as

$$m_{\text{light}} = \mathsf{diag}(m_1, m_2, m_3)$$

we find

$$M_n = U m_{\text{light}} U^T = \frac{1}{3}\begin{pmatrix} 2m_1 + m_2 & -m_1 + m_2 & -m_1 + m_2 \\ -m_1 + m_2 & \frac{1}{2}(m_1 + 2m_2 + 3m_3) & \frac{1}{2}(m_1 + 2m_2 - 3m_3) \\ -m_1 + m_2 & \frac{1}{2}(m_1 + 2m_2 - 3m_3) & \frac{1}{2}(m_1 + 2m_2 + 3m_3) \end{pmatrix}$$

In our case

$$M_n = (vY_\nu)\frac{1}{M}(vY_\nu)^T$$

$$\Downarrow$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ -\frac{a}{2} & b & 0 \\ -\frac{a}{2} & b & 0 \end{pmatrix} \quad \begin{array}{l} m_1 = -3a^2\frac{v^2}{M_1} \\ m_2 = -6b^2\frac{v^2}{M_2} \\ m_3 = 0 \end{array} \quad \text{and } Y_\nu = \begin{pmatrix} a & b & 0 \\ a & -\frac{b}{2} & 0 \\ a & -\frac{b}{2} & 0 \end{pmatrix} \quad \begin{array}{l} m_1 = -3b^2\frac{v^2}{M_2} \\ m_2 = -6a^2\frac{v^2}{M_1} \\ m_3 = 0 \end{array}$$

Does $Y_\varphi \neq 0$ imply $\varphi \to n_i n_j$ decays?

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h' & 0 \end{pmatrix}, \; Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \;\Rightarrow\; \varphi \to N^\star_{1,2} N_3 \to \underbrace{n_{1,2,3}\, h}_{N^\star_{1,2}} \; N_3$$

that can be kinematically forbidden by requiring $M_3 > m$.

Summary

- The addition of N_φ real scalar singlets φ_i to the SM may ameliorate the little hierarchy problem (by lifting the cutoff Λ to $\sim 4-9$ TeV range). Some fine tuning remains.

- It also provides a realistic candidate for DM if $m_\varphi \sim 1-3$ TeV (depending on N_φ).

- For appropriate choices of $\mathbb{Z}_2$ charges, the $\mathbb{Z}_2$ symmetry implies one massless neutrino and light-neutrino mass matrix consistent with the tri-bimaximal lepton mixing.