Non-Supersymmetric New Physics at the Dawn of LHC

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- Introduction to the Standard Model
  - Theory (gauge and Higgs sectors, fermions, mixing, . . .)
  - Experiments (LEP, . . .)
- Reasons to go beyond the Standard Model
- Models of New Physics
- Future perspectives at LHC
- Summary

♠ Gauge symmetry: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L} \supset -\frac{1}{4}\underbrace{F_a^{\mu\nu}F_{a\,\mu\nu}}_{SU(3)_C} - \frac{1}{4}\underbrace{W_i^{\mu\nu}W_{i\,\mu\nu}}_{SU(2)_L} - \frac{1}{4}\underbrace{B^{\mu\nu}B_{\mu\nu}}_{U(1)_Y}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$G_a^\mu|_{a=1,\ldots 8} \qquad\qquad W_\mu^\pm, Z_\mu, A_\mu$$

♠ The Higgs sector:

- The minimal choice $H = \begin{pmatrix} G^+ \\ (H + iG^0)/\sqrt{2} \end{pmatrix}$ for $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

$$\mathcal{L} \supset (D_\mu H)^\dagger D^\mu H - V(H)$$

for $D_\mu \equiv \partial_\mu + igW_\mu^i T^i + ig'\frac{1}{2}YB_\mu$ and $V(H) = \mu^2|H|^2 + \lambda|H|^4$ with $Y_H = \frac{1}{2}$

- If $\mu^2 < 0$ then $\langle 0||H|^2|0\rangle = -\frac{1}{2}\frac{\mu^2}{\lambda} \equiv \frac{v^2}{2}$ (spontaneous symmetry breaking, the origin of mass), $v \simeq 246$ GeV
- Boson masses: $m_h = \sqrt{2\lambda}v$, $m_{W^\pm} = \frac{1}{2}gv$ and $m_Z = m_W/c_W$, for $c_W \equiv \cos\theta_W = g/(g^2 + g'^{\,2})^{1/2}$

♠ Fermions $\times 3$ (3 generations)

fermion	T	T_3	$\frac{1}{2}Y$	Q
$\nu_{i\,L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0
$l_{i\,L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
$u_{i\,L}$	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}$
$d_{i\,L}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}$
$l_{i\,R}$	0	0	-1	-1
$u_{i\,R}$	0	0	$\frac{2}{3}$	$\frac{2}{3}$
$d_{i\,R}$	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$\nu_{i\,R}$	0	0	0	0

$i = 1, \ldots, N_f = 3,\ \psi_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$ (parity violation), $Q = T_3 + \frac{1}{2}Y$

Neutrino masses:

- Dirac mass: $f_{ij}\ \bar{L}_{i\,L}\nu_{j\,R}\ \tilde{H} + \text{H.c.}$ for $\tilde{H} \equiv i\tau_2 H^\star$
- Majorana mass: $\frac{1}{2}M_{ij}\ \nu_{i\,R}\ \mathbf{C}\ \nu_{j\,R} + \text{H.c.}$

Gauge transformations:

$$\psi(x) \to \exp\left\{-igT^i\theta_i(x) - ig'\frac{1}{2}Y\beta(x)\right\}\psi(x)$$

Gauge interactions:

$$\mathcal{L} \supset \sum_{\psi} \bar{\psi} i\gamma^\mu D_\mu \psi \qquad \text{for} \qquad D_\mu \equiv \partial_\mu + igW^i_\mu T^i + ig'\frac{1}{2}YB_\mu$$

Yukawa interactions: $\mathcal{L} \supset -\sum_{i,j=1}^{3}\left(\tilde{\Gamma}_{ij}\bar{u}_{i\,R}\tilde{H}^\dagger Q_{j\,L} + \Gamma_{ij}\bar{d}_{i\,R}H^\dagger Q_{j\,L} + \text{H.c.}\right)$

$$\Downarrow$$

if $\langle H \rangle \neq 0$ then $m_q \neq 0$

$$\mathcal{L}_{\text{q mass}} = -\sum_{i,j=1}^{3}\left(\bar{u}_{i\,R}\mathcal{M}^u_{ij}u_{j\,L} + \bar{d}_{i\,R}\mathcal{M}^d_{ij}d_{j\,L} + \text{H.c.}\right)$$

for

$$\mathcal{M}^u \propto v\tilde{\Gamma}, \qquad \mathcal{M}^d \propto v\Gamma$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

$$U_R^\dagger \mathcal{M}^u U_L = \mathrm{diag}(m_u, m_c, m_t) \qquad D_R^\dagger \mathcal{M}^d D_L = \mathrm{diag}(m_d, m_s, m_b)$$

$$\Downarrow$$

$$\tilde{\Gamma}, \Gamma \qquad \text{diagonal}$$

- charged currents: $\sum \bar{u}_{i\,L} \gamma^\mu d_{i\,L} = (\bar{u}, \bar{c}, \bar{t})_L \underbrace{U_L^\dagger D_L}_{U_{CKM}} \gamma^\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$
- neutral currents: $\sum \bar{u}_{i\,L} \gamma^\mu u_{i\,L}$, $\sum \bar{d}_{i\,L} \gamma^\mu d_{i\,L}$ remain unchanged upon $U_{L,R}$, $D_{L,R}$ transformations

U_{CKM}:

- unitary complex $N \times N$ matrix, $q_{i\,L} \to e^{i\alpha_i} q_{i\,L} \;\Rightarrow\; \frac{1}{2}(N-1)(N-2)$ phases in U_{CKM}

- $N \geq 3 \qquad \Rightarrow \qquad$ CP violation in charged currents

♠ Masses in the SM: $\quad m_V \propto g v \qquad m_h \propto \lambda^{1/2} v \qquad m_f \propto g_f v$

Leptons:

$$\begin{array}{lll} m_{\nu_e} \lesssim 3 \text{ eV} & m_{\nu_\mu} \lesssim 0.2 \text{ MeV} & m_{\nu_\tau} \lesssim 18 \text{ MeV} \\ m_e = 0.5 \text{ MeV} & m_\mu = 105.5 \text{ MeV} & m_\tau = 1.78 \text{ GeV} \end{array}$$

Quarks:

$$\begin{array}{lll} m_u \simeq 2 \text{ MeV} & m_c \simeq 1.2 \text{ GeV} & m_t \simeq 174 \text{ GeV} \\ m_d = 5 \text{ MeV} & m_s = 0.1 \text{ GeV} & m_b = 4.3 \text{ GeV} \end{array}$$

Bosons:

$$m_{W^\pm} = 80.4 \text{ GeV} \qquad m_Z = 91.2 \text{ GeV} \qquad m_h \geq 115 \text{ GeV}$$

$$\Downarrow$$

Fine tuning:

$$\frac{m_{\nu_e}}{m_t} \lesssim 0.5 \cdot 10^{-11} \qquad \Rightarrow \qquad \frac{g_{\nu_e}}{g_t} \lesssim 0.5 \cdot 10^{-11}$$

