

# Towards simple natural extensions of the Standard Model

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- B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068.
- B.G., O.M. Ogreid, P. Osland, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80:055013,2009.
- B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009.

## The little hierarchy problem

$$m_h^2 = m_h^{(B)2} + \delta^{(SM)} m_h^2 + \dots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2} \left[ \frac{3}{2} m_t^2 - \frac{1}{8} (6m_W^2 + 3m_Z^2) - \frac{3}{8} m_h^2 \right]$$

$$m_h = 130 \text{ GeV} \Rightarrow \delta^{(SM)} m_h^2 \simeq m_h^2 \quad \text{for} \quad \Lambda \simeq 600 \text{ GeV}$$

- For  $\Lambda \gtrsim 600 \text{ GeV}$  there must be a cancellation between the tree-level Higgs mass<sup>2</sup>  $m_h^{(B)2}$  and the 1-loop leading correction  $\delta^{(SM)} m_h^2$ :

$$m_h^{(B)2} \sim \delta^{(SM)} m_h^2 > m_h^2$$

⇓

the perturbative expansion is breaking down.

- The SM cutoff is very low!

Solutions to the little hierarchy problem:

♠ **Suppression of corrections growing with  $\Lambda^2$  at the 1-loop level:**

- The Veltman condition, no  $\Lambda^2$  terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}(6m_W^2 + 3m_Z^2) - \frac{3}{8}m_h^2 = 0 \quad \Longrightarrow \quad m_h \simeq 310 \text{ GeV}$$

- SUSY:

$$\delta^{(SUSY)}m_h^2 \sim m_{\tilde{t}}^2 \frac{3g_t^2}{8\pi^2} \ln \left( \frac{\Lambda^2}{m_{\tilde{t}}^2} \right)$$

then for  $\Lambda \sim 10^{16-18}$  GeV one gets  $m_{\tilde{t}}^2 \lesssim 1 \text{ TeV}^2$  in order to have  $\delta^{(SUSY)}m_h^2 \sim m_h^2$ .

♠ **Increase of the allowed value of  $m_h$ :**

- The inert Higgs model by Barbieri, Hall, Rychkov, Phys.Rev.D74:015007,2006, (Ma, Phys.Rev.D73:077301,2006)  $\Rightarrow m_h \sim 400 - 600 \text{ GeV}$ , ( $\ln m_h$  terms in  $T$  parameter canceled by  $m_{H^\pm}, m_A, m_S$  contributions).

## The Strategy

- The SM 1-loop quadratic divergences are dominated by the top (fermionic) contribution, so to suppress them we are going to introduce extra scalars (as the SM Higgs would need to be too heavy to do the job).
- We will look for a model which allows for relatively heavy lightest Higgs boson (in order to suppress  $\delta M_i^2/M_i^2$  even more). Note also that within the SM fit to the precision data there is a tension caused by the lightness of the Higgs.
- DM candidate is mandatory.
- CPV will be desirable.

## Natural Models

♠ Less divergence + DM  $\Rightarrow$  SM +  $N_\varphi$  scalar singlets

B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009.

- $N_\varphi$  extra gauge singlets  $\varphi_i$  with  $\langle \varphi_i \rangle = 0$ ,
- Symmetries  $\mathbb{Z}_2^{(i)}$ :  $\varphi_i \rightarrow -\varphi_i$  (to eliminate  $|H|^2\varphi_i$  couplings).

$$V(H, \varphi_i) = -\mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_\varphi^2 \sum_{i=1}^{N_\varphi} \varphi_i^2 + \frac{\lambda_\varphi}{24} \sum_{i=1}^{N_\varphi} \varphi_i^4 + \lambda_x |H|^2 \sum_{i=1}^{N_\varphi} \varphi_i^2$$

with  $O(N_\varphi)$  symmetry imposed

$$\langle H \rangle = \frac{v}{\sqrt{2}}, \quad \langle \varphi_i \rangle = 0 \quad \text{for} \quad \mu_\varphi^2 > 0$$

then  $m_h^2 = 2\mu_H^2$  and  $m^2 = 2\mu_\varphi^2 + \lambda_x v^2$

$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2} \left[ \Lambda^2 - m^2 \ln \left( 1 + \frac{\Lambda^2}{m^2} \right) \right]$$

$$|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2| = D_t m_h^2$$

$$\Downarrow$$

$$\lambda_x = \lambda_x(m, m_h, D_t, \Lambda, N_\varphi)$$

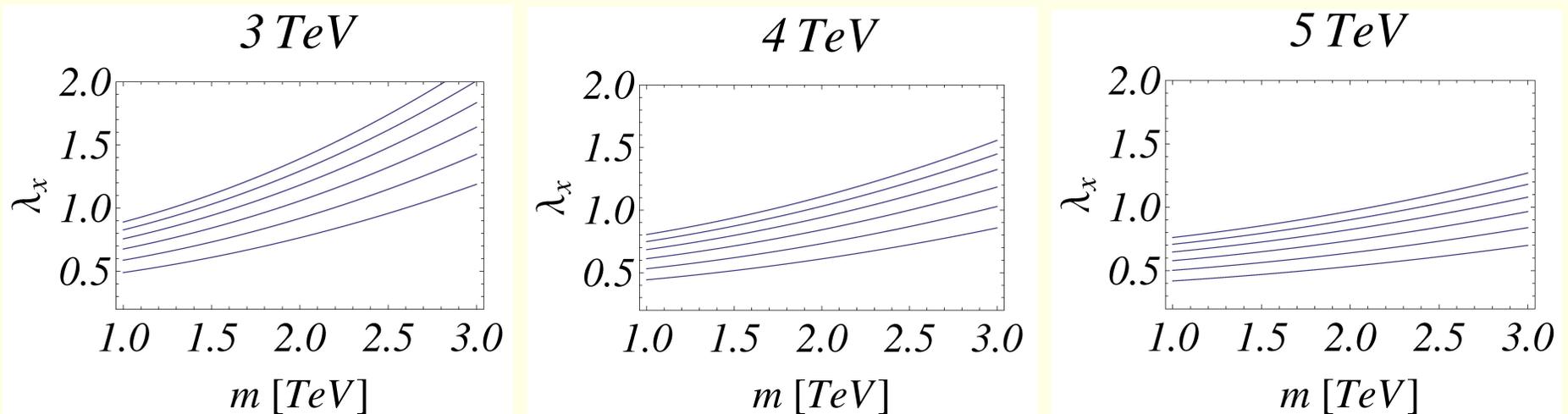


Figure 1: Plots of  $\lambda_x$  as a function of  $m$  for  $N_\varphi = 6$ ,  $D_t = 0$  and various choices of  $\Lambda = 3, 4, 5$  shown above each panel. The curves correspond to  $m_h = 130, 150, 170, 190, 210, 230$  GeV (starting with the uppermost curve).

## Stability of the fine tuning

$$\begin{aligned}\delta^{(SM)} m_h^2 &= \frac{\Lambda^2}{16\pi^2} \left( 12g_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2 - 12\lambda_H \right) \\ \delta^{(\varphi)} m_h^2 &= -N_\varphi \frac{\lambda_x}{8\pi^2} \left[ \Lambda^2 - m^2 \ln \left( 1 + \frac{\Lambda^2}{m^2} \right) \right]\end{aligned}$$

In general

$$\delta m_h^2 = \underbrace{\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2}_{\simeq 0} + 2\Lambda^2 \sum_{n=1}^{\infty} f_n(\lambda, \dots) \ln^n \left( \frac{\Lambda}{\mu} \right)$$

where (Jack & Jones, 1990)

$$(n+1)f_{n+1} = \mu \frac{\partial}{\partial \mu} f_n = \beta_i \frac{\partial}{\partial \lambda_i} f_n \quad \text{and} \quad f_n \propto \left( \frac{N_\varphi \lambda_x}{16\pi^2} \right)^{n+1}$$

From 1-loop condition ( $n = 0$ )

$$\delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 \simeq 0$$

we have

$$\lambda_x = \frac{1}{N_\varphi} \left\{ 4.8 - 3 \left( \frac{m_h}{v} \right)^2 + 2D_t \left[ \frac{2\pi}{\Lambda/\text{TeV}} \right]^2 \right\} \left[ 1 - \frac{m^2}{\Lambda^2} \ln \left( \frac{m^2}{\Lambda^2} \right) \right] + \mathcal{O} \left( \frac{m^4}{\Lambda^4} \right).$$

Therefore at the 2-loop ( $n = 1$ )

$$D_t \equiv \frac{\delta m_h^2}{m_h^2} \simeq \left( \frac{N_\varphi \lambda_x}{16\pi^2} \right)^2 \frac{\Lambda^2}{m_h^2} \ln \left( \frac{\Lambda}{m_h} \right) \simeq \left( \frac{4}{16\pi^2} \right)^2 \frac{\Lambda^2}{m_h^2} \ln \left( \frac{\Lambda}{m_h} \right)$$

for  $D_t \lesssim 1$

$$\Lambda \lesssim 3 - 5 \text{ TeV} \quad \text{for} \quad m_h = 130 - 230 \text{ GeV}$$

## Singlet DM

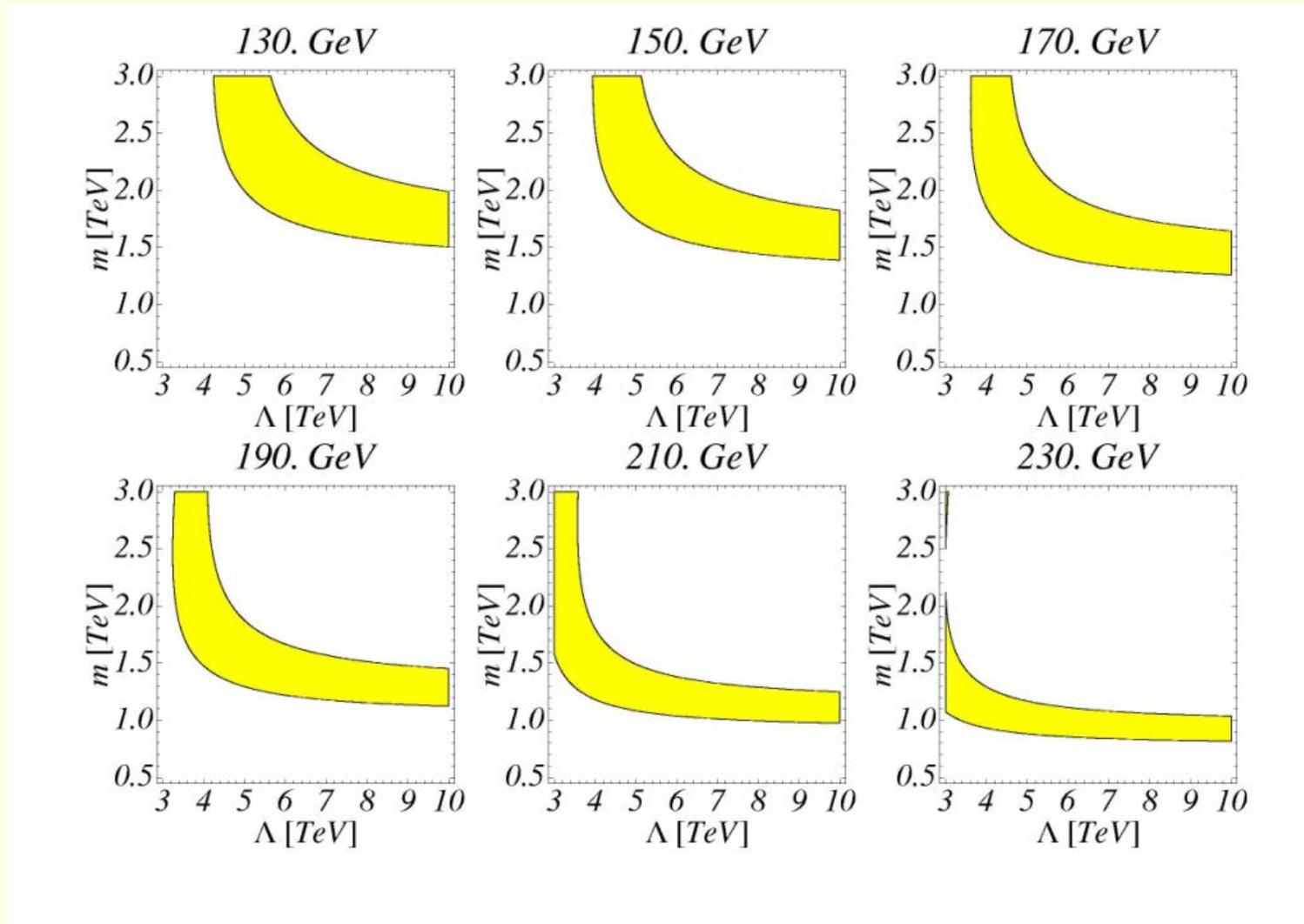


Figure 2: Allowed regions in the space  $(m, \Lambda)$  are plotted for  $D_t(m) = 0$ ,  $N_\varphi = 6$  and assuming that each  $\varphi_i$  contributes the same to the total  $\Omega_{DM}$  at the  $3\sigma$  level:  $\Omega_\varphi h^2 = 0.106 \pm 0.008$  for the Higgs mass shown above each panel;  $m_h = 130, 150, 170, 190, 210, 230$  GeV.

- ♠ **Less divergence + DM + CPV**  $\Rightarrow$   $\begin{cases} 2\text{HDM (CPV)} + \text{Inert singlet (DM)} \\ 2\text{HDM (CPV)} + \text{Inert doublet (DM)} \end{cases}$
- The Inert Doublet Model with no quadratic divergences

$$\mathbb{Z}_2 : \quad \phi_2 \rightarrow -\phi_2$$

$$V(\phi_1, \phi_2) = -\frac{1}{2}m_{11}^2\phi_1^\dagger\phi_1 - \frac{1}{2}m_{22}^2\phi_2^\dagger\phi_2 + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\ + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2} \left[ \lambda_5(\phi_1^\dagger\phi_2)^2 + \text{H.c.} \right].$$

$$m_{22}^2 < 0 \quad \text{and} \quad m_{11}^2 > 0 \quad \Rightarrow \quad \langle \phi_1 \rangle = v/\sqrt{2} \quad \text{and} \quad \langle \phi_2 \rangle = 0$$

Cancellation of quadratic divergences for  $\phi_1$  and  $\phi_2$  (Newton & Wu, 1994):

$$\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2} \left( \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4 \right) = 3m_t^2,$$

$$\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2} \left( \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4 \right) = 0.$$

## Comments on the inert 2HDM:

- Motivations:
  - To allow for heavy SM-like Higgs boson in order to weaken the little hierarchy problem,
  - To provide a candidate for DM.
- It turns out that for the inert 2HDM there is no CPV in the potential (as implied by the exact  $\mathbb{Z}_2$ ).
- Also, the vacuum stability conditions turn out to be inconsistent with the cancellation of quadratic divergences for realistic top mass.



- Allow for  $m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.}$  (CPV),
- Allow for  $\langle \phi_2 \rangle \neq 0$ ,
- The price: no DM candidate!

- The Non-Inert Doublet Model with no quadratic divergences

B.G., P. Osland, "A Natural Two-Higgs-Doublet Model", e-Print: arXiv:0910.4068

The cancellation of quadratic divergences

$$\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2} \left( \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4 \right) = 3\frac{m_b^2}{c_\beta^2},$$
$$\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2} \left( \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4 \right) = 3\frac{m_t^2}{s_\beta^2},$$

where  $v^2 \equiv v_1^2 + v_2^2$ ,  $\tan \beta \equiv v_2/v_1$



For a given choice of the mixing angles  $\alpha_i$ 's ( $i = 1, 2, 3$ ), the neutral-Higgs masses  $M_1^2$ ,  $M_2^2$  and  $M_3^2$  can be determined from the cancellation conditions in terms of  $\tan \beta$ ,  $\mu^2 \equiv \mathbf{Re}(m_{12}^2)/(2s_\beta c_\beta)$  and  $M_{H^\pm}^2$ .

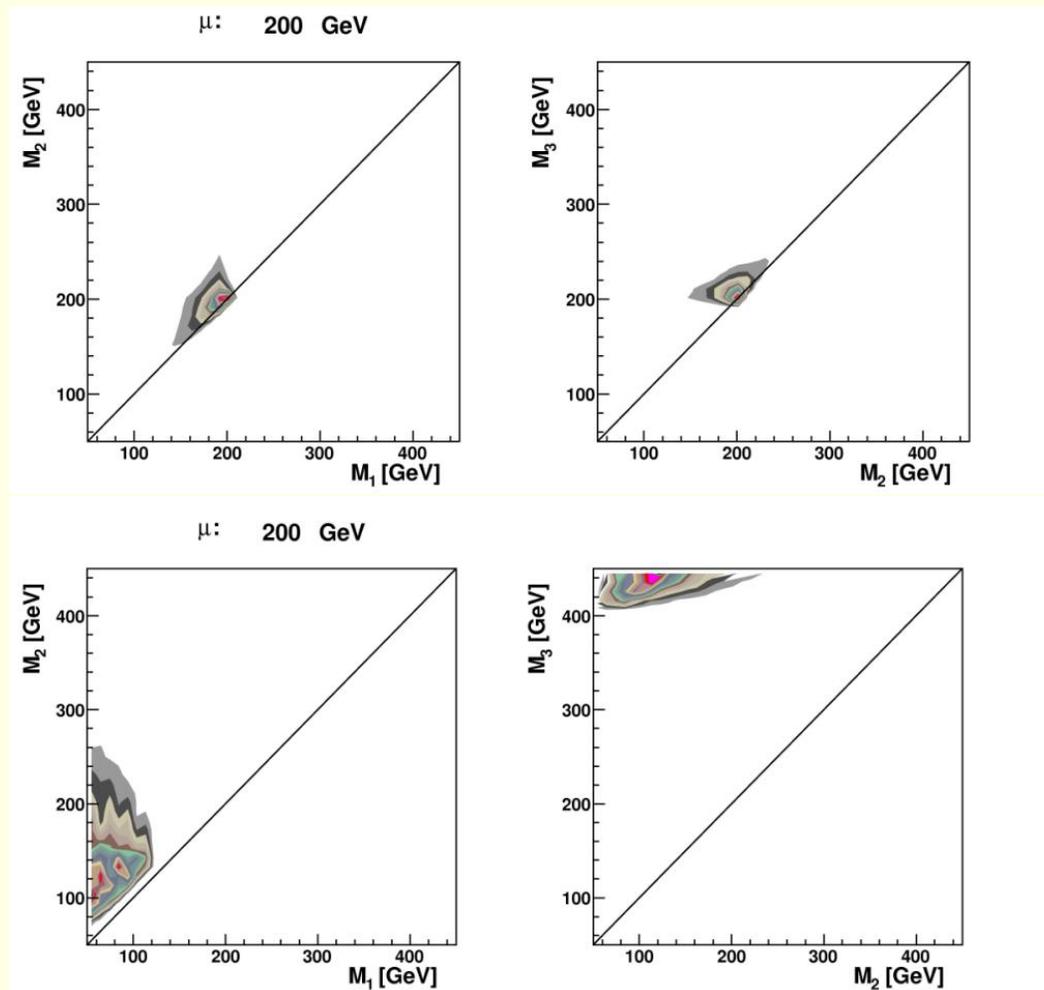


Figure 3: Distributions of allowed masses  $M_2$  vs  $M_1$  (left panels) and  $M_3$  vs  $M_2$  (right), resulting from a scan over the full range of  $\alpha_i$ ,  $\tan\beta \in (0.5, 50)$  and  $M_{H^\pm} \in (300, 700)$  GeV, for  $\mu = 200$  GeV. No constraints are imposed other than the cancellation of quadratic divergences,  $M_i^2 > 0$  and  $M_1 < M_2 < M_3$ . Two ranges of  $\tan\beta$ -values are displayed: bottom panels:  $0.5 \leq \tan\beta \leq 1$ , top panels:  $40 \leq \tan\beta \leq 50$ . The color coding indicates increasing density of allowed points as one moves inward from the boundary.

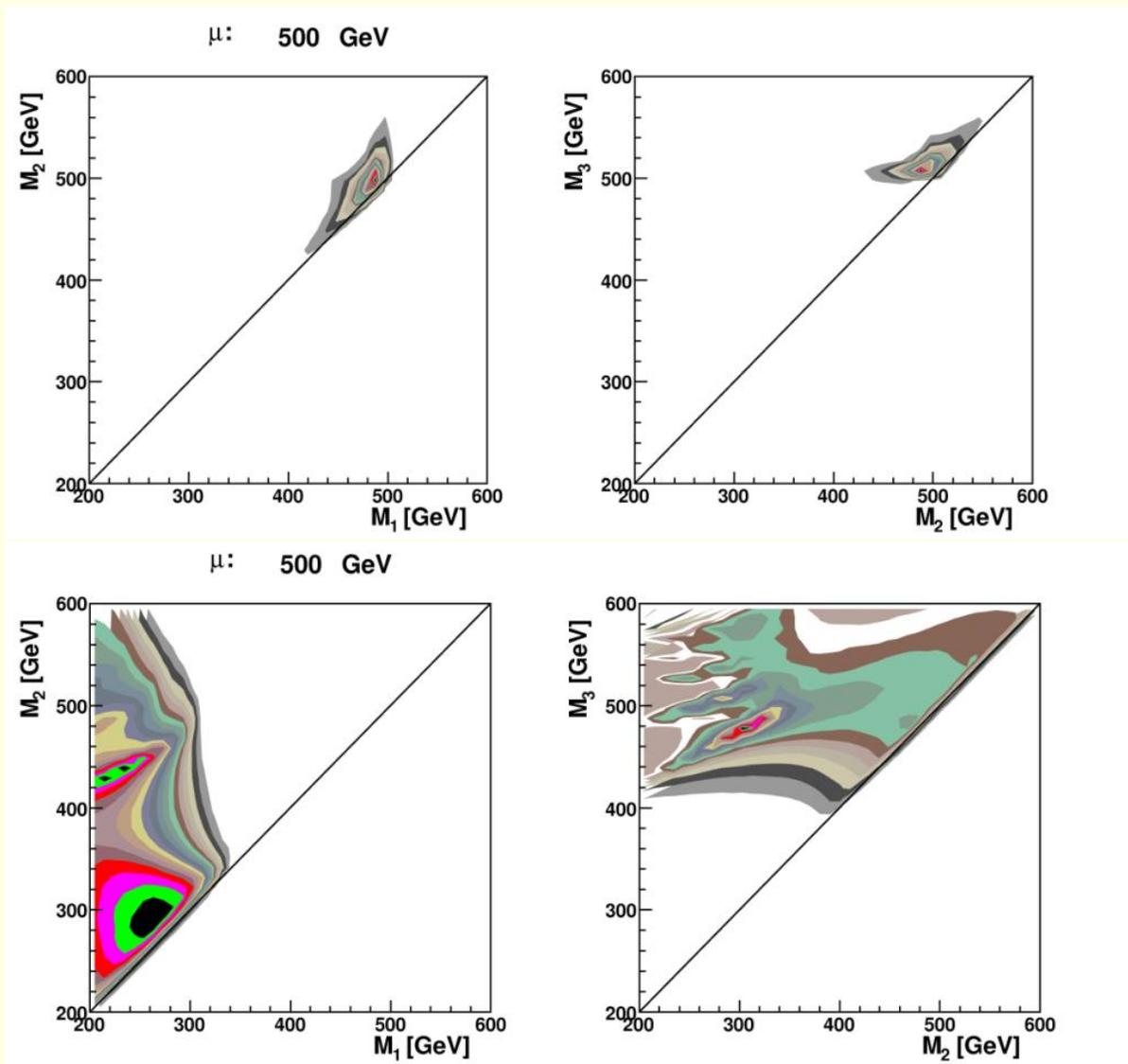


Figure 4: Similar to Fig. 3, for  $\mu = 500$  GeV.

$$\begin{aligned}
M_1^2 - M_2^2 &= \frac{1}{\tan \beta} \frac{R_{33}}{R_{12}R_{22}} [-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2] + \mathcal{O}\left(\frac{1}{\tan^2 \beta}\right) \\
M_3^2 &= -\frac{M_1^2 R_{12}R_{13} + M_2^2 R_{22}R_{23}}{R_{32}R_{33}} + \mathcal{O}\left(\frac{1}{\tan \beta}\right).
\end{aligned}$$

where  $R_{ij}$  are elements of the orthogonal rotation matrix for the neutral scalars and  $\bar{m}^2 \equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2$ .

↓

$$\tan \beta \gtrsim 40 \quad \Longrightarrow \quad M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$$

## Advantages:

- No 1-loop quadratic divergences (so,  $\delta M_i^2/M_i^2$  suppressed),
- Large  $H_1$  mass allowed (so,  $\delta M_i^2/M_i^2$  suppressed),
- A chance for substantial CPV,
- DM candidate easily accommodated by adding singlets  $\varphi_i$ -like.

## The following experimental constraints are imposed:

- The oblique parameters  $T$  and  $S$
- $B_0 - \bar{B}_0$  mixing
- $B \rightarrow X_s \gamma$
- $B \rightarrow \tau \bar{\nu}_\tau X$
- $B \rightarrow D \tau \bar{\nu}_\tau$
- LEP2 Higgs-boson non-discovery
- $R_b$
- The muon anomalous magnetic moment
- Electron electric dipole moment

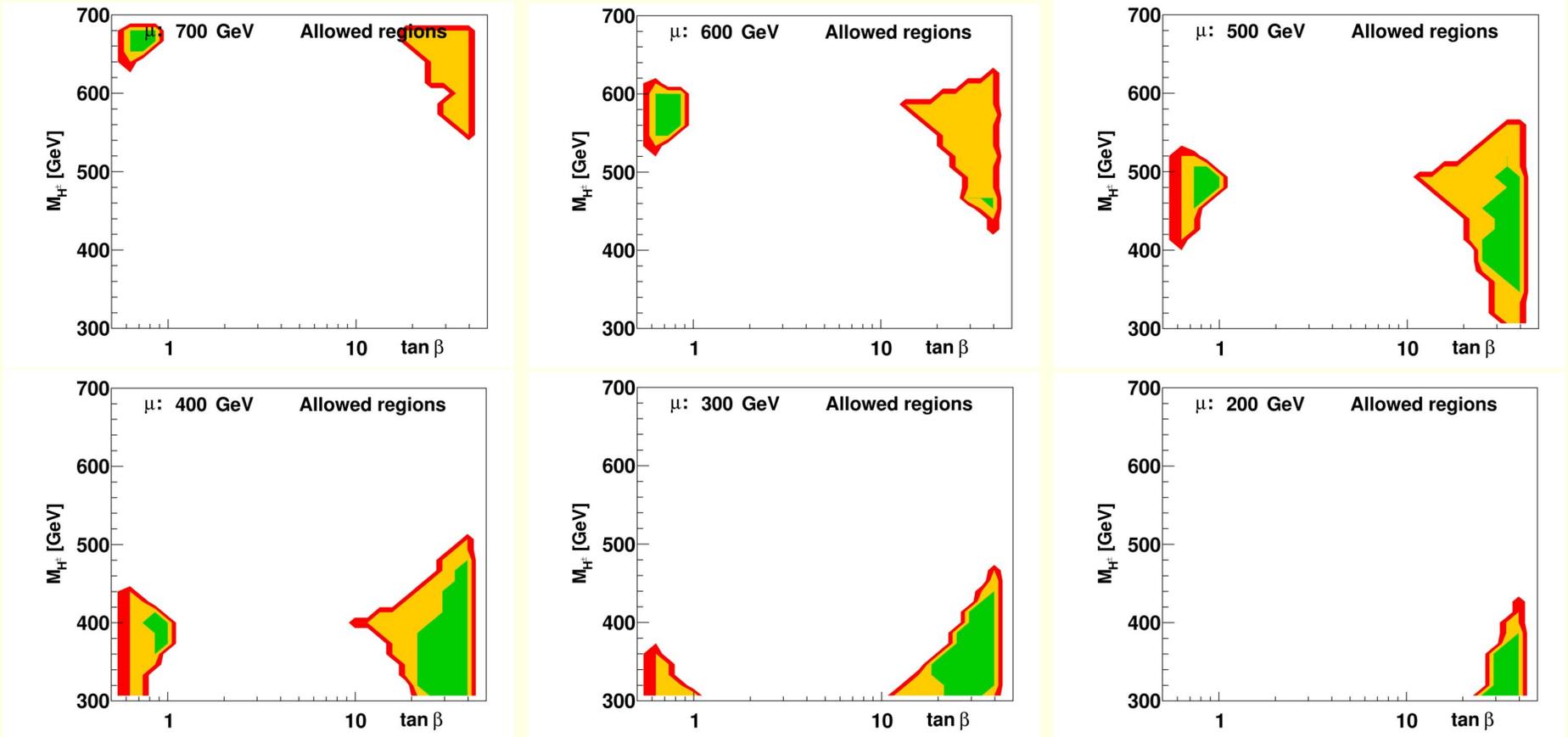


Figure 5: Allowed regions in the  $\tan \beta - M_{H^\pm}$  plane, for  $\mu = 200, 300, 400, 500, 600$  and  $700$  GeV (as indicated). Red: Positivity is satisfied; yellow: positivity and unitarity both satisfied; green: also experimental constraints satisfied at the 95% C.L., as specified in the text.

## Violation of CP

$$\Im J_1 = -\frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) \Im \lambda_5,$$

$$\begin{aligned} \Im J_2 = & -\frac{v_1^2 v_2^2}{v^8} \left[ ((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_1^4 + 2(\lambda_1 - \lambda_2) \Re \lambda_5 v_1^2 v_2^2 \right. \\ & \left. - ((\lambda_2 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_2^4 \right] \Im \lambda_5, \end{aligned}$$

$$\Im J_3 = \frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) (\lambda_1 + \lambda_2 + 2\lambda_4) \Im \lambda_5.$$

For  $\tan \beta \gtrsim 40$

$$\Im J_i \sim \frac{\Im \lambda_5}{\tan^2 \beta}$$

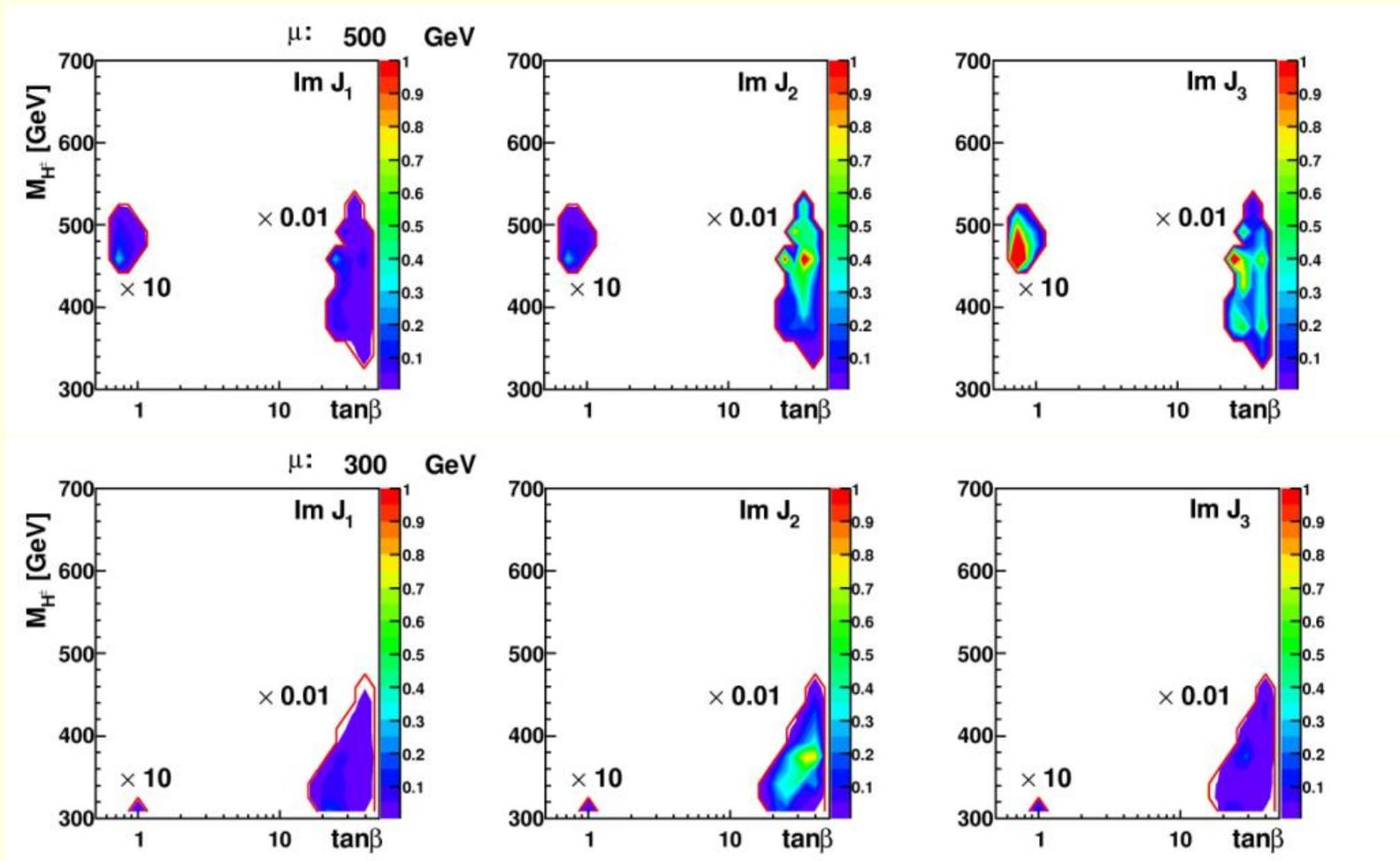


Figure 6: Imaginary parts of the rephasing invariants  $|\Im J_i|$ , for  $\mu = 500$  GeV (top) and  $\mu = 300$  GeV (bottom). The colour coding is given along the right vertical axis. At low  $\tan\beta$  the values should be rescaled by a factor of 10, at high  $\tan\beta$  by a factor 0.01.

## Stability of the cancellation condition

$$\delta M_i^2 = \Lambda^2 \sum_{n=0} f_n^{(i)}(\lambda) \left[ \ln \left( \frac{\Lambda}{v} \right) \right]^n ,$$

The coefficients  $f_n^{(i)}(\lambda)$  can be determined recursively, however here a simple estimate is sufficient:

$$f_n^{(i)}(\lambda) \sim \left( \frac{\lambda}{16\pi^2} \right)^{n+1} \sim \left( \frac{4\pi}{16\pi^2} \right)^{n+1} \sim \left( \frac{1}{4\pi} \right)^{n+1}$$

Requiring that the 2-loop contribution does not exceed  $M_1^2$  one finds:

$$\Lambda^2 \ln \left( \frac{\Lambda}{v} \right) \sim (4\pi M_1)^2$$

Then, e.g. for  $M_1 = 200(500)$  GeV the cutoff is at  $\Lambda \sim 1.8(3.8)$  TeV.

- DM in the Non-Inert Doublet Model with no quadratic divergences

$$\begin{aligned}
 V(\phi_1, \phi_2) = & -\frac{1}{2} \left\{ m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + \left[ m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.} \right] \right\} \\
 & + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1)^2 + \frac{1}{2} \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\
 & + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{1}{2} \left[ \lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{H.c.} \right] \\
 & + \mu_\varphi^2 \varphi^2 + \frac{1}{24} \lambda_\varphi \varphi^4 + \varphi^2 (\eta_1 \phi_1^\dagger \phi_1 + \eta_2 \phi_2^\dagger \phi_2)
 \end{aligned}$$

The cancellation conditions:

$$\begin{aligned}
 \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{v^2}{2} \left( \frac{1}{2} \eta_1 + \frac{3}{2} \lambda_1 + \lambda_3 + \frac{1}{2} \lambda_4 \right) &= 3 \frac{m_b^2}{c_\beta^2}, \\
 \frac{3}{2} m_W^2 + \frac{3}{4} m_Z^2 + \frac{v^2}{2} \left( \frac{1}{2} \eta_2 + \frac{3}{2} \lambda_2 + \lambda_3 + \frac{1}{2} \lambda_4 \right) &= 3 \frac{m_t^2}{s_\beta^2}, \\
 \frac{\lambda_\varphi}{2} + 4(\eta_1 + \eta_2) &= 8 \text{Tr}\{Y_\varphi Y_\varphi^\dagger\}
 \end{aligned}$$

where  $\mathcal{L}_Y = -\varphi \overline{(\nu_R)^c} Y_\varphi \nu_R + \text{H.c.}$ .

$$\mathcal{L} = -\varphi^2(\kappa_i v H_i + \lambda_{ij} H_i H_j + \lambda_{\pm} H^+ H^-)$$

with

$$\kappa_i = \eta_1 R_{i1} c_{\beta} + \eta_2 R_{i2} s_{\beta},$$

$$\lambda_{ij} = \frac{1}{2} \left[ \eta_1 (R_{i1} R_{j1} + s_{\beta}^2 R_{i3} R_{j3}) + \eta_2 (R_{i2} R_{j2} + c_{\beta}^2 R_{i3} R_{j3}) \right],$$

$$\lambda_{\pm} = \eta_1 s_{\beta}^2 + \eta_2 c_{\beta}^2$$

Assumption:  $M_1 \ll M_{2,3}$  so that DM annihilation is dominated by  $H_1$  exchange.

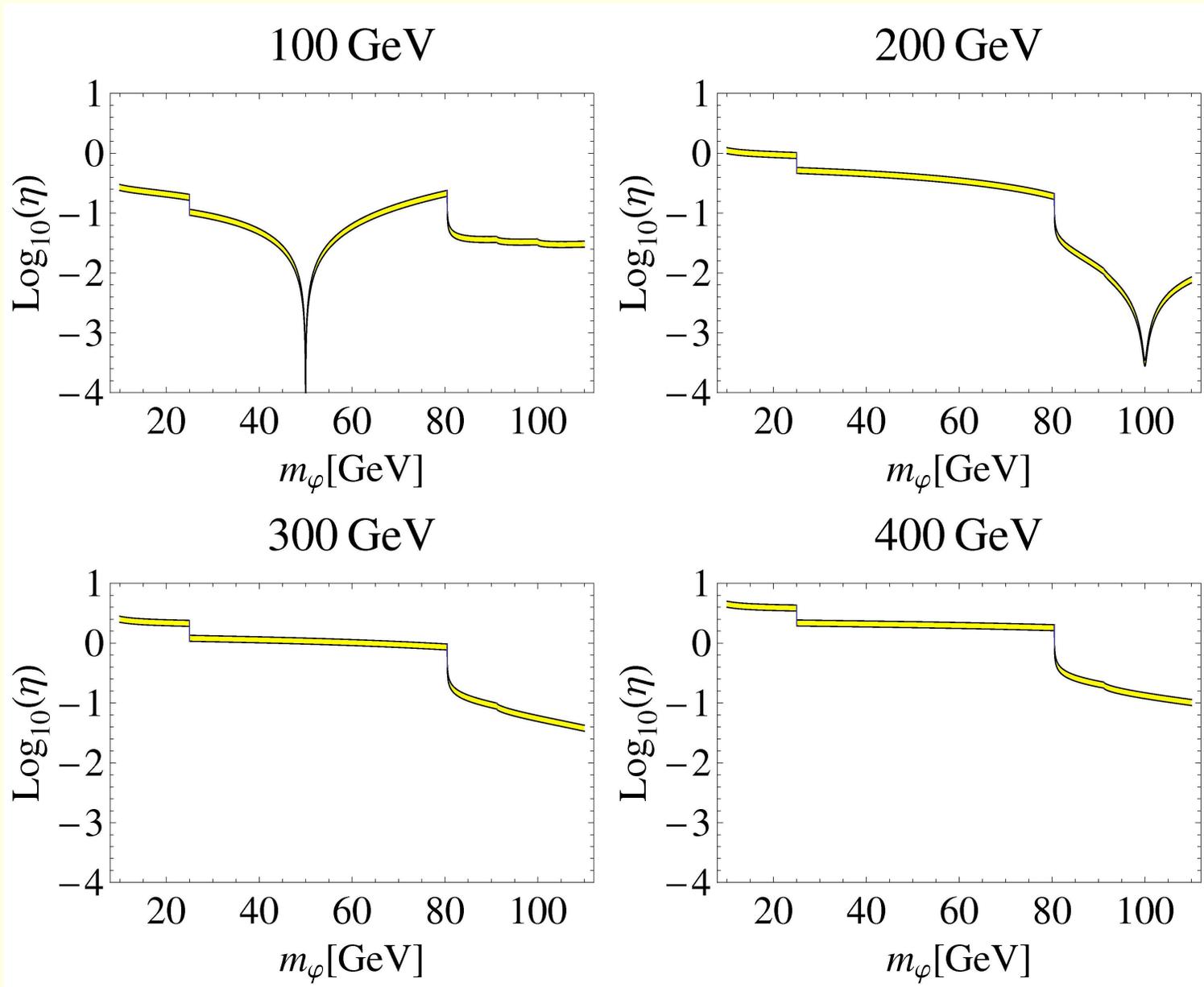


Figure 7: Inert-scalar coupling  $\eta$  (vs  $m_\phi$ ) required by the observed DM abundance  $\Omega_{DM}h^2 = 0.106 \pm 0.008$  within a  $3\text{-}\sigma$  band. As indicated above each panel, the lightest Higgs-boson mass ranges from  $M_1 = 100$  to  $400$  GeV. It was assumed that  $2\lambda_{11} = \kappa_1 \equiv \eta$ .

- 2HDM (CPV) + Inert doublet (DM)

B.G., O.M. OGREID, P. OSLAND, "Natural Multi-Higgs Model with Dark Matter and CP Violation", Phys.Rev.D80:055013,2009.

$$V(\Phi_1, \Phi_2, \eta) = V_{12}(\Phi_1, \Phi_2) + V_3(\eta) + V_{123}(\Phi_1, \Phi_2, \eta)$$

where

$$\begin{aligned} V_{12}(\Phi_1, \Phi_2) = & -\frac{1}{2} \left\{ m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right] \right\} \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \frac{1}{2} \left[ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right], \end{aligned}$$

$$V_3(\eta) = m_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2} (\eta^\dagger \eta)^2,$$

$$\begin{aligned} V_{123}(\Phi_1, \Phi_2, \eta) = & \lambda_{1133} (\Phi_1^\dagger \Phi_1) (\eta^\dagger \eta) + \lambda_{2233} (\Phi_2^\dagger \Phi_2) (\eta^\dagger \eta) \\ & + \lambda_{1331} (\Phi_1^\dagger \eta) (\eta^\dagger \Phi_1) + \lambda_{2332} (\Phi_2^\dagger \eta) (\eta^\dagger \Phi_2) \\ & + \frac{1}{2} \left[ \lambda_{1313} (\Phi_1^\dagger \eta)^2 + \text{H.c.} \right] + \frac{1}{2} \left[ \lambda_{2323} (\Phi_2^\dagger \eta)^2 + \text{H.c.} \right] \end{aligned}$$

## Summary

- The SM could be easily extended so that the little hierarchy problem is ameliorated, DM candidate is provided and also CP is violated in the extra sector:
  - The addition of  $N_\varphi$  real scalar singlets  $\varphi_i$  to the SM lifts the cutoff  $\Lambda$  to  $\sim 4 - 9$  TeV. It also provides a realistic candidate for DM if  $m_\varphi \sim 1 - 3$  TeV (depending on  $N_\varphi$ ).
  - To accommodate CPV in the Higgs potential the SM scalar sector should be replaced by 2 Higgs doublets (non-inert). Cancellation of quadratic divergences could be arranged within the 2HDM. Heavy lightest Higgs additionally suppresses  $\delta M_i^2 / M_i^2$ . Adding extra inert scalar singlet or doublet offers a DM candidate.
  - CPV in the Higgs potential with the SM doublet and singlets only?
- Some fine tuning always remains.