

# Gravitational Dark Matter

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- ◇ A. Ahmed, B.G., Anna Socha, "Gravitational Vector Dark Matter", in preparation

## Motivations

The existence of DM has been inferred only from gravitational effects.

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{int}} \right],$$

with  $\kappa \equiv M_{Pl}^{-1} = \sqrt{8\pi G_N}$

$$\mathcal{L}_{\text{DM}} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu,$$

where  $X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$ .

## Quantization of a vector field in a curved background

The action for an Abelian vector fields reads:

$$\mathcal{L}_{DM} = \int d^4x \sqrt{-g} \left( -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} X_{\mu\nu} X_{\alpha\beta} + \frac{1}{2} m_X^2 g^{\mu\nu} X_\mu X_\nu \right).$$

The background metric is the FLRW:

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2.$$

Extremizing the above action with respect to  $X_\mu$  one obtains

$$\partial_\mu (g^{\mu\rho} g^{\nu\sigma} X_{\rho\sigma}) + \frac{1}{\sqrt{-g}} (\partial_\mu \sqrt{-g}) g^{\mu\rho} g^{\nu\sigma} X_{\rho\sigma} + m_X^2 g^{\mu\nu} X_\mu = 0,$$

so that

$$\begin{aligned} \vec{\nabla} \cdot \dot{\vec{X}} - \nabla^2 X_0 + m_X^2 a^2 X_0 &= 0, \\ \ddot{\vec{X}} + H \dot{\vec{X}} - \frac{1}{a^2} \left[ \nabla^2 \vec{X} - \vec{\nabla} (\vec{\nabla} \cdot \vec{X}) \right] + m_X^2 \vec{X} &= \vec{\nabla} (\dot{X}_0 + H X_0). \end{aligned}$$

That could be simplified:

$$\vec{\nabla} \cdot \dot{\vec{X}} - \nabla^2 X_0 + m_X^2 a^2 X_0 = 0,$$
$$\ddot{\vec{X}} + H \dot{\vec{X}} - \frac{1}{a^2} \nabla^2 \vec{X} + m_X^2 \vec{X} = -2H \nabla X_0.$$

It is convenient to adopt the Fourier transform

$$X_\mu(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \tilde{X}_\mu(t, \vec{k}) e^{i\vec{k} \cdot \vec{x}},$$

where the reality of the  $X_\mu(t, \vec{x})$  field implies  $\tilde{X}_\mu(t, \vec{k}) = \tilde{X}_\mu^*(t, -\vec{k})$ .

Then we get

$$\tilde{X}_0 = \frac{-i\vec{k} \cdot \partial_t \vec{\tilde{X}}}{k^2 + a^2 m_X^2},$$
$$\partial_t^2 \vec{\tilde{X}} + H \partial_t \vec{\tilde{X}} + \left( \frac{k^2}{a^2} + m_X^2 \right) \vec{\tilde{X}} = -2\vec{k} \frac{\vec{k} \cdot \partial_t \vec{\tilde{X}}}{k^2 + m_X^2 a^2} H.$$

Note that the  $X_0$  is an auxiliary field and has no dynamics.

The  $\vec{\tilde{X}}$  can be decomposed in a basis of helicity states:

$$\vec{\tilde{X}}(t, \vec{k}) = \sum_{\lambda=\pm, L} \vec{\epsilon}_\lambda(\vec{k}) \tilde{X}_\lambda(t, \vec{k}),$$

where  $\tilde{X}_\pm$  and  $\tilde{X}_L$  denote two transversely-polarized modes and a single longitudinally-polarized mode, respectively. Then

$$\begin{aligned} \ddot{\tilde{X}}_\pm + H \dot{\tilde{X}}_\pm + \left( \frac{k^2}{a^2} + m_X^2 \right) \tilde{X}_\pm &= 0, \\ \ddot{\tilde{X}}_L + H \left( 1 + \frac{2k^2}{k^2 + a^2 m_X^2} \right) \dot{\tilde{X}}_L + \left( \frac{k^2}{a^2} + m_X^2 \right) \tilde{X}_L &= 0. \end{aligned}$$

Switching to the conformal time ( $dt = a(\tau)d\tau$ ) we get

$$\begin{aligned} \tilde{X}_\pm'' + \overbrace{(k^2 + a^2 m_X^2)}^{\omega_\pm^2} \tilde{X}_\pm &= 0, \\ \tilde{X}_L'' + \frac{2k^2}{k^2 + a^2 m_X^2} \frac{a'}{a} \tilde{X}_L' + (k^2 + a^2 m_X^2) \tilde{X}_L &= 0. \end{aligned}$$

For the longitudinal mode,  $\tilde{X}_L$ , it is convenient to perform a field redefinition

$$\tilde{X}_L = \frac{\sqrt{k^2 + a^2 m_X^2}}{a m_X} \mathcal{X}_L,$$

so that

$$\mathcal{X}_L'' + \omega_L^2(\tau) \mathcal{X}_L = 0,$$

with

$$\omega_L^2(\tau) \equiv k^2 + m_X^2 a^2 - \frac{k^2}{k^2 + m_X^2 a^2} \frac{a''}{a} + 3 \frac{k^2 m_X^2 a'^2}{(k^2 + m_X^2 a^2)^2}.$$

Quantization:

$$\hat{\vec{X}}_L(\tau, \vec{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left\{ \vec{\epsilon}_L(\vec{k}) \hat{a}_{\vec{k}} \tilde{\mathcal{X}}_L(\tau, \vec{k}) e^{i\vec{k}\cdot\vec{x}} + \vec{\epsilon}_L(\vec{k}) \hat{a}_{\vec{k}}^\dagger \tilde{\mathcal{X}}_L^*(\tau, \vec{k}) e^{-i\vec{k}\cdot\vec{x}} \right\},$$

$$\hat{\vec{X}}_{\pm}(\tau, \vec{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left\{ \vec{\epsilon}_{\pm}(\vec{k}) \hat{b}_{\vec{k}, \pm} \tilde{\mathcal{X}}_{\pm}(\tau, \vec{k}) e^{i\vec{k}\cdot\vec{x}} + \vec{\epsilon}_{\pm}^*(\vec{k}) \hat{b}_{\vec{k}, \pm}^\dagger \tilde{\mathcal{X}}_{\pm}^*(\tau, \vec{k}) e^{-i\vec{k}\cdot\vec{x}} \right\}.$$

and

$$[\hat{\vec{X}}_L(\tau, \vec{x}), \hat{\vec{\Pi}}_L(\tau, \vec{y})] = i\delta^{(3)}(\vec{x} - \vec{y}),$$

$$[\hat{\vec{X}}_{\pm}(\tau, \vec{x}), \hat{\vec{\Pi}}_{\pm}(\tau, \vec{y})] = i\delta^{(3)}(\vec{x} - \vec{y}),$$

with

$$[\hat{a}_{\vec{k}}, \hat{a}_{\vec{k}'}^\dagger] = \delta^{(3)}(\vec{k} - \vec{k}')$$
$$[\hat{b}_{\vec{k}, \lambda}, \hat{b}_{\vec{k}', \lambda'}^\dagger] = \delta_{\lambda\lambda'} \delta^{(3)}(\vec{k} - \vec{k}')$$

Note that the Wronskian:

$$W[v, v^*] \equiv v'v^* - v'^*v$$

is time-independent and normalized as follows

$$W[\tilde{\chi}_L, \tilde{\chi}_L^*] = W[\tilde{\chi}_\pm, \tilde{\chi}_\pm^*] = -i. \quad (1)$$

To solve equations of motion for the two transversely-polarized mode and the single longitudinally-polarized mode we impose the Bunch-Davies initial conditions:

$$\lim_{t \rightarrow -\infty} \tilde{\chi}_L(t, \vec{k}) = \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad \lim_{t \rightarrow -\infty} \tilde{\chi}_\pm(t, \vec{k}) = \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$

## Gravitational production of dark matter

The action for the vector DM in a background metric  $\bar{g}_{\mu\nu}$  is given by

$$\mathcal{L}_{\text{DM}} = \int d^4x \sqrt{-\bar{g}} \left( -\frac{1}{4} \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} X_{\mu\nu} X_{\alpha\beta} - \frac{1}{2} m_X^2 \bar{g}^{\mu\nu} X_\mu X_\nu \right),$$

where the background metric is of the FLRW form with the line element

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2.$$

The energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-\bar{g}}} \frac{\delta(\sqrt{-\bar{g}} \mathcal{L}_{\text{DM}})}{\delta \bar{g}_{\mu\nu}},$$

one can find the energy density of the vector DM as

$$\rho_X = \frac{1}{2a^2} (|\dot{\vec{X}} - \nabla X_0|^2 + \frac{1}{a^2} |\vec{\nabla} \times \vec{X}|^2 + a^2 m_X^2 X_0^2 + m_X^2 \vec{X}^2).$$



$$\langle \rho_L \rangle = \frac{1}{2\pi^2 a^4} \int dk k^2 \left[ |\tilde{\chi}'_L|^2 + (\tilde{\chi}'_L \tilde{\chi}_L^* + \tilde{\chi}'_L{}^* \tilde{\chi}_L) + \frac{A'(\tau)}{A(\tau)} \left( \frac{A'(\tau)^2}{A^2(\tau)} + k^2 + a^2 m_X^2 \right) |\tilde{\chi}_L|^2 \right],$$

$$\langle \rho_{\pm} \rangle = \frac{1}{2\pi^2 a^4} \int dk k^2 \left[ |\chi'_{\pm}|^2 + (k^2 + a^2 m_X^2) |\chi_{\pm}|^2 \right],$$

where  $\langle \rho_L \rangle \equiv \langle 0 | \rho_L | 0 \rangle$  and  $\langle \rho_{\pm} \rangle \equiv \langle 0 | \rho_{\pm} | 0 \rangle$  and

$$A(\tau) \equiv \frac{\sqrt{k^2 + a^2(\tau) m_X^2}}{a(\tau) m_X}.$$

Equations of motion for longitudinal and transversely polarized Fourier modes:

$$\tilde{\chi}_L'' + \omega_L^2(\tau) \tilde{\chi}_L = 0, \quad \chi_{\pm}'' + \omega_{\pm}^2(\tau) \chi_{\pm} = 0,$$

where the time-dependent frequencies are given by

$$\omega_L^2(\tau) = k^2 + a^2 m_X^2 - \frac{k^2}{k^2 + a^2 m_X^2} \left( \frac{a''}{a} - \frac{3a^2 m_X^2}{k^2 + a^2 m_X^2} \frac{a'^2}{a^2} \right),$$

$$\omega_{\pm}^2(\tau) = k^2 + a^2 m_X^2.$$

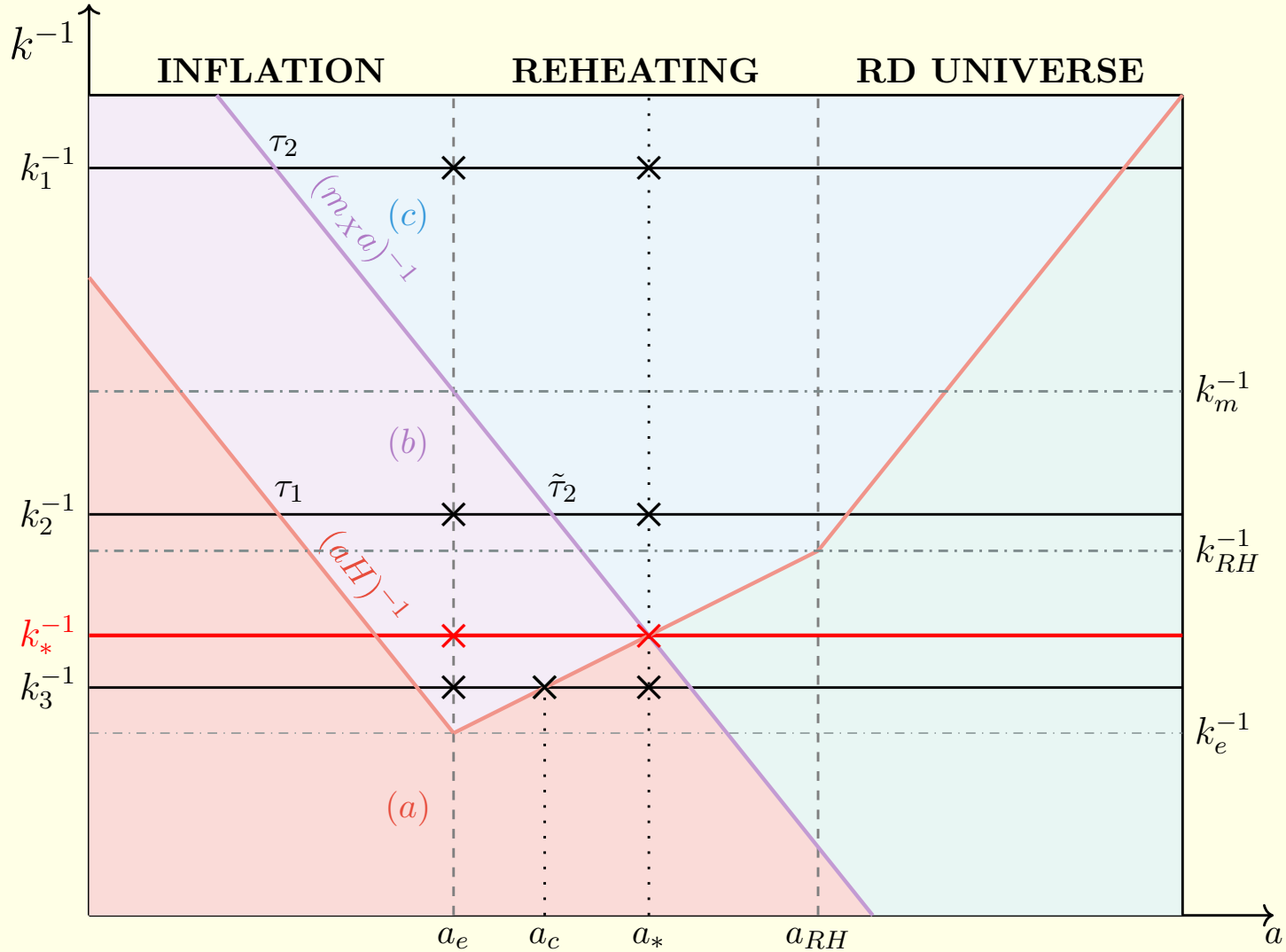


Figure 1: Scaling of the energy density as a function of the scale factor for heavy DM i.e  $H_{RH} < m_X$ . The main contribution to the total energy density comes from the mode  $k_* \equiv a(\tau : H = m_X)m_X$ . Here  $k_e \equiv a_e H_e$ ,  $k_m \equiv a_e m_X$  and  $k_{RH} \equiv a_{RH} H_{RH}$ .

- For vector DM mass  $H_{\text{RH}} < m_X < H_I$ ,

$$\frac{d\langle n_L(\tau: H = m_X) \rangle}{d \ln k} = \frac{1}{8\pi^2} \begin{cases} H_I^{\frac{2(3w^2+3w+2)}{(w+1)(3w+1)}} m_X^{\frac{2}{1+w}} \left(\frac{a_e}{k}\right)^{\frac{3(1-w)}{1+3w}}, & \text{for } k_* < k < k_e, \\ H_I^{\frac{2(1+3w)}{3(1+w)}} m_X^{\frac{1-3w}{3(1+w)}} \left(\frac{k}{a_e}\right)^2, & \text{for } k < k_*. \end{cases}$$

- For vector DM mass  $H_{\text{RME}} < m_X < H_{\text{RH}}$ ,

$$\frac{d\langle n_L(\tau: H = m_X) \rangle}{d \ln k} = \frac{1}{8\pi^2} \begin{cases} m_X^{3/2} H_I^{\frac{3(w+3)}{2(3w+1)}} \gamma^{\frac{1-3w}{1+w}} \left(\frac{a_e}{k}\right)^{\frac{3(1-w)}{1+3w}}, & \text{for } k_{\text{RH}} < k < k_e, \\ m_X^{3/2} H_I^{5/2} \gamma^{\frac{-1+3w}{3(1+w)}} \left(\frac{a_e}{k}\right), & \text{for } k_* < k < k_{\text{RH}}, \\ H_I \gamma^{\frac{2(1-3w)}{3(1+w)}} \left(\frac{k}{a_e}\right)^2, & \text{for } k < k_*. \end{cases}$$

$$\gamma \equiv \sqrt{\frac{H_{\text{RH}}}{H_I}} \quad \text{and} \quad p = w\rho$$

Note that the number density per ln frequency has a peak structure if and only if  $w \in (-\frac{1}{3}, 1)$ . In this case,  $d\langle n_L(H = m_X) \rangle / d \ln k$  is dominated by modes with  $k = k_* \equiv a(\tau : H = m_X) m_X$  and  $k_e \equiv a_e H_I$ .

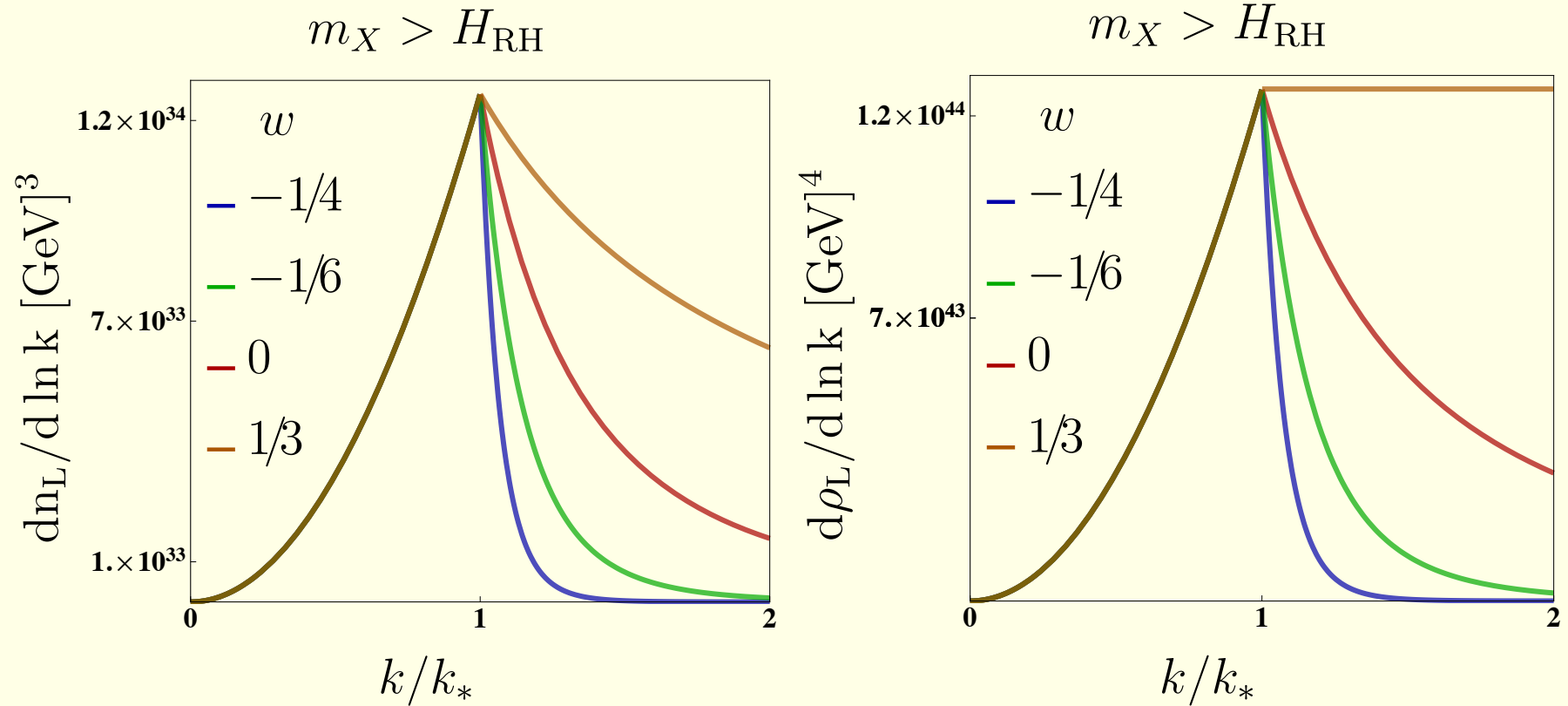


Figure 2: The energy density momentum distribution for different values of the equation-of state parameter  $w$ , for  $m_X > H_{RH}$ ,  $k_* \equiv a(\tau : H = m_X) m_X$ .

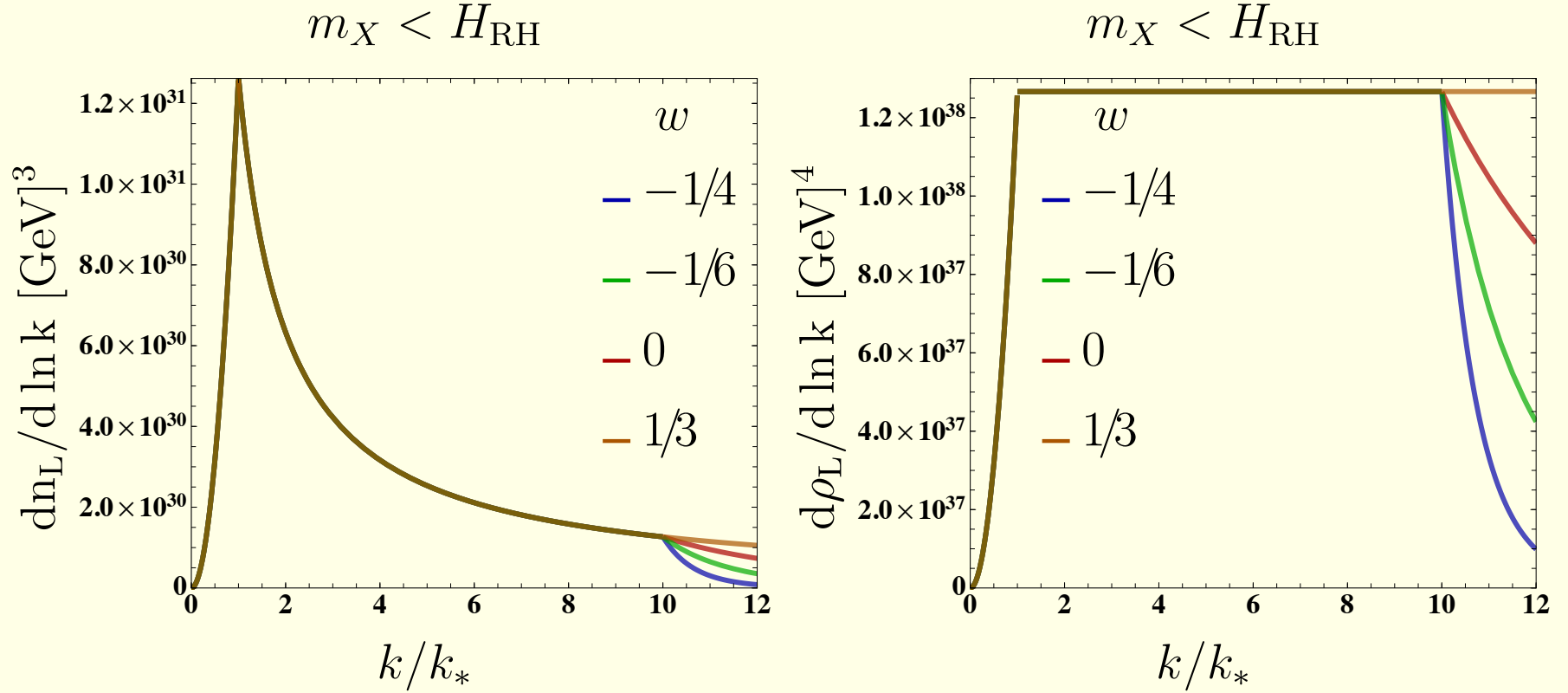


Figure 3: The energy density momentum distribution for different values of the equation-of state parameter  $w$ , for  $m_X < H_{RH}$ ,  $k_* \equiv a(\tau : H = m_X)m_X$ .

Finally the DM present-day relic abundance can be calculated as,

$$\begin{aligned}
 \Omega_X h^2 &= T_* \frac{s_0 h^2}{4\rho_c \kappa^{-2}} \frac{n_*}{m_X} \\
 &= \frac{1}{8\pi^2} \left( \frac{45}{128\pi^2 g_*(T_{\text{RH}})} \right)^{1/4} \frac{s_0 h^2}{\kappa^{-3/2} \rho_c} \\
 &\quad \times \begin{cases} \left( \frac{1}{2} + \frac{1+3w}{3(1-w)} \right) \gamma^{1/2} H_I^{9/4} m_X^{1/4}, & H_{\text{RH}} < m_X < H_I \\ \left( \frac{3}{2} H_I^2 m_X^{1/2} + \frac{1+3w}{3(1-w)} H_I^{3/2} m_X \gamma^{-1} \right), & H_{\text{RME}} < m_X < H_{\text{RH}} \end{cases} \quad (4)
 \end{aligned}$$

where  $s_0 = 2970 \text{ cm}^{-3}$ ,  $\rho_c = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$ ,  $\kappa^{-1} = M_{Pl} = 2.435 \times 10^{18} \text{ GeV}$  and  $\Omega_X^{\text{obs}} h^2 = 0.1198 \pm 0.002$ . Furthermore, we assume that  $g_*(T_{\text{RH}}) \simeq 106$ , i.e. no extra d.o.f. beyond the SM.

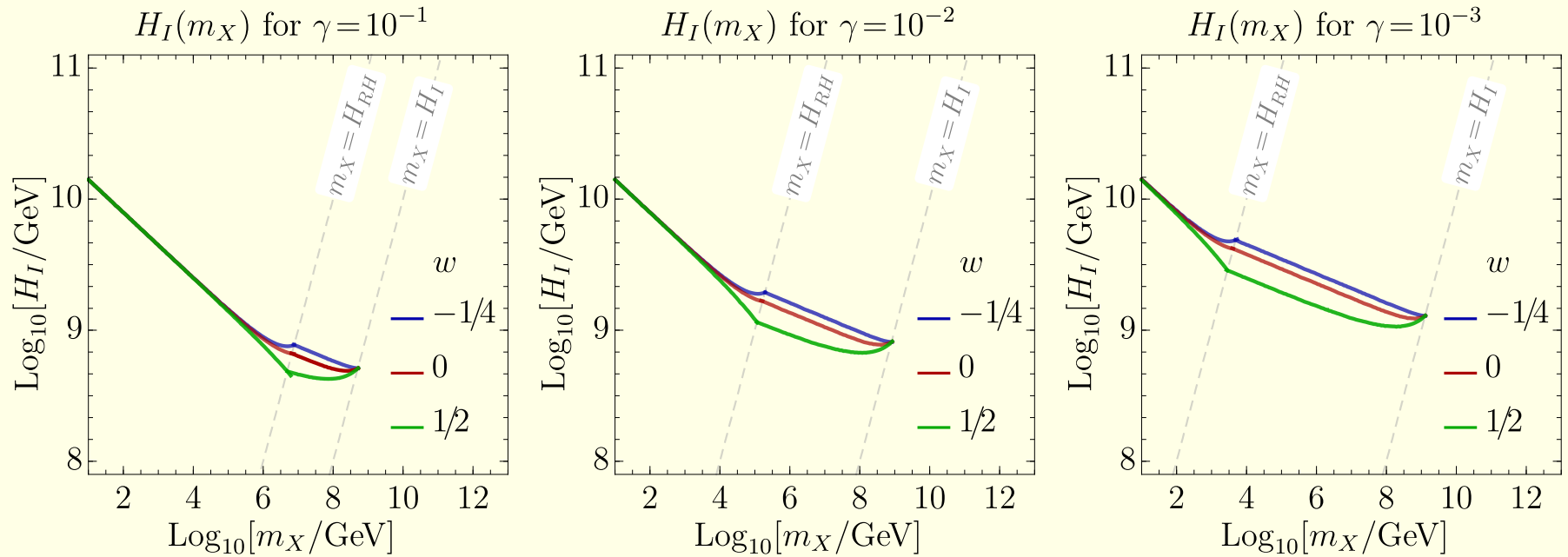


Figure 4: Relations between the Hubble rate at the end of inflation  $H_I$  and vector DM mass  $m_X$  that reproduces the observed relic abundance  $\Omega_{\text{DM}}h^2$  for the gravitational production only.

# Reheating and perturbative production of dark matter

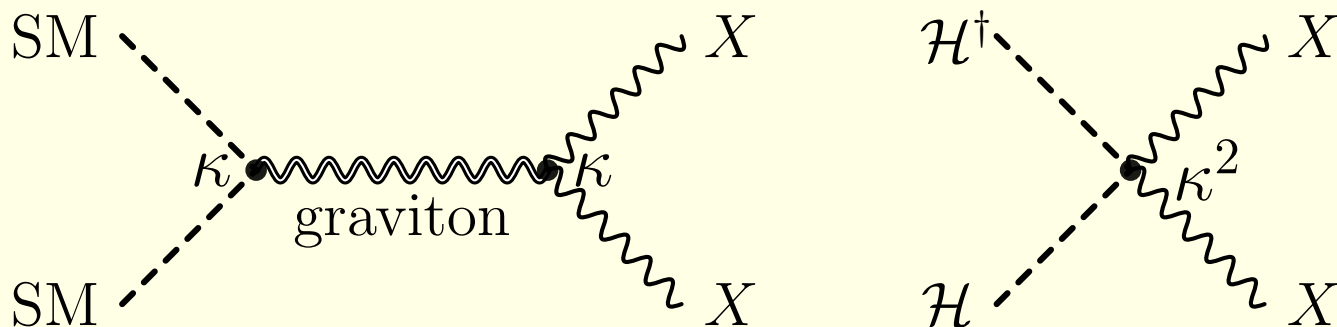
$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{int}} \right] = \int d^4x \mathcal{L}_{\text{eff}},$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}^{(4)} + \mathcal{L}_{\text{DM}}^{(4)} + \mathcal{L}_{\text{int}}^{(5)} + \mathcal{L}_{\text{int}}^{(6)} + \mathcal{O}(\kappa^3) \quad \text{for} \quad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\mathcal{L}_{\text{DM}}^{(4)} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu,$$

$$\mathcal{L}_{\text{int}}^{(5)} = \frac{\kappa}{2} h^{\mu\nu} \left[ T_{\mu\nu}^{\text{SM}} + T_{\mu\nu}^{\text{DM}} \right],$$

$$\mathcal{L}_{\text{int}}^{(6)} = \frac{\kappa^2}{2} C_X m_X^2 |\mathcal{H}|^2 X_\mu X^\mu \quad \leftarrow \quad C_X \kappa^2 (D^\mu \Phi)^* (D_\mu \Phi) \mathcal{H}^\dagger \mathcal{H}$$





Amplitudes for the annihilation of SM particles (with spin  $i=0, 1/2, 1$  before SSB, i.e.  $m_i=0$ ) to VDM squared and summed over all spins read

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}_{0 \rightarrow 1}|^2 &= \frac{\kappa^4}{16s^2} \left[ 3(m_X^2 - t)^2 (m_X^2 - s - t)^2 + 4C_X^2 s^2 (12m_X^4 - 4m_X^2 s + s^2) \right. \\ &\quad \left. + 4C_X s (6m_X^6 - 5m_X^4 s - 12m_X^4 t + m_X^2 s^2 + 4m_X^2 st + 6m_X^2 t^2 + s^2 t + st^2) \right], \\ \sum_{\text{spins}} |\mathcal{M}_{1/2 \rightarrow 1}|^2 &= -\frac{\kappa^4}{32s^2} \left[ 12m_X^8 - 12m_X^6 (s + 4t) + m_X^4 (5s^2 + 48st + 72t^2) \right. \\ &\quad \left. - 2m_X^2 (2s^3 + 11s^2 t + 30st^2 + 24t^3) + t(s + t) (5s^2 + 12st + 12t^2) \right], \\ \sum_{\text{spins}} |\mathcal{M}_{1 \rightarrow 1}|^2 &= \frac{\kappa^4}{8s^2} \left[ (m_X^4 - 2m_X^2 t + s^2 + st + t^2) (3m_X^4 - 6m_X^2 t + s^2 + 3st + 3t^2) \right], \end{aligned}$$

The freeze-in:

$$\sigma(s)_{1 \rightarrow 1} = \frac{\kappa^4}{7680\pi\sqrt{s}\sqrt{s-4m_X^2}} \left[ 6[40\mathcal{C}_X(6\mathcal{C}_X - 1) + 3]m_X^4 + \right. \\ \left. 2[40\mathcal{C}_X(1 - 6\mathcal{C}_X) + 3]m_X^2s + [20\mathcal{C}_X(6\mathcal{C}_X - 1) + 3]s^2 \right],$$
$$\sigma(s)_{1/2 \rightarrow 1} = \frac{\kappa^4 (48m_X^4 + 56m_X^2s + 13s^2)}{15360\pi\sqrt{s}(s - 4m_X^2)},$$
$$\sigma(s)_{1 \rightarrow 1} = \frac{\kappa^4 (48m_X^4 + 56m_X^2s + 13s^2)}{3840\pi\sqrt{s}\sqrt{s - 4m_X^2}}.$$

The corresponding thermally averaged cross sections:

$$\begin{aligned} \langle \sigma v \rangle_{1 \rightarrow 1} = & \frac{\kappa^4}{23040\pi K_2^2(m_X/T)} \left\{ K_2^2 \left( \frac{m_X}{T} \right) [6(m_X^2 + 4T^2) + 160T^2 \mathcal{C}_X (6\mathcal{C}_X - 1)] + \right. \\ & + m_X K_1 \left( \frac{m_X}{T} \right) \left[ m_X (40(6\mathcal{C}_X - 1)\mathcal{C}_X + 9) K_1 \left( \frac{m_X}{T} \right) + \right. \\ & \left. \left. + 4T(20(6\mathcal{C}_X - 1)\mathcal{C}_X + 3) K_2 \left( \frac{m_X}{T} \right) \right] \right\} \end{aligned}$$

$$\begin{aligned} \langle \sigma v \rangle_{1/2 \rightarrow 1} = & \frac{\kappa^4}{11520\pi K_2^2(m_X/T)} \left\{ K_2^2 \left( \frac{m_X}{T} \right) [9m_X^2 + 26T^2] + \right. \\ & \left. + m_X K_1 \left( \frac{m_X}{T} \right) \left[ 11m_X + 13TK_2 \left( \frac{m_X}{T} \right) \right] \right\} \end{aligned}$$

$$\begin{aligned} \langle \sigma v \rangle_{1 \rightarrow 1} = & \frac{\kappa^4}{2880\pi K_2^2(m_X/T)} \left\{ K_2^2 \left( \frac{m_X}{T} \right) [9m_X^2 + 26T^2] + \right. \\ & \left. + m_X K_1 \left( \frac{m_X}{T} \right) \left[ 11m_X + 13TK_2 \left( \frac{m_X}{T} \right) \right] \right\} = 4 \langle \sigma v \rangle_{1/2 \rightarrow 0} \end{aligned}$$

The total cross-section for the vector DM production can be written as a sum of those three contributions from all the SM particles as,

$$\langle\sigma v\rangle = N_0 \langle\sigma v\rangle_{0\rightarrow 1} + N_{1/2} \langle\sigma v\rangle_{1/2\rightarrow 1} + N_1 \langle\sigma v\rangle_{1\rightarrow 1},$$

where  $N_0 = 4$ ,  $N_{1/2} = 45$ , and  $N_1 = 12$  denote numbers of degrees of freedom (before SSB).

### Inflaton decay to DM

$$\mathcal{L}_{\text{int}}^\phi = -\mathcal{C}_{\phi X} \kappa \frac{m_X^2}{2} \phi X_\mu X^\mu \quad \leftarrow \quad \mathcal{C}_{\phi X} \kappa (D^\mu \Phi)^* (D_\mu \Phi) \phi$$

⇓

$$\Gamma_{\phi \rightarrow XX} = \frac{\kappa^2 \mathcal{C}_{\phi X}^2}{128\pi m_\phi} \sqrt{1 - 4 \frac{m_X^2}{m_\phi^2}} (m_\phi^4 - 4m_X^2 m_\phi^2 + 12m_X^4),$$

The time evolution of the energy density  $\rho_i$  and number density  $n_i$  of each considered species is encoded in the following system of three coupled Boltzmann equations

$$\begin{aligned}\dot{\rho}_\phi + 3(1+w)H\rho_\phi &= -\left(\langle\Gamma_R^\phi\rangle + \langle\Gamma_X^\phi\rangle\right)\rho_\phi, \\ \dot{\rho}_R + 4H\rho_R &= \langle\Gamma_R^\phi\rangle\rho_\phi + 2\langle E_X\rangle\langle\sigma|v|\rangle\left(n_X^2 - \bar{n}_X^2\right), \\ \dot{n}_X + 3Hn_X &= \langle\Gamma_X^\phi\rangle\frac{\rho_\phi}{m_\phi} - \langle\sigma|v|\rangle\left(n_X^2 - \bar{n}_X^2\right), \\ H^2 &= \frac{\kappa^2}{3}\left(\rho_\phi + \rho_X + \rho_R\right),\end{aligned}$$

where  $\bar{n}_X$  denotes the equilibrium number density of the vector DM  $X_\mu$ , while  $w$  parametrizes the equation of state of the inflaton field.

We introduce rescaled dimensionless variables to solve the above Boltzmann equations,

$$\Phi = \rho_\phi \frac{a^{3(1+w)}}{T_{\text{RH}}^4}, \quad \mathcal{R} = \rho_R \frac{a^4}{T_{\text{RH}}^4}, \quad \mathcal{N}_X = n_X \frac{a^3}{T_{\text{RH}}^3}.$$

It is convenient to use the scale factor, rather than time:

$$\frac{da}{aH} = dt.$$

The Hubble parameter expressed in terms of the new variables is given by

$$H^2 = \frac{\kappa^2 T_{\text{RH}}^4}{3 a^3} \left[ \Phi a^{-3w} + \mathcal{R} a^{-1} + \mathcal{N}_X \frac{\langle E_X \rangle}{T_{\text{RH}}} \right].$$

We can also rewrite Boltzmann equations as follows

$$\begin{aligned} \frac{d\Phi}{da} &= -\frac{\langle \Gamma_R^\phi \rangle + \langle \Gamma_X^\phi \rangle}{aH} \Phi, \\ \frac{d\mathcal{R}}{da} &= \frac{\langle \Gamma_R^\phi \rangle}{H} \Phi a^{-3w} + \frac{\langle \sigma v \rangle}{H a^3} 2 \langle E_X \rangle T_{\text{RH}}^2 (\mathcal{N}_X^2 - \bar{\mathcal{N}}_X^2) \\ \frac{d\mathcal{N}_X}{da} &= \frac{\langle \Gamma_X^\phi \rangle T_{\text{RH}}}{H m_\phi} \Phi a^{-(1+3w)} - \frac{\langle \sigma v \rangle}{H a^4} (\mathcal{N}_X^2 - \bar{\mathcal{N}}_X^2). \end{aligned} \tag{5}$$

We adopt the following assumptions:

- (i) For the inflaton decay, the dominant decay channel is  $\phi \rightarrow RR$ , while  $\phi \rightarrow XX$  is sub-dominant, i.e.  $\Gamma_R^\phi \gg \Gamma_X^\phi$ , so that the standard cosmology is not affected substantially,
- (ii) During the reheating, i.e. period between  $H_I^{-1}$  and  $\Gamma_R^\phi^{-1}$  the dominant part of the total energy density was in the form of the inflaton field.
- (iii) The reheating period is followed by the RD epoch during which the total energy density is dominated by the  $\rho_R$ .

The second assumption allows us to fix the initial conditions for the set of new variables,

$$\Phi_I = \frac{3H_I^2}{\kappa^2 T_{\text{RH}}^4}, \quad \mathcal{R}_I = 0, \quad \mathcal{N}_{X_I} = 0.$$

## Temperature – scale factor relation

$$\mathcal{R} = \frac{\pi^2 g_*(T_{\text{RH}})}{30\gamma^2} \begin{cases} \frac{2}{5-3w} \left( a^{\frac{5}{2}(1-3w/5)} - 1 \right), & \text{for } w \neq \frac{5}{3} \\ \ln a, & \text{for } w = \frac{5}{3} \end{cases}$$

with  $\gamma \equiv \sqrt{\frac{H_{\text{RH}}}{H_I}}$ .

The temperature of the system is related to radiation energy density  $\mathcal{R}$  as follows

$$T = \left( \frac{30}{\pi^2 g_*(T)} \right)^{1/4} T_{\text{RH}} \mathcal{R}^{1/4} a^{-1}.$$

Therefore, we can express the temperature in terms of the scale factor as,

$$T(a) = \left( \frac{90}{\pi^2 g_*(T)} \right)^{1/4} \sqrt{\frac{\gamma H_I}{\kappa}} \begin{cases} \left( \frac{2}{5-3w} \right)^{1/4} \left( a^{-\frac{3}{2}(1+w)} - a^{-4} \right)^{1/4}, & \text{for } w \neq \frac{5}{3}, \\ \left( \frac{\ln a}{a^4} \right)^{1/4}, & \text{for } w = \frac{5}{3}. \end{cases}$$



## The Hubble rate – scale factor relation

$$H(a) = \begin{cases} H_I a^{-\frac{3}{2}(1+w)}, & \text{for } a < a_{\text{RH}} \\ H_{\text{RH}} \left(\frac{a_{\text{RH}}}{a}\right)^2, & \text{for } a > a_{\text{RH}} \end{cases}$$

## DM relic abundance

- (i) the DM production dominated by annihilation of SM particles in the thermal bath, i.e. via annihilation freeze-in mechanism,
- (ii) the DM mostly produced through inflaton decays.

(i)

$$\frac{d\mathcal{N}_X^{\text{FI}}}{da} \simeq \frac{\langle\sigma|v|\rangle}{Ha^4} \bar{\mathcal{N}}_X^{\text{FI}},$$
$$\mathcal{N}_{X\infty}^{\text{FI}} \simeq \frac{1}{T_{\text{RH}}^3} \int_1^{a_{\text{RH}}} da \frac{a^2}{H(a)} \langle\sigma v\rangle \bar{n}_X^2 + \frac{1}{T_{\text{RH}}^3} \int_{a_{\text{RH}}}^{a_f} da \frac{a^2}{H(a)} \langle\sigma v\rangle \bar{n}_X^2,$$

(ii)

$$\frac{d\mathcal{N}_X^\phi}{da} = 3 \frac{\langle \Gamma_X^\phi \rangle H_I}{\kappa^2 m_\phi} T_{\text{RH}}^{-3} a^{\frac{1}{2}(1-3w)},$$

$$\begin{aligned} \mathcal{N}_{X_{\text{RH}}}^\phi &= \frac{\mathcal{C}_{\phi X}^2}{64\pi(1-w)} \left( \frac{\pi^2 g_*(T_{\text{RH}})}{90} \right)^{3/4} \sqrt{1 - 4 \frac{m_X^2}{m_\phi^2} (m_\phi^4 - 4m_X^2 m_\phi^2 + 12m_X^4)} \\ &\quad \times \frac{\kappa^{3/2}}{\gamma^3 H_I^{1/2} m_\phi^2} \left( a_{\text{RH}}^{\frac{3}{2}(1-w)} - 1 \right). \end{aligned}$$

⇓

$$\Omega_X h^2 = \frac{45}{2\pi^2 g_*(T_{\text{RH}})} \frac{s_0 h^2}{\rho_c} \gamma^{\frac{4}{1+w}} m_X \mathcal{N}_X(T_0).$$

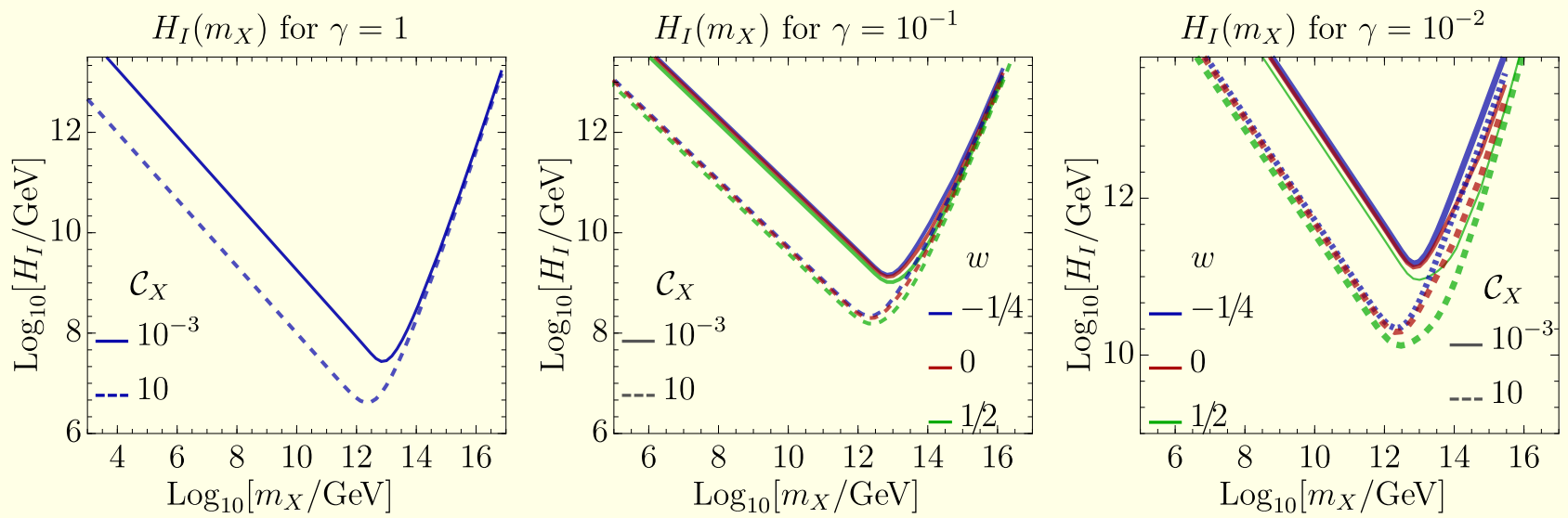


Figure 5: Relations between the Hubble rate at the end of inflation  $H_I$  and vector DM mass  $m_X$  that reproduces the observed relic abundance  $\Omega_{\text{DM}}h^2$  for the freeze-in through the dim-6 operator.

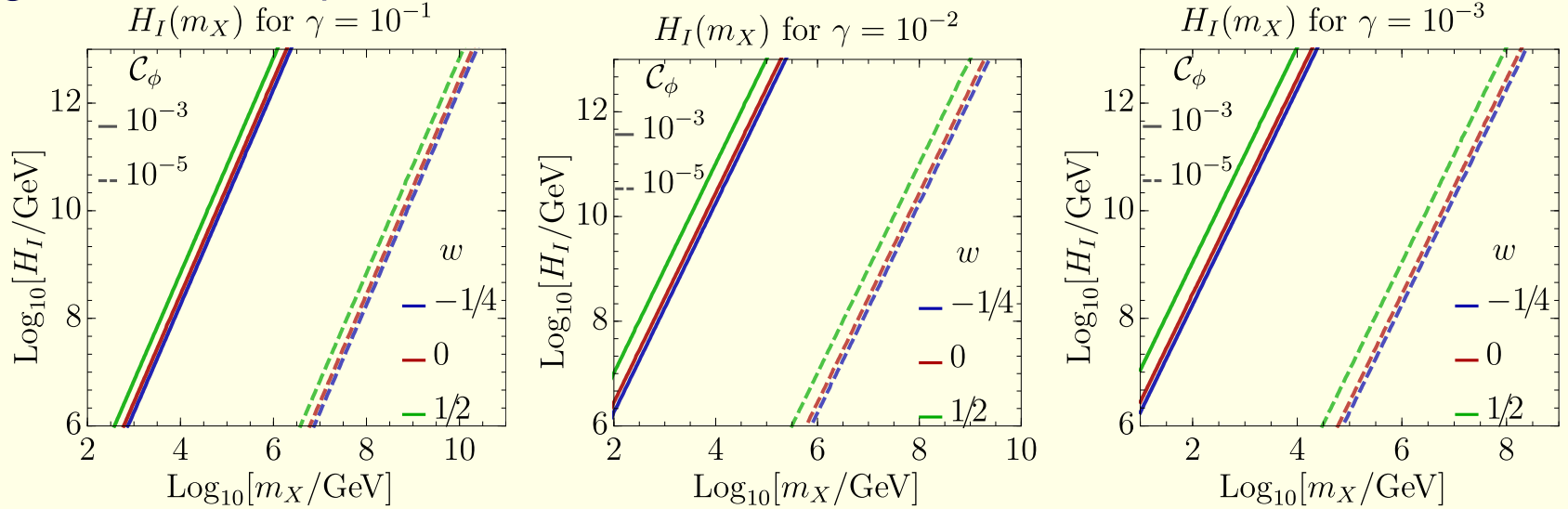


Figure 6: Relations between the Hubble rate at the end of inflation  $H_I$  and vector DM mass  $m_X$  that reproduces the observed relic abundance  $\Omega_{\text{DM}}h^2$  for the inflaton decay through the dim-5 operator.

## Summary

- Evidence for DM is only of gravitational origin.
  - DM that interacts with the SM exclusively through gravity is a viable option consistent with the observed DM abundance.
  - DM production mechanisms:
    - non-perturbative - gravitational production,
    - perturbative - freeze-in via graviton exchange or dim-6 Planck mass suppressed effective operator, inflaton decays to DM pairs via dim-5 Planck mass suppressed effective operator.
- have been discussed.
- Effects of modified equation of state during reheating have been discussed.

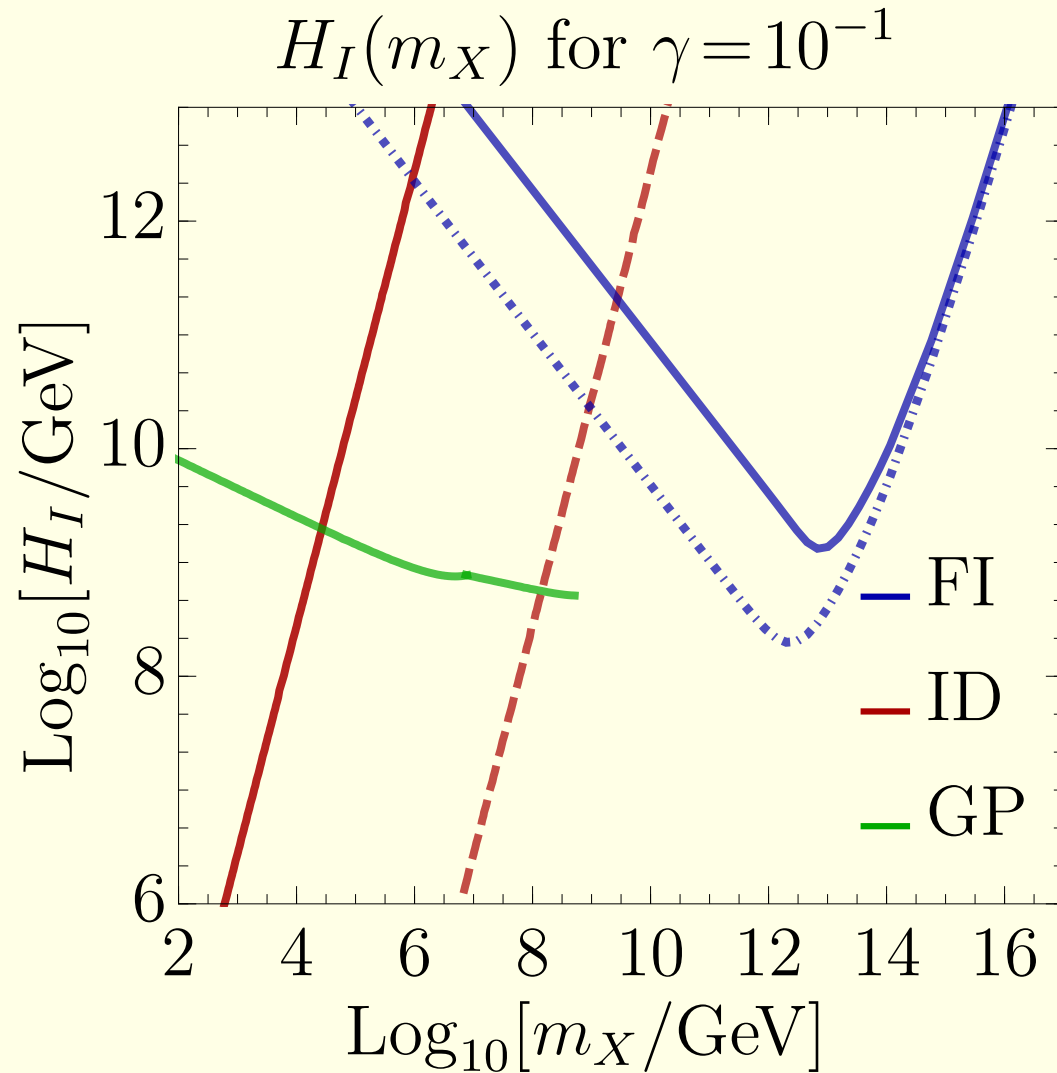


Figure 7: Comparison of the gravitational production, the inflaton decay and dim-6 operator annihilation. The observed relic abundance  $\Omega_{\text{DM}}h^2$  is reproduced. For solid and dashed lines we used  $\mathcal{C}_X = 10^{-3}, 10$  and  $\mathcal{C}_\phi = 10^{-3}, 10^{-5}$ , respectively.