♠ Theoretical limits on the SM Higgs boson mass (Why LHC?):

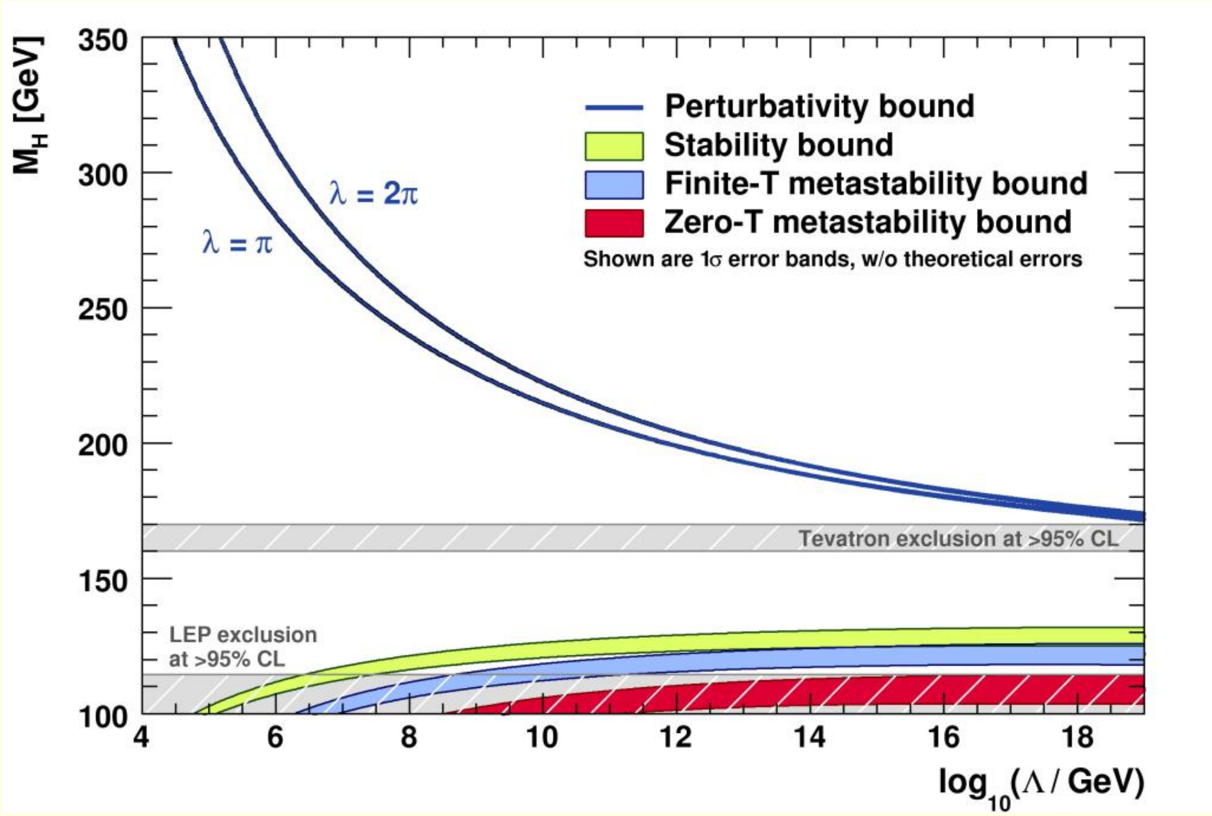
- Unitarity:

$$a_0(W_L^+W_L^- \to W_L^+W_L^-) \overset{s \gg m_h^2}{\longrightarrow} \frac{1}{8\pi^2}\left(\frac{m_h}{v}\right)^2 \quad \Longrightarrow \quad m_h \lesssim 700 \text{ GeV}$$

- Perturbativity:

$$\lambda \propto \left(\frac{m_h}{v}\right)^2 \quad \Longrightarrow \quad m_h \lesssim 700 \text{ GeV}$$

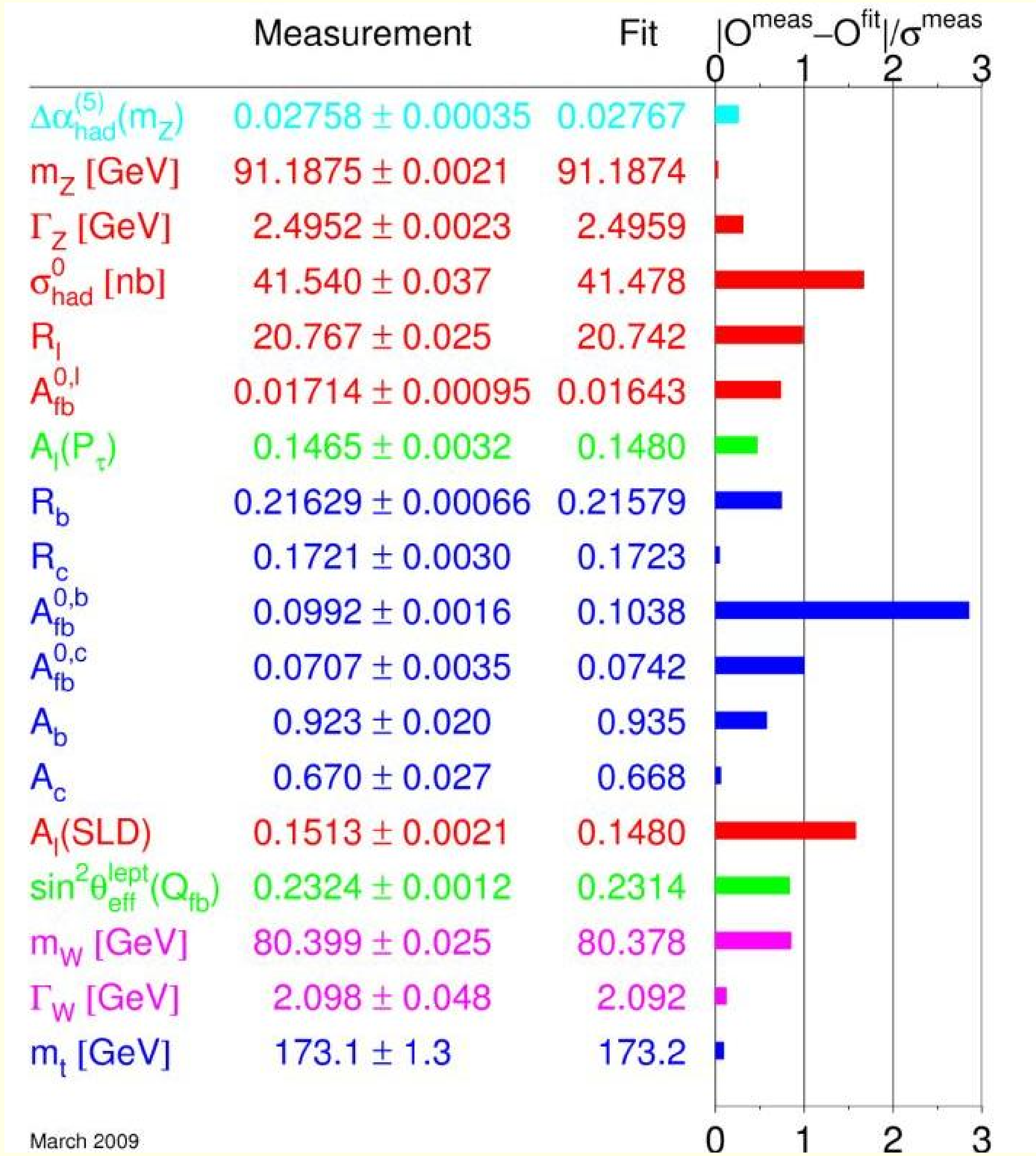
- Triviality and vacuum stability:



Ellis *et al.* 2009

B.G. and J. Wudka, "Bounds on the Higgs-Boson mass in the presence of nonstandard interactions", Phys.Rev.Lett.88:041802,2002

- Perfect agreement with the existing data



- The scalar sector weakly constrained
  - Higgs-boson representation:

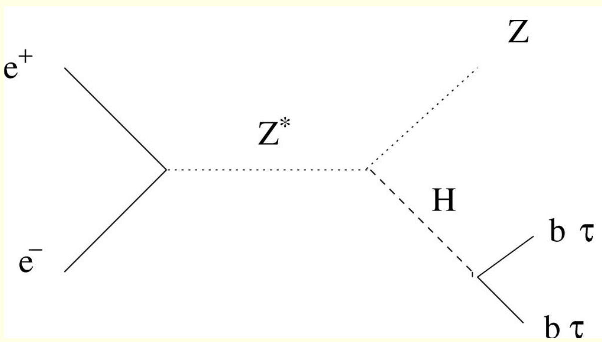
$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \qquad \text{SM} \qquad \Rightarrow \qquad \rho = 1 + \mathcal{O}(\alpha)$$

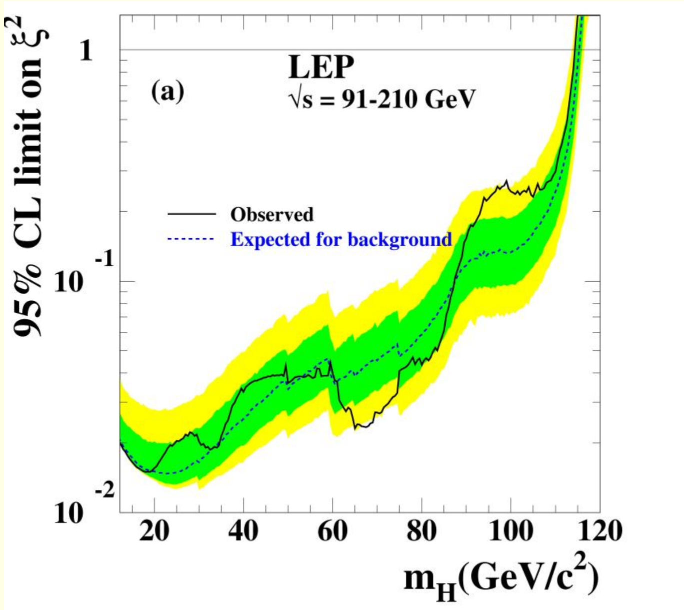
for general Higgs multiplets: $\rho = \frac{\sum_i \left[T_i(T_i+1) - T_{i3}^2\right] v_i^2}{\sum_i 2T_{i3}^2 v_i^2}$

data: $\rho = 1.0002 \begin{cases} +0.0024 \\ -0.0009 \end{cases} \qquad \Rightarrow \qquad T = \frac{1}{2} \qquad$ (doublets are favored)

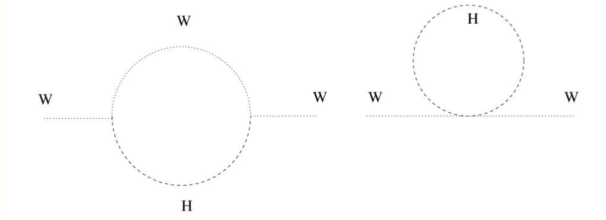
  - Higgs-boson interactions: no direct tests of the scalar potential

- Higgs-boson production at LEP: $m_h \gtrsim 115$ GeV

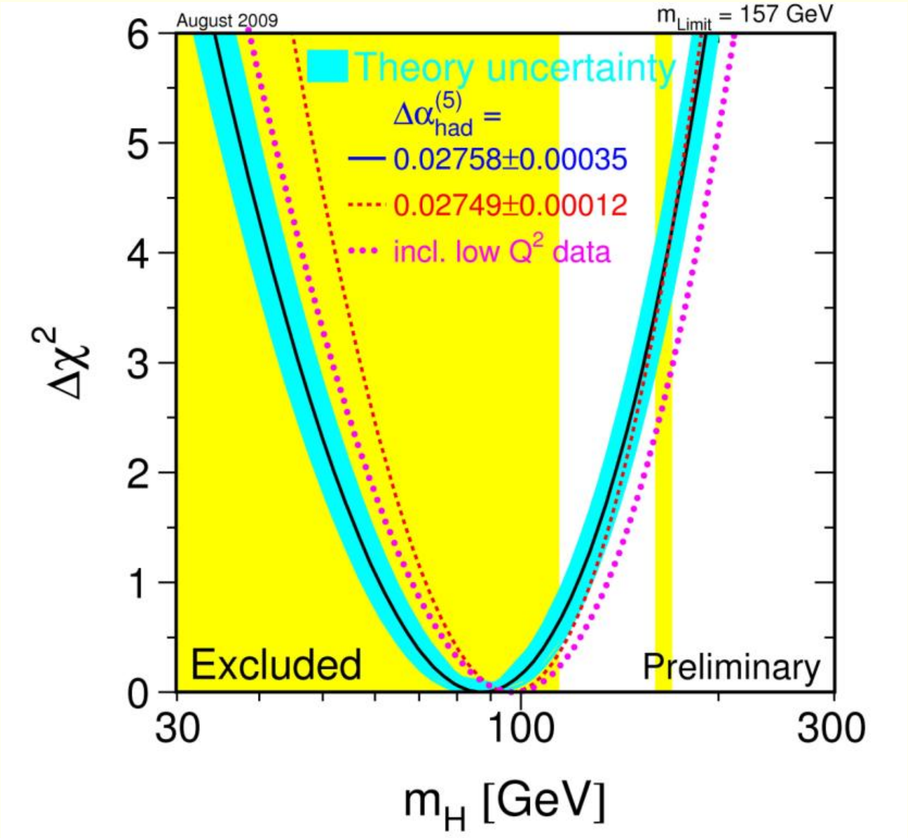




- Indirect Higgs-boson-mass limits (through radiative corrections):



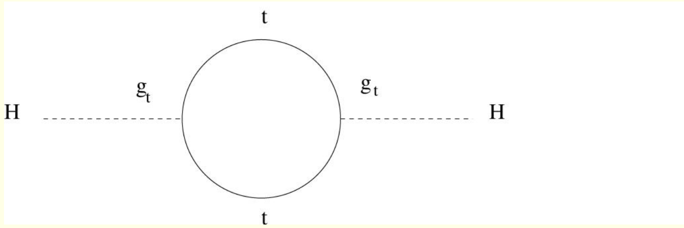
$+\cdots \qquad \propto \qquad \ln\left(\frac{m_h^2}{v^2}\right)$ (Veltman's screening)



$\Rightarrow \qquad m_h \lesssim 200$ GeV

♠ Gauge-Higgs sector:

- The hierarchy problem A:



$$\Rightarrow \qquad m_h^2 = m_h^{(\mathrm{tree})\,2} - c\cdot\Lambda^2$$

if $m_h \simeq 100$ GeV and $c \simeq \frac{g_t^2}{4\pi^2} \simeq \frac{1}{40}$ then $1 = \left(\frac{10 m_h^{(\mathrm{tree})}}{1\ \mathrm{TeV}}\right)^2 - \left(\frac{1.6\Lambda}{1\ \mathrm{TeV}}\right)^2$
If $\Lambda \gg 1$ TeV then a fine tuned cancellation is needed to obtain $m_h \simeq 100$ GeV, e.g. for $\Lambda = M_{Pl} = 10^{18}$ GeV one has

$$1 = \left(\frac{10 m_h^{(\mathrm{tree})}}{1\ \mathrm{TeV}}\right)^2 - 2.5\cdot 10^{30} \;\Rightarrow\; \text{to avoid fine tuning} \qquad \Lambda \lesssim 5 m_h \lesssim 1\ \mathrm{TeV}$$

The New Physics is expected at $E \simeq 1$ TeV

- The hierarchy problem B: Why $v \ll M_{Pl}$?

- Why is there only one Higgs boson?
  - The Higgs field was introduced just to make the model renormalizable (unitary)
  - There exist many fermions and vector bosons, so why only one scalar? Why, for instance, not a dedicated scalar for each fermion family?

- The strong CP problem:
  - symmetries of the SM allow for

$$\mathrm{tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(F_{\mu\nu}F_{\alpha\beta}\right) \xrightarrow{P} -\mathrm{tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

  - odd under CP

$$\mathcal{L}_\theta = \theta\frac{g_s^2}{32\pi^2}F^{a\,\mu\nu}\tilde{F}^a_{\mu\nu} \qquad \Rightarrow \qquad \mathrm{neutron-EDM} \qquad D_n \simeq 2.7\cdot 10^{-16}\theta\ \mathrm{e\ cm}$$

$$\Downarrow$$

data: $D_n \lesssim 1.1\cdot 10^{-25}\ \mathrm{e\ cm} \qquad \Rightarrow \qquad \theta \lesssim 3\cdot 10^{-10}$

The strong CP problem: why is θ so small?

♠ The flavour sector:

- parity violation:

$$W^{+\,\mu}\,\bar{u}_i\gamma_\mu(1-\gamma_5)d_j \quad \xrightarrow{P} \quad W^{+\,\mu}\,\bar{u}_i\gamma_\mu(1+\gamma_5)d_j$$

Maximal parity violation, why?

- Charge quantization, why $q_u = \frac{2}{3}$, $q_d = -\frac{1}{3}$ and $q_l = -1$?

- Number of generations, why $N = 3$?

- Why is the top quark so heavy ($m_t \simeq 174$ GeV while $m_b \simeq 4.3$ GeV) ?

$$m_t \simeq v = \langle 0|H|0\rangle \simeq 246 \text{ GeV}$$
$$\Downarrow$$

top quark is very different (possibly sensitive to the spontaneous symmetry breaking)

♠ Fine tuning of Yukawa couplings:

$$\frac{m_{\nu_e}}{m_t} \lesssim 0.5 \cdot 10^{-11} \qquad \Rightarrow \qquad \frac{g_{\nu_e}}{g_t} \lesssim 0.5 \cdot 10^{-11}$$

♠ Parameters of the SM:

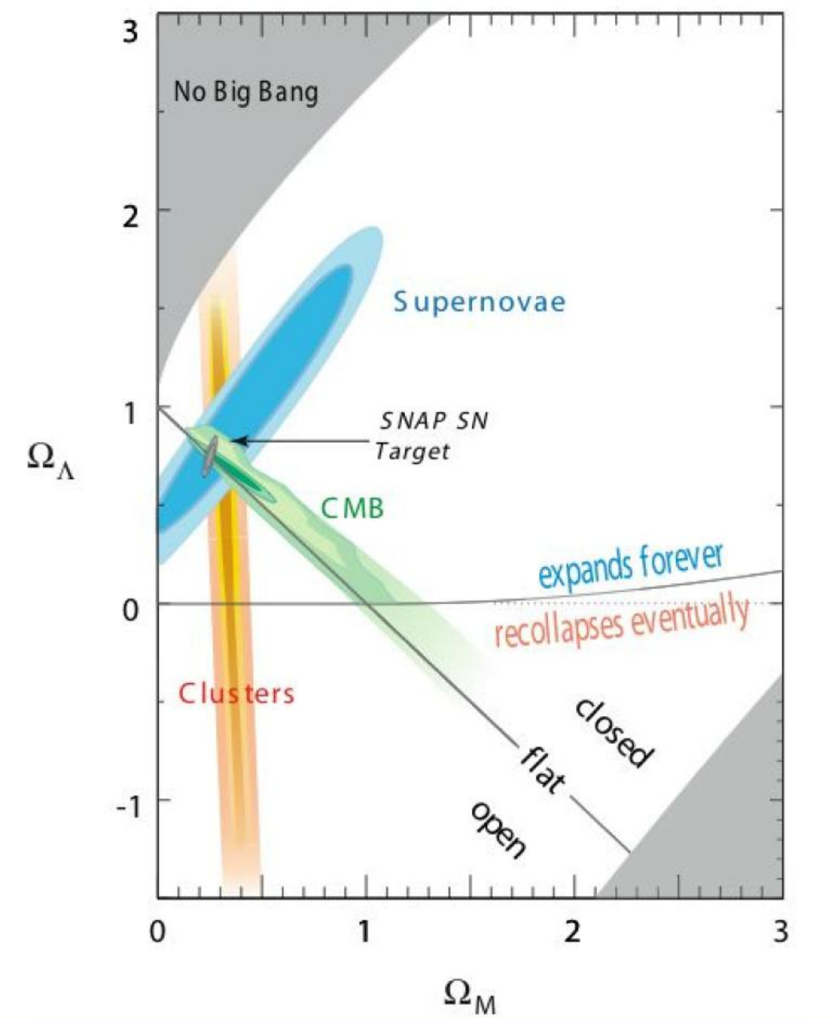
$$\begin{matrix} m_e & m_\mu & m_\tau & m_u & m_c & m_t \\ m_{\nu_e} & m_{\nu_\mu} & m_{\nu_\tau} & m_d & m_s & m_b \end{matrix}$$

$$\underbrace{g, g'}_{(\alpha_{QED}, \sin\theta_W)}, \underbrace{g_s}_{(\alpha_{QCD})}, \underbrace{m_h,\ \lambda}_{(\mu,\lambda)}, \underbrace{U_{CKM}}_{\theta_{1,2,3},\delta_{CP}}$$

21 parameters !

♠ Cosmology:

- Dark matter and dark energy



$$\Omega_i \equiv \frac{\rho_i}{\rho_c} \qquad \text{for} \qquad \rho_c \equiv \frac{3H_0^2}{8\pi G_N}$$

data $\Rightarrow$ $\Omega_\Lambda = \frac{\Lambda}{3H_0^2} \simeq 70\%$, $\Omega_{DM} \simeq 27\%$ and $\Omega_B \simeq 3\%$

- SM has no candidate for dark matter

- $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \simeq 0.7 \quad \Rightarrow \quad \rho_\Lambda \simeq 10^{-120} M_{Pl}^4 = (10^{-3} \text{ eV})^4$ while typical scale of the SM is $\mathcal{O}(100 \text{ GeV})$! Fine tuning again!

- Inflation: period of fast expansion of the very early Universe, $a(t) \simeq \exp\left(\sqrt{\frac{\Lambda}{3}}\, t\right)$
  Again the SM has no means to explain the inflation. For a typical inflaton $m_\varphi \sim 10^{13}$ GeV and $\lambda \sim 10^{-13}$, so probably the SM Higgs boson is not an inflaton (however see discussion on $\xi H^\dagger H R$).

- Baryogenesis and SM CP violation $\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq \frac{n_b}{n_\gamma} \simeq 6 \cdot 10^{-10}$
  The Sakharov's necessary conditions for baryogenesis:
  - B number violation
  - C and CP violation
  - Departure from thermal equilibrium

  SM:

  - B number violation: **OK**
  - C and CP violation: too weak CP violation $\propto \mathbf{Im} Q$, for $Q \equiv U_{ud} U_{cb} U^\star_{ub} U^\star_{cd}$
  - Departure from equilibrium: no electroweak phase transition for $m_h \gtrsim 73$ GeV

  Conclusion: the SM does not explain the baryogenesis

♠ Extra Higgs bosons

- Multi-doublet models favored by the ρ measurement and CP violation

- An example of extra Higgs boson scenario: the 2 Higgs Doublet Model:

$$\begin{aligned} V(\phi_1,\phi_2) &= m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 + m_3^2(e^{i\delta_3}\phi_1^\dagger\phi_2 + e^{-i\delta_3}\phi_2^\dagger\phi_1) + \\ &+\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \\ &+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_5\left[e^{i\delta_5}(\phi_1^\dagger\phi_2)^2 + \text{H.c.}\right] + \\ &+\lambda_6(\phi_1^\dagger\phi_1)\left[e^{i\delta_6}\phi_1^\dagger\phi_2 + \text{H.c.}\right] + \lambda_7(\phi_2^\dagger\phi_2)\left[e^{i\delta_7}\phi_1^\dagger\phi_2 + \text{H.c.}\right] \end{aligned}$$

where m_i^2, λ_i and δ_i real

$$\text{under CP: } \phi_i(t,\vec{x}) \quad \xrightarrow{CP} \quad e^{i\alpha_i}\phi_i^\star(t,-\vec{x}) \quad \text{for} \quad i=1,2$$

- explicit CP violation: $\delta_i \neq 0$

$$\phi_1^\dagger\phi_2 \quad \xrightarrow{CP} \quad e^{i(\alpha_2-\alpha_1)}\phi_2^\dagger\phi_1$$

- spontaneous CP violation ($\delta_i = 0$)

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \qquad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_1 e^{i\theta}}{\sqrt{2}} \end{pmatrix}$$

Difficulties of extra-Higgs-boson scenarios:

- many new parameters ($m_i^2, \lambda_i, \delta_i$)

- tree-level FCNC to be suppressed

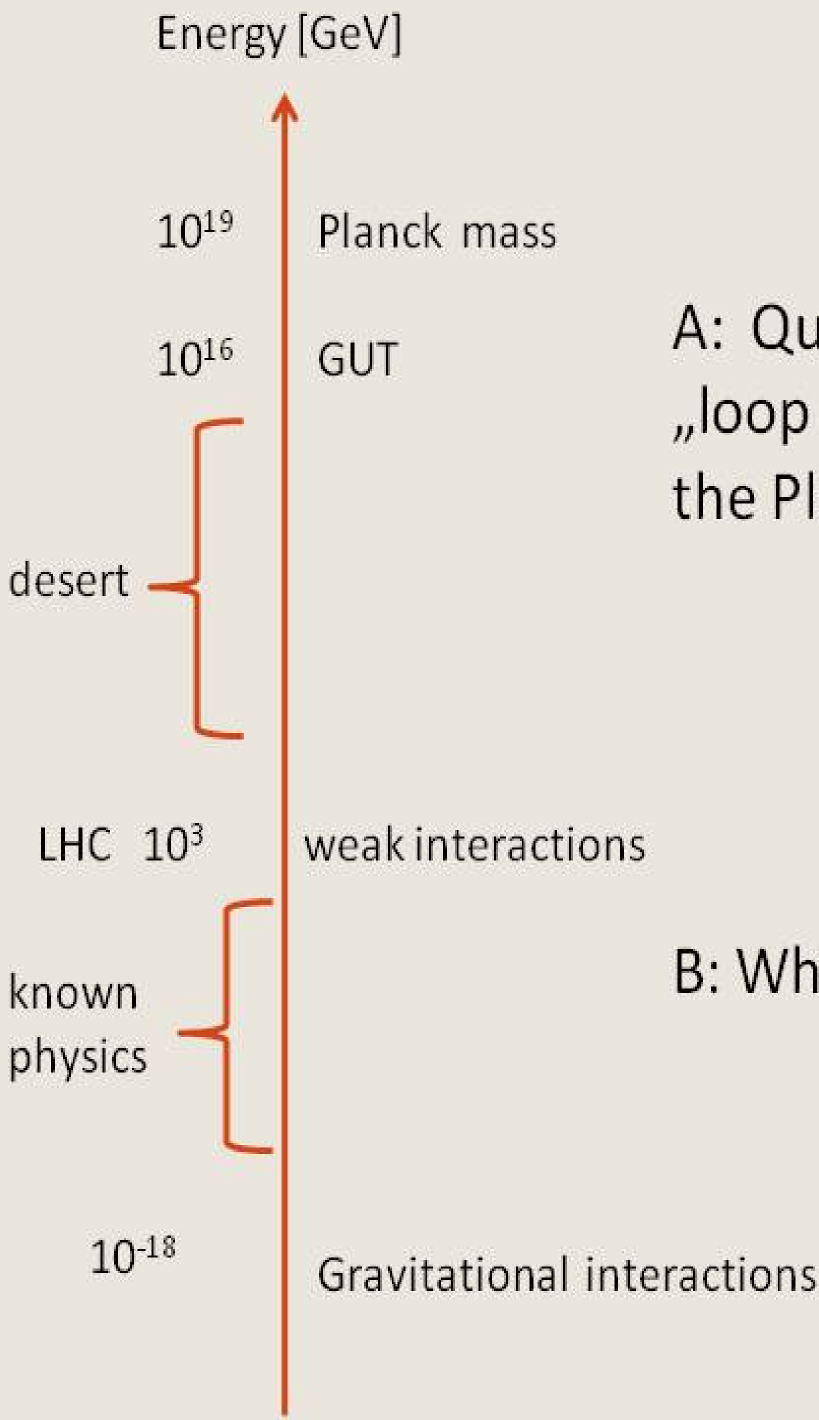
$$\mathcal{M}^u_{ij} = \sum_{\alpha=1}^{N_H} \tilde{\Gamma}^{(\alpha)}_{ij} \frac{v_\alpha}{\sqrt{2}} \propto \tilde{\Gamma}^{(1)}_{ij} v_1 + \tilde{\Gamma}^{(2)}_{ij} v_2 + \cdots \,, \;\; \mathcal{M}^d_{ij} = \sum_{\alpha=1}^{N_H} \Gamma^{\alpha}_{ij} \frac{v_\alpha}{\sqrt{2}} \propto \Gamma^{(1)}_{ij} v_1 + \Gamma^{(2)}_{ij} v_2 + \cdots$$

♠ Extra gauge symmetries

- GUTs, e.g. $SU(5)$: unification of gauge couplings, . . .

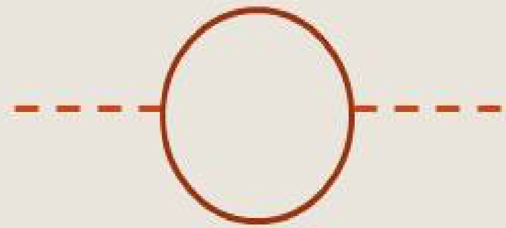
- $L - R$ symmetry, $SU(2)_L \times SU(2)_R \times U(1)$: spontaneous parity violation

- $SU(2)_L \times U(1) \times U(1)'$: just extra Z'



Hierarchy Problem

A: Quantum Field Theory:
„loop corrections” pull scalar masses to the Planck mass

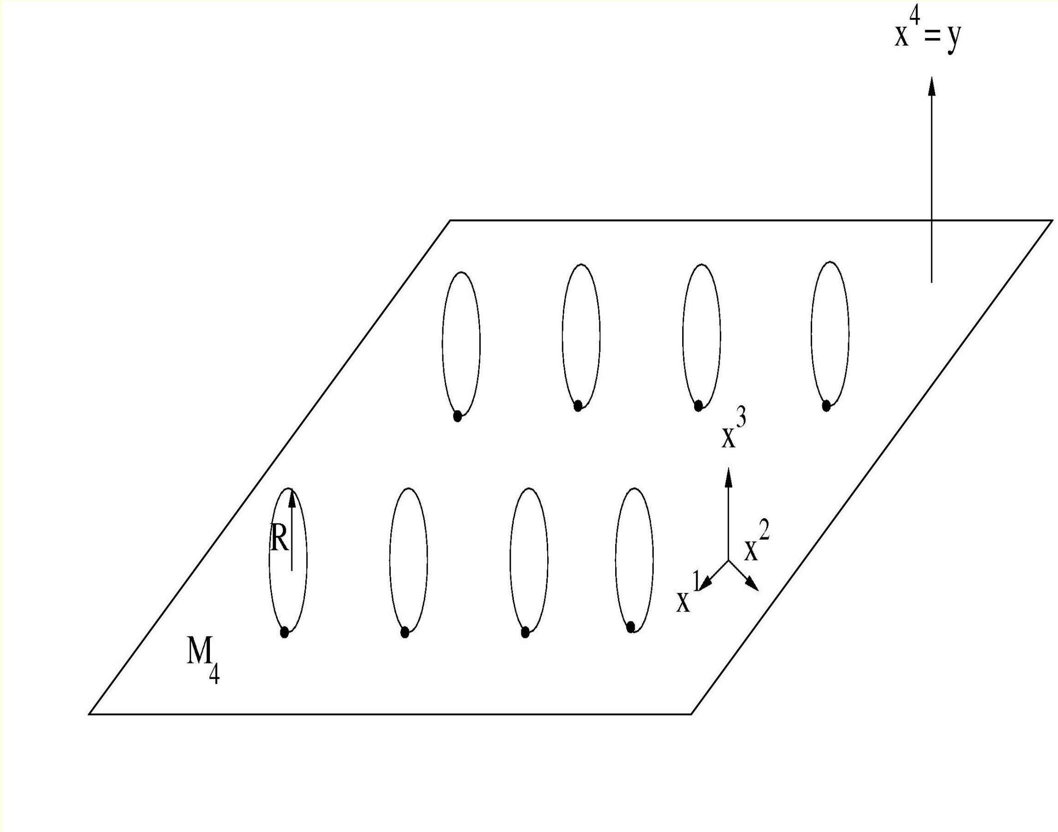


B: Why $v \ll M_{Pl}$?

♠ Extra dimensions (more spatial dimensions)

Motivations:

- Unification of gravity and gauge interactions in g_{AB} (T. Kaluza 1921)
- Quantization of gravity (strings)
- Solution (amelioration) of the hierarchy problem



$$\phi(x,y) = \phi(x, y+2\pi L) \quad \Rightarrow \quad \phi(x,y) = \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{i\frac{n}{L}y}$$

where $\phi_n(x)$ are called Kaluza-Klein modes.
Equation of motion (momentum along 5th D $\rightarrow$ mass in 4D):

$$(\partial^\mu \partial_\mu + \partial^y \partial_y)\phi(x,y) = 0 \;\Rightarrow\; \left[\partial^\mu \partial_\mu + \left(\frac{n}{L}\right)^2\right]\phi_n(x) = 0 \;\Rightarrow\; m_n = \frac{n}{L}$$

- $n = 0 \rightarrow$ massless modes: e.g. gravity & electromagnetism (Kaluza & Klein),
- $n > 0 \rightarrow$ massive modes: “high” scale physics.

An example of Kaluza-Klein expansion: scalar field for $\delta = 1$:

$$\mathcal{L} = \frac{1}{2}\eta^{AB}\partial_A\phi\partial_B\phi \qquad \text{for} \qquad A = 0,1,2,3,5$$

$$\phi(t,\vec{x},y) = \phi(x,y) \qquad \text{space time} \qquad M_4 \times S^1 \qquad \Rightarrow \qquad \phi(x,y+2\pi L) = \phi(x,y)$$

$$\Downarrow$$

$$\phi(x,y) = \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{in\frac{y}{L}}$$

$$\mathcal{L} = \frac{1}{2}\sum_{n,m=-\infty}^{+\infty}\left[\partial_\mu\phi_n\partial^\mu\phi_m + \frac{nm}{L^2}\phi_n\phi_m\right]e^{i(n+m)\frac{y}{L}}$$

$$\begin{aligned}\mathcal{S} &= \int d^4x \int_0^{2\pi L} dy\,\mathcal{L} = \frac{2\pi L}{2}\int d^4x \sum_{n=-\infty}^{\infty}\left[\partial_\mu\phi_n\partial^\mu\phi_n^\star - \left(\frac{n}{L}\right)^2\phi_n\phi_n^\star\right] = \\ &= \int d^4x\left\{\left[\frac{1}{2}\partial_\mu\varphi_0\partial^\mu\varphi_0\right] + \sum_{n=0}^{\infty}\left[\partial_\mu\varphi_n\partial^\mu\varphi_n^\star - m_n^2\varphi_n\varphi_n^\star\right]\right\}\end{aligned}$$

for $\varphi_n \equiv \sqrt{2\pi L}\phi_n$ and $m_n^2 \equiv \left(\frac{n}{L}\right)^2$

- The lightest KK particle $(n \neq 0)$ is stable $\Longrightarrow$ a candidate for DM.

⋆ Large extra dimensions (ADD): a solution to the hierarchy problem B

Arkani-Hamed, Dimopoulos & Dvali, 1998

$$S_{ADD} = \frac{M^{2+\delta}}{2} \int d^4x \underbrace{\int_0^{2\pi L} \cdots \int_0^{2\pi L}}_{\delta} d^\delta y \sqrt{-g_{4+\delta}} R_{4+\delta} + \int d^4x \sqrt{-g_4} \mathcal{L}_{SM}$$

- SM localized on the 3-brane (M_4)
- All (δ) extra dimensions of size L
- Only gravity propagates in the bulk ($ds^2 = g_{4\,\mu\nu}(x) dx^\mu dx^\nu - \delta_{ab} dy^a dy^b$)

$$\frac{M^{2+\delta}}{2} \int d^4x \underbrace{\int_0^{2\pi L} \cdots \int_0^{2\pi L}}_{\delta} d^\delta y \sqrt{-g_{4+\delta}} R_{4+\delta} \quad \longrightarrow \quad \frac{M^{2+\delta}(2\pi L)^\delta}{2} \int d^4x \sqrt{-g_4} R_4 + \cdot$$

$$\Downarrow$$

$$M_{Pl}^2 = M^{2+\delta}(2\pi L)^\delta$$

Trade: the hierarchy for the volume

The $4+\delta$ gravity scale M, can be as low as 1 TeV, then $L = 10^{-17+30/\delta}$ cm

$$L = \begin{cases} 10^{13}\text{cm} & \delta = 1 \quad \text{excluded} \\ 10^{-2}\text{cm} & \delta = 2 \quad \text{allowed} \end{cases}$$

Graviton interactions:

- $$g_{MN}(x,y) = \eta_{MN} + \frac{h_{MN}(x,y)}{M^{\delta/2+1}}$$

- KK graviton modes

$$h_{MN}(x,y) = \sum_{\vec{n}} h^{\vec{n}}_{MN}(x)\exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{L}\right)$$

with $m_{\vec{n}} = \frac{\sqrt{\vec{n}^2}}{L}$ for $\vec{n} = (n_1, \ldots, n_\delta)$.

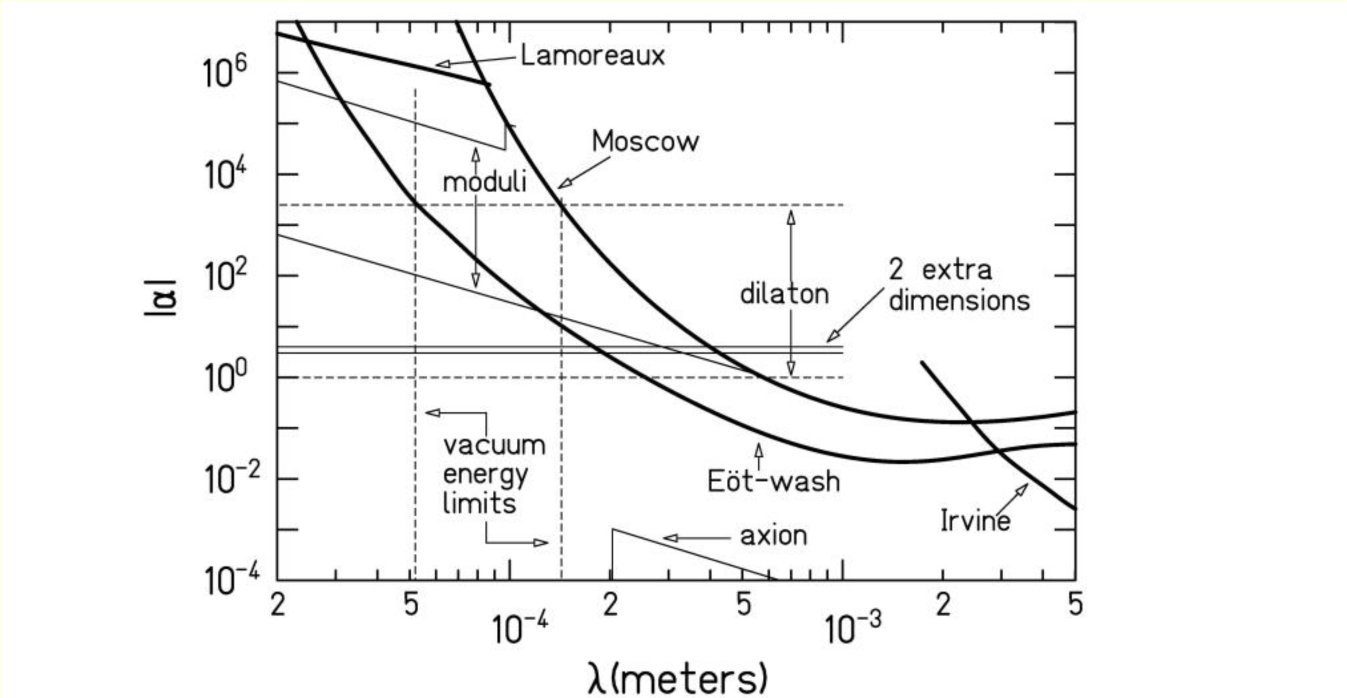
- KK – $T_{\mu\nu}$ coupling

$$\mathcal{L}^{\vec{n}}_{\rm int} = -\frac{1}{2M_{Pl}}\left[\left(h^{\mu\nu,\vec{n}} + h^{\mu\nu,-\vec{n}}\right)T_{\mu\nu}\right] + \cdots$$

δ	2	6
L	10^{-4} m	10 fm $= 10^{-14}$ m
L^{-1}	$2 \cdot 10^{-3}$ eV	20 MeV

Gravity modified at distances $\simeq L$

$$V(r) = \begin{cases} -\frac{m_1 m_2}{M_S^{\delta+2} L^\delta} \frac{1}{r} \sim -\frac{G_N m_1 m_2}{r} & \text{for} \quad r \gg L \\ -\frac{m_1 m_2}{M^{2+\delta}} \frac{1}{r^{1+\delta}} & \text{for} \quad r \ll L \end{cases}$$



The Eot-Wash Group: submillimeter tests of gravity

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha\, e^{-\frac{r}{\lambda}}\right)$$

ADD with $\delta = 2 \qquad \Rightarrow \qquad \lambda = L\,, \qquad \alpha = 4.$

Experimental signature:

- Virtual graviton-exchange effects, e.g. $g + g \to h_n^\star \to \mu^+\mu^-$:

$$\mathcal{L}_{\rm eff} \;\propto\; \frac{T_{\mu\nu}T^{\mu\nu}}{\Lambda^4}$$

- Direct graviton production, e.g. $q\bar{q} \to h_n g$ (monojet plus missing transverse energy):

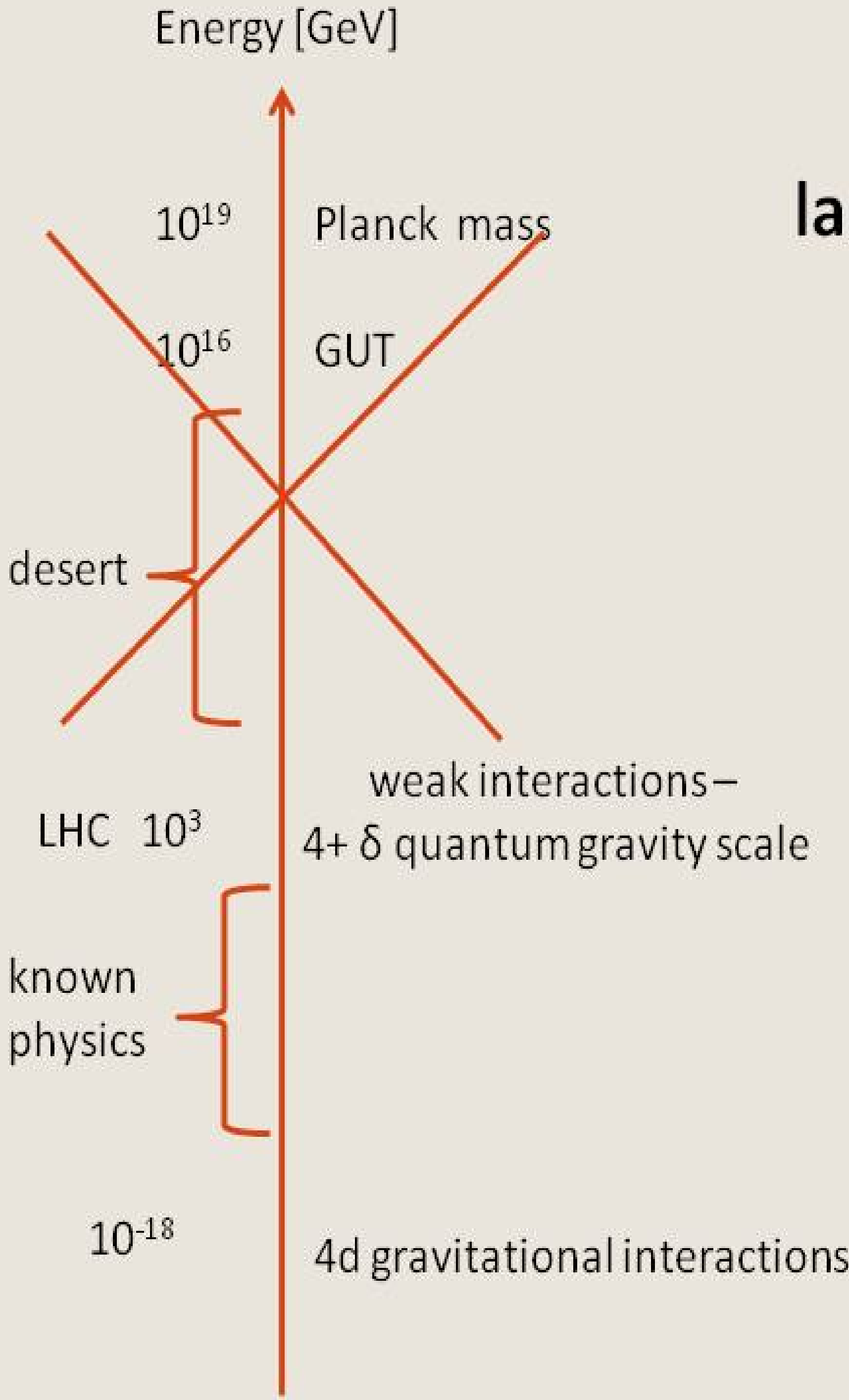
$$\sigma_{\rm tot} \propto \sum_n \sigma(pp \to h_n + g) \propto \frac{1}{M^2} \qquad \text{for} \qquad \sigma(pp \to h_n + g) \propto \frac{1}{M_{Pl}^2}$$

  – Number of KK modes that are kinematically available in $q\bar{q} \to h_n g$: $N \sim (EL)^\delta$, so for $\delta = 2$ and $E = 1$ TeV, $N \simeq 10^{30}$.
  – Graviton lifetime:

$$\tau_{\vec{n}} \sim \frac{M_{Pl}^2}{m_{\vec{n}}^3} \sim \left(\frac{\text{TeV}}{m_{\vec{n}}}\right)^3 10^3\,\text{sec}$$

  – LHC-sensitivity to M in TeV for $\sqrt{s} = 14$ TeV (95% CL for exclusion)

process	$\sqrt{s}$ TeV	$\mathcal{L}$ fb^{-1}	$\delta = 2$	$\delta = 4$	$\delta = 6$
$pp \to h_n + g$	14	100	4-8.9	4.5-6.8	5.0-5.8



Hierarchy Problem in large extra dimensions - ