Multi-Higgs-doublet physics

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Difficulties of the SM

- Lack of the DM candidate within the SM
  - Strong experimental evidence for DM:
    - Galaxy rotation curves
    - Gravitational lensing
    - Cosmic microwave background
    - Structure formation
  - Modified Newtonian Dynamics (MOND/TeVeS) not an attractive possibility, it is not sufficient to explain data (DM is still needed)
- Unexplained baryon asymmetry
  The Sakhrov conditions:
  - B-violation
  - C- and CP-violation
  - Thermal inequilibrium

- The strong CP problem
  - symmetries of the SM allow for

$$\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) \xrightarrow{P} -\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

  - odd under CP

$$\mathcal{L}_\theta = \theta\frac{g_s^2}{32\pi^2}F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \quad \Rightarrow \quad \text{neutron-EDM} \qquad D_n \simeq 2.7\cdot 10^{-16}\theta \text{ e cm}$$

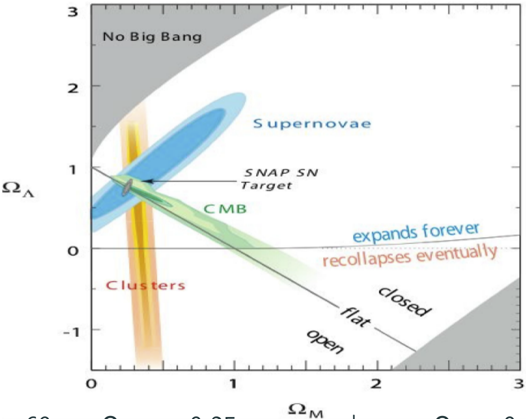
$$\Downarrow$$

$$D_n \lesssim 1.1\cdot 10^{-25} \text{ e cm} \qquad \Rightarrow \qquad \theta \lesssim 3\cdot 10^{-10}$$

The strong CP problem: why is θ so small?

- Dark energy

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$
$$\rho_c = \frac{3H_0^2}{8\pi G_N}$$



$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} \simeq .69, \qquad \Omega_{DM} \simeq 0.25 \qquad \text{and} \qquad \Omega_B \simeq 0.05$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \simeq 0.7 \qquad \Rightarrow \qquad \rho_\Lambda \simeq 10^{-46}\ \text{GeV}^4$$

The gravitational prediction: $\rho_\Lambda \sim M_{\rm Pl}^4 \sim 10^{76}\ \text{GeV}^4$

- Parameters of the SM

$$\begin{matrix} m_e & m_\mu & m_\tau & m_u & m_c & m_t \\ m_{\nu_e} & m_{\nu_\mu} & m_{\nu_\tau} & m_d & m_s & m_b \end{matrix}$$

$$\underbrace{g \quad , \quad g'}_{(\alpha_{QED}, \sin\theta_W)} , \underbrace{g_s}_{(\alpha_{QCD})} , \underbrace{m_h, \ \lambda}_{(\mu,\lambda)} , \underbrace{U_{CKM}}_{\theta_{1,2,3},\delta_{CP}}$$

21 parameters !

- Why only one Higgs boson?
    - The Higgs field was introduced just to make the model renormalizable (unitary)
    - There exist many fermions and vector bosons, so why only one scalar? Why, for instance, not a dedicated scalar for each fermion?

The goal

The model with DM candidate and explained baryon asymmetry.

Interpretation of the LHC Higgs data

SM as an effective field theory:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c}{\Lambda_{UV}}\mathcal{O}_5 + \sum_i \frac{c_i}{\Lambda_{UV}^2}\mathcal{O}_i$$

The 125-GeV Higgs boson is SM-like

•

$$H_{125} \simeq H_{SM}$$

$$\Downarrow$$

$$\Lambda_{UV} \gg v = 246 \text{ GeV}$$

No new physics in the TeV energy/mass range!

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- n-Higgs doublet model (nHDM) in the alignment limit: $\Lambda_{UV} \sim 300$ GeV

Fundamental (renormalizable) extensions of the SM

♠ Extra gauge symmetries

- GUTs, e.g. $SU(5)$: unification of gauge couplings, ...
- $L - R$ symmetry, $SU(2)_L \times SU(2)_R \times U(1)$: spontaneous parity violation
- $SU(2)_L \times U(1) \times U(1)'$: just extra Z'

♠ Extra fermions

- vector-like quarks

Extra Higgs bosons

- SM-like Higgs-boson discovery by ATLAS and CMS at the LHC announced on 4 July 2012

$$m_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

- SM single Higgs doublet is rather unnatural, why only one?
  - Higgs-boson representation:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \qquad \text{SM} \qquad \Rightarrow \qquad \rho = 1 + \mathcal{O}(\alpha)$$

for general Higgs multiplets:

$$\rho = \frac{\sum_i \left[T_i(T_i+1) - T_{i\,3}^2\right] v_i^2}{\sum_i 2T_{i\,3}^2 v_i^2}$$

data: $\rho = 1.0002 \begin{cases} +0.0024 \\ -0.0009 \end{cases} \qquad \Rightarrow \qquad T = \frac{1}{2}$

Doublets (nHDM) and

♠ Extra Higgs bosons

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Doublets (nHDM) and extra singlets (real or complex) are favored.

- Scalar $SU(2)$ singlets:
  - real (Z_2-symmetric) $\Rightarrow$ DM
  - complex (softly broken $U(1)$) $\Rightarrow$ extra Higgs boson $\bigoplus$ pseudo-Goldstone DM with suppressed DM-nucleon coupling
- Scalar $SU(2)$ doublets: extra sources of CPV from the scalar potential and from Yukawas (for baryogenesis)

n-singlet models

Real singlet scalar S (DM) $\bigoplus$ SM

$$V = -\mu_\phi^2|\phi|^2 + \lambda_\phi|\phi|^4 - \mu_S^2 S^2 + \lambda_S S^4 + \kappa S^2|\phi|^2$$

- ϕ is the SM Higgs doublet
- Z_2 symmetry $S \to -S$, S is DM candidate

B.G. and J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics," Phys. Rev. Lett. **103**, 091802 (2009)

A. Drozd, B.G. and J. Wudka, "Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson," JHEP **04**, 006 (2012)

Complex singlet (extra Higgs and DM) $\bigoplus$ SM

$$S \xrightarrow{\text{U(1)}} e^{i\alpha} S$$

$$V = -\mu_\phi^2|\phi|^2 + \lambda_\phi|\phi|^4 - \mu_S^2|S|^2 + \lambda_S|S|^4 + \kappa|S|^2|\phi|^2 + \mu^2(S^2 + S^{*\,2})$$

$$S = \frac{1}{\sqrt{2}}(v_S + \phi_S + iA)$$

Symmetries:

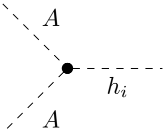
- weak basis choice: $\langle S \rangle = \frac{v_S}{\sqrt{2}}$,
- dark charge conjugation C: $S \to S^\star$ $\Rightarrow$ stability of $\mathrm{Im}\, S$ and $\mathrm{Im}\,\mu = 0$,
- $U(1)$ softly broken by $\mu^2(S^2 + S^{*\,2})$ $\Rightarrow$ pseudo-Goldstone boson,
- $U(1)$ softly broken by $\mu^2(S^2 + S^{*\,2})$ $\Rightarrow$ residual symmetry: $S \xrightarrow{Z_2} -S$.

D. Azevedo, M. Duch, B.G., D. Huang, M. Iglicki and R. Santos, “Testing scalar versus vector dark matter,” Phys. Rev. D **99**, no.1, 015017 (2019), “One-loop contribution to dark-matter-nucleon scattering in the pseudo-scalar dark matter model,” JHEP **01**, 138 (2019)

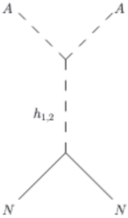
Direct detection

The DM direct detection signals are naturally suppressed in the pGDM model.

$$V \supset \frac{A^2}{2v_S}(\sin\alpha\, m_1^2 h_1 + \cos\alpha\, m_2^2 h_2)\,,$$

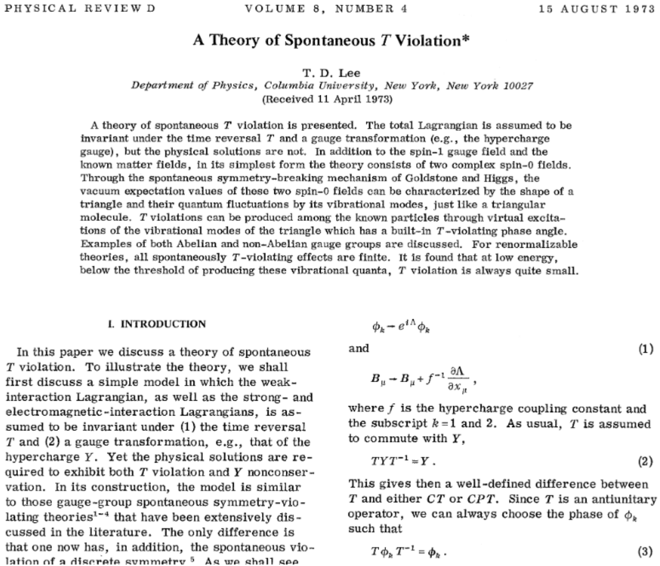


$$= -iR_{2i}\frac{m_i^2}{v_S}$$



$$i\mathcal{M} = -i\frac{\sin 2\alpha f_N m_N}{2vv_S}\left(\frac{m_1^2}{q^2 - m_1^2} - \frac{m_2^2}{q^2 - m_2^2}\right)\bar{u}_N(p_4)u_N(p_2) \to 0$$

Properties of nHDM


PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

A Theory of Spontaneous T Violation*

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(Received 11 April 1973)

A theory of spontaneous T violation is presented. The total Lagrangian is assumed to be invariant under the time reversal T and a gauge transformation (e.g., the hypercharge gauge), but the physical solutions are not. In addition to the spin-1 gauge field and the known matter fields, in its simplest form the theory consists of two complex spin-0 fields. Through the spontaneous symmetry-breaking mechanism of Goldstone and Higgs, the vacuum expectation values of these two spin-0 fields can be characterized by the shape of a triangle and their quantum fluctuations by its vibrational modes, just like a triangular molecule. T violations can be produced among the known particles through virtual excitations of the vibrational modes of the triangle which has a built-in T-violating phase angle. Examples of both Abelian and non-Abelian gauge groups are discussed. For renormalizable theories, all spontaneously T-violating effects are finite. It is found that at low energy, below the threshold of producing these vibrational quanta, T violation is always quite small.

I. INTRODUCTION

In this paper we discuss a theory of spontaneous T violation. To illustrate the theory, we shall first discuss a simple model in which the weak-interaction Lagrangian, as well as the strong- and electromagnetic-interaction Lagrangians, is assumed to be invariant under (1) the time reversal T and (2) a gauge transformation, e.g., that of the hypercharge Y. Yet the physical solutions are required to exhibit both T violation and Y nonconservation. In its construction, the model is similar to those gauge-group spontaneous symmetry-violating theories[1-4] that have been extensively discussed in the literature. The only difference is that one now has, in addition, the spontaneous violation of a discrete symmetry.[5] As we shall see

$$\phi_k \to e^{i\Lambda}\phi_k$$

and (1)

$$B_\mu \to B_\mu + f^{-1}\frac{\partial\Lambda}{\partial x_\mu},$$

where f is the hypercharge coupling constant and the subscript $k=1$ and 2. As usual, T is assumed to commute with Y,

$$TYT^{-1} = Y. \quad (2)$$

This gives then a well-defined difference between T and either CT or CPT. Since T is an antiunitary operator, we can always choose the phase of ϕ_k such that

$$T\phi_k T^{-1} = \phi_k. \quad (3)$$

$$
\begin{aligned}
&V(\Phi_1, \Phi_2) \\
&= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right]\right\} \\
&+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&+\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + \text{H.c.}\right\}
\end{aligned}
$$

In a general basis, the vacuum may be complex:

$$
\Phi_j = e^{i\xi_j}\begin{pmatrix} \varphi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix}, \quad j = 1, 2,
$$

where v_j are real numbers, so that $v_1^2 + v_2^2 = v^2$.

Spontaneous or explicit violation of CP is possible

The phase difference between the two vevs is defined as

$$\xi \equiv \xi_2 - \xi_1.$$

Next, let's determine the Goldston bosons G_0 and $G^\pm$ by an orthogonal rotation

$$\begin{pmatrix} G_0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \varphi_1^\pm \\ \varphi_2^\pm \end{pmatrix}$$

where $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$ for $\tan\beta \equiv v_2/v_1$. Then G_0 and $G^\pm$ become the massless Goldston fields. $H^\pm$ are the charged scalars.

The model contains three neutral scalar mass-eigenstates, which are linear compositions of the η_i,

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},$$

with the 3×3 orthogonal rotation matrix R satisfying

$$R \mathcal{M}^2 R^{\mathrm{T}} = \mathcal{M}^2_{\mathrm{diag}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2),$$

and with $M_1 \leq M_2 \leq M_3$. A convenient parametrization of the rotation matrix R is

$$R = R_3 R_2 R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_1\, c_2 & s_1\, c_2 & s_2 \\ -(c_1\, s_2\, s_3 + s_1\, c_3) & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, s_2\, c_3 + s_1\, s_3 & -(c_1\, s_3 + s_1\, s_2\, c_3) & c_2\, c_3 \end{pmatrix}$$

Physical/observable input parameter set:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

$$\begin{aligned} e_i \equiv v_1 R_{i1} + v_2 R_{i2} \quad &\propto \quad H_i W^+ W^-, H_i ZZ \\ q_i \quad &\propto \quad H_i H^+ H^- \end{aligned}$$

$$e_1^2 + e_2^2 + e_3^2 = v^2 \equiv v_1^2 + v_2^2$$

Weak-bases transformation:

$$\begin{pmatrix} \bar{\Phi}_1 \\ \bar{\Phi}_2 \end{pmatrix} = \underbrace{e^{i\psi} \begin{pmatrix} \cos\theta & e^{-i\tilde{\xi}} \sin\theta \\ -e^{i\chi} \sin\theta & e^{i(\chi-\tilde{\xi})} \cos\theta \end{pmatrix}}_{U(2)} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

Observables are weak-basis independent

Flavor-Changing Neutral Currents in nHDM

$$\mathcal{L} \supset -\sum_{\alpha=1}^{n} \sum_{i,j=1}^{3} \left(\tilde{\Gamma}_{ij}^{\alpha} \bar{u}_{i\,R} \tilde{\phi}^{\alpha\,\dagger} Q_{j\,L} + \Gamma_{ij}^{\alpha} \bar{d}_{i\,R} \phi^{\alpha\,\dagger} Q_{j\,L} + \text{H.c.} \right)$$

$$\mathcal{M}_{ij}^{u} = \sum_{\alpha=1}^{n} \tilde{\Gamma}_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \sum_{\alpha=1}^{n} \Gamma_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}$$

$n = 2$

$$\mathcal{M}_{ij}^{u} = \tilde{\Gamma}_{ij}^{1} \frac{v_1}{\sqrt{2}} + \tilde{\Gamma}_{ij}^{2} \frac{v_2}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \Gamma_{ij}^{1} \frac{v_1}{\sqrt{2}} + \Gamma_{ij}^{2} \frac{v_2}{\sqrt{2}}$$

$$Z_2 : \phi_2 \to -\phi_2, u_{i\,R} \to -u_{i\,R}$$

$$\mathcal{M}_{ij}^{u} = \cancel{\tilde{\Gamma}_{ij}^{1} \frac{v_1}{\sqrt{2}}} + \tilde{\Gamma}_{ij}^{2} \frac{v_2}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \Gamma_{ij}^{1} \frac{v_1}{\sqrt{2}} + \cancel{\Gamma_{ij}^{2} \frac{v_2}{\sqrt{2}}}$$

no: FCNC

Gauge couplings:

$$H_i Z_\mu Z_\nu : \quad \frac{ig^2}{2\cos^2\theta_W} e_i \, g_{\mu\nu}, \qquad H_i W^+_\mu W^-_\nu : \quad \frac{ig^2}{2} e_i \, g_{\mu\nu}$$

where

$$e_i \equiv v_1 R_{i1} + v_2 R_{i2}$$

In terms of the mixing angles

$$\begin{aligned} e_1 &= v \cos\alpha_2 \cos(\beta - \alpha_1) \\ e_2 &= v[\cos\alpha_3 \sin(\beta - \alpha_1) - \sin\alpha_2 \sin\alpha_3 \cos(\beta - \alpha_1)] \\ e_3 &= -v[\sin\alpha_3 \sin(\beta - \alpha_1) + \sin\alpha_2 \cos\alpha_3 \cos(\beta - \alpha_1)] \end{aligned}$$

Note that

$$e_1^2 + e_2^2 + e_3^2 = v^2$$

The Higgs alignment

$$H_{125} \simeq H_{SM}$$

$$\Downarrow$$

$$e_1 \simeq v$$

$$\Downarrow$$

$$2HDM: \; e_1 = v, e_2 = e_3 = 0 \qquad (e_1^2 + e_2^2 + e_3^2 = v^2)$$

$$\Downarrow$$

$$e_1 = v\cos(\alpha_2)\cos(\alpha_1 - \beta) = v,$$

where $\tan\beta = v_2/v_1$.

$$\Downarrow$$

$$\alpha_1 = \beta, \quad \alpha_2 = 0$$

Weak-basis CPV invariants

$$\begin{aligned}
\mathrm{Im}\, J_1 &= \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk} M_i^2 e_i e_k q_j \\
&= \frac{1}{v^5}[M_1^2 e_1(e_3 q_2 - e_2 q_3) + M_2^2 e_2(e_1 q_3 - e_3 q_1) + M_3^2 e_3(e_2 q_1 - e_1 q_2)] \\
\mathrm{Im}\, J_2 &= \frac{2}{v^9}\sum_{i,j,k}\epsilon_{ijk} e_i e_j e_k M_i^4 M_k^2 \\
&= \frac{2 e_1 e_2 e_3}{v^9}(M_1^2 - M_2^2)(M_2^2 - M_3^2)(M_3^2 - M_1^2) \\
\mathrm{Im}\, J_{30} &= \frac{1}{v^5}\sum_{i,j,k}\epsilon_{ijk} q_i M_i^2 e_j q_k
\end{aligned}$$

found by Lavoura, Silva and Botella (1994, 1995)

CP is conserved if and only if $\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = \mathrm{Im}\, J_{30} = 0$

Alignment: e_1=v, e_2=e_3=0

$$\begin{aligned}
\mathrm{Im}\, J_1 &= 0, \\
\mathrm{Im}\, J_2 &= 0, \\
\mathrm{Im}\, J_{30} &= \frac{q_2 q_3}{v^4}(M_3^2 - M_2^2)
\end{aligned}$$

- $e_1 = v$ implies no CP violation in the couplings to gauge bosons ($\mathrm{Im}\, J_1 = \mathrm{Im}\, J_2 = 0$), the only possible CP violation may appear in cubic scalar couplings q_2 and q_3.
- The necessary condition for CP violation is that both $(H_2 H^+ H^-)$ *and* $(H_3 H^+ H^-)$ couplings must exist *together* with a non-zero ZH_2H_3 vertex ($\propto e_1$).
- If $\lambda_6 = \lambda_7 = 0$ (Z_2-symmetric 2HDM), then $\mathrm{Im}\, J_{30} = 0$, so no CPV!

B.G., O. M. Ogreid and P. Osland, "Measuring CP violation in Two-Higgs-Doublet models in light of the LHC Higgs data", JHEP **11**, 084 (2014)

The Inert Higgs Doublet model (IDM)

$Z_2 : \phi_2 \to -\phi_2$ unbroken, i.e. $v_2 = 0$:

$$V(\Phi_1, \Phi_2) =$$

$$-\frac{1}{2}\left\{ m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[\cancel{m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}}\right]\right\}$$

$$+\frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger \Phi_2)^2 + \cancel{[\lambda_6(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2)](\Phi_1^\dagger \Phi_2)} + \text{H.c.}\right\}$$

$$\Phi_2 = \cancel{e^{i\xi_2}} \begin{pmatrix} \varphi_2^+ \\ (\cancel{v_2} + \eta_2 + i\chi_2)/\sqrt{2} \end{pmatrix},$$

$$\Downarrow$$

DM candidates: η_2, χ_2

2HDM conclusions

- Violation of CP in the scalar potential
- Weak-basis invariance of observables
- Flavor-Changing Neutral Currents ($Z_2 : \phi_2 \to -\phi_2$)
- Symmetries of the potential (e.g. Z_2 extended to Yukawa invariance)
- Inert Doublet Model (IDM): exactly Z_2-symmetric ($\phi_2 \to -\phi_2$) 2HDM, DM candidates, no CPV
- No CPV in the potential in Z_2-symmetric models in the alignment limit

$$\Downarrow$$

For CPV in the potential the Z_2 must be abandoned, so FCNC appear

Minimal models with CPV and DM

- minimal: real or complex singlet (DM) $\oplus$ 2HDM (CPV)

Minimal models with CPV and DM

- minimal: real or complex singlet (DM) $\oplus$ 2HDM (CPV)
- next to minimal: $\underbrace{\text{inert doublet (DM)} \oplus \text{2HDM (CPV)}}_{3HDM}$

Real singlet (DM) $\oplus$ 2HDM (CPV)

$Z_2 \times Z_2'$ ($\phi_2 \to -\phi_2$, $\varphi \to -\varphi$) symmetry, Z_2 softly broken

$$
\begin{aligned}
V(\phi_1,\phi_2,\varphi) \; = \; & -\frac{1}{2}\left\{m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \text{H.c.}\right]\right\} \\
& + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\
& + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] \\
& + \mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \varphi^2(\eta_1\phi_1^\dagger\phi_1 + \eta_2\phi_2^\dagger\phi_2).
\end{aligned}
$$

φ - DM candidate

B.G. and P. Osland, “Tempered Two-Higgs-Doublet Model”, Phys.Rev.D82, 125026 (2010)

Singlet Complex Scalar ⊕ General 2HDM

$$S \xrightarrow{\text{U}(1)} e^{i\alpha} S$$

The $U(1)$ softly broken ($Z_2 : S \to -S$):

$$V = -\frac{1}{2}\left[m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \left(m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right)\right] + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4$$
$$+ \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_2)|\Phi_1|^2 + \lambda_7(\Phi_1^\dagger\Phi_2)|\Phi_2|^2 +\right.$$
$$- \mu_s^2|S|^2 + \frac{\lambda_s}{2}|S|^4 + (\mu^2 S^2 + \text{H.c.}) + |S|^2\left[\kappa_1|\Phi_1|^2 + \kappa_2|\Phi_2|^2 + \left(\frac{\kappa_3}{2}\Phi_1^\dagger\Phi_2 + \text{H.c.}\right)\right]$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + \eta_1 + i\chi_1}{\sqrt{2}} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + \eta_2 + i\chi_2}{\sqrt{2}} \end{pmatrix}, \qquad S = \frac{v_s + s + iA}{\sqrt{2}}$$

In the general 2HDM with pGDM the tree-level DM-quark amplitude also vanishes at the zero momentum transfer limit.

N. Darvishi and B.G. “Pseudo-Goldstone dark matter model with CP violation,” JHEP **06**, 092 (2022)

Inert doublet (DM) $\oplus$ 2HDM (CPV)

$Z_2 \times Z_2'$ $(\phi_2 \to -\phi_2, \eta \to -\eta)$ symmetry, Z_2 softly broken:

$$V(\Phi_1, \Phi_2, \eta) = V_{12}(\Phi_1, \Phi_2) + V_3(\eta) + V_{123}(\Phi_1, \Phi_2, \eta)$$

where

$$\begin{aligned}
V_{12}(\Phi_1, \Phi_2) &= -\frac{1}{2}\left\{m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}\right]\right\} \\
&+ \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
&+ \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2}\left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}\right], \\
V_3(\eta) &= m_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2}(\eta^\dagger \eta)^2, \\
V_{123}(\Phi_1, \Phi_2, \eta) &= \lambda_{1133}(\Phi_1^\dagger \Phi_1)(\eta^\dagger \eta) + \lambda_{2233}(\Phi_2^\dagger \Phi_2)(\eta^\dagger \eta) \\
&+ \lambda_{1331}(\Phi_1^\dagger \eta)(\eta^\dagger \Phi_1) + \lambda_{2332}(\Phi_2^\dagger \eta)(\eta^\dagger \Phi_2) \\
&+ \frac{1}{2}\left[\lambda_{1313}(\Phi_1^\dagger \eta)^2 + \text{H.c.}\right] + \frac{1}{2}\left[\lambda_{2323}(\Phi_2^\dagger \eta)^2 + \text{H.c.}\right]
\end{aligned}$$

B.G., O.M. Ogreid, P. Osland, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80, 055013 (2009)

Summary and conclusions

- The SM is not perfect (DM, Λ, θ, BA, 21, etc.)
- The SM should be extended: extra scalar doublets and singlets are likely/favored
- Extra scalars (real or complex singlets, $SU(2)$-doublets) $\Rightarrow$ DM
- Complex scalar with soft breaking of a global symmetry (e.g. $U(1)$) $\Rightarrow$ pGDM with suppressed $DM - nucleus$ coupling.
- $2HDM$, $nHDM$ $\Rightarrow$ CP violation (explicit or spontaneous)
- No CPV in Z_2 symmetric 2HDM in the alignment limit ($e_1 = 1, e_2 = e_3 = 0$) $\Rightarrow$ generic 2HDM with FCNC in Yukawa interactions
- Minimal "pragmatic" models (CPV & DM):
  - minimal: real or complex singlet (DM) $\oplus$ 2HDM (CPV), but FCNC
  - next to minimal: inert doublet (DM) $\oplus$ 2HDM (CPV), but FCNC (?)
- $3HDM$ e.g. inert doublet (DM) $\oplus$ 2HDM (CPV), $N_f = N_h = 3$

3HDM of Weinberg:

- CPV in $H^{\pm}$ interactions.
- Natural flavour conservation by a Z_2

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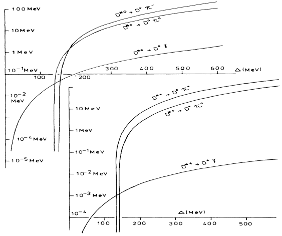

FIG. 1. Decay rates $\Gamma_{D^* \to D\pi}$ and $\Gamma_{D^* \to D\gamma}$ in the charmed quark model: $\Delta \equiv D^{*0} - D^0$.

For $\Delta > 200$ MeV one gets

$$\Gamma_{D^{*0} \to D^+\pi^-} : \Gamma_{D^{*0} \to D^0\pi^0} : \Gamma_{D^{*0} \to D^0\gamma} \simeq 1:0.5:10^{-2},$$
$$\Gamma_{D^{*+} \to D^0\pi^+} : \Gamma_{D^{*+} \to D^+\pi^0} : \Gamma_{D^{*+} \to D^+\gamma} \simeq 1:0.5:3\times 10^{-3}. \quad (17)$$

In view of the fact that, experimentally, Δ appears to be about 140 MeV, very nearly a pion mass, and is not measured with high accuracy, one may not be able to calculate the decay rates of $D^* \to D\pi$ very accurately.

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Gauge Theory of *CP* Nonconservation*

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It is proposed that *CP* nonconservation arises purely from the exchange of Higgs bosons.

Ever since the discovery[1] that *CP* conservation is not exact, the mystery has been why it is so feebly violated.[2] In many proposed theories,[3] one must arrange to have *CP* to be approximately conserved, by making the appropriate constants in the Lagrangian to have sufficiently small values. However, one would prefer a more natural explanation.

Renormalizable gauge theories[4] of the weak and electromagnetic interactions provide a mechanism which could violate *CP* conservation with about the right strength: the Higgs boson. The the Fermi coupling constant), so that the exchange of a Higgs boson of mass m_H produces an effective Fermi interaction with coupling of order $G_F m^2/m_H^2$. For reasonable mass values,[5] this is "milliweak." However, in order for the Higgs exchange to appear as a natural explanation for a feeble *CP* nonconservation, one must understand why *CP* conservation is strongly violated in the Higgs exchange, *and nowhere else*. In this paper, I wish to present a realistic gauge theory, in which *CP* nonconservation automatically arises in just this way.[6]

- explanation of weak mixing angles through horizontal symmetries

$$\mathcal{L} \supset -\sum_{i,j=1}^{3} \sum_{\alpha=1}^{N_H} \left(\tilde{\Gamma}_{ij}^{\alpha} \bar{u}_{i\,R} \tilde{H}^{\alpha\,\dagger} Q_{j\,L} + \Gamma_{ij}^{\alpha} \bar{d}_{i\,R} H^{\alpha\,\dagger} Q_{j\,L} + \text{H.c.} \right)$$

$$H^{\alpha} \to \mathcal{H}_{\beta}^{\alpha} H^{\beta}\,, \qquad u_{i\,R} \to \mathcal{U}_i^j u_{j\,R}\,, \qquad d_{i\,R} \to \mathcal{D}_i^j d_{j\,R}\,, \qquad Q_{i\,L} \to \mathcal{Q}_i^j Q_{j\,L}$$

$$\Downarrow$$

constraints on fermion mass-matrices:

$$\mathcal{M}_{ij}^{u} = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}, \qquad \mathcal{M}_{ij}^{d} = \sum_{\alpha=1}^{N_H} \Gamma_{ij}^{\alpha} \frac{v_\alpha}{\sqrt{2}}$$

$$U_R^{\dagger} \mathcal{M}^{u} U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^{\dagger} \mathcal{M}^{d} D_L = \mathrm{diag}(m_d, m_s, m_b)$$

If $\mathcal{M}^{u,d}$ constrained, then $U^{CKM} \equiv U_L^{\dagger} D_L = U^{CKM}(m_q/m_{q'})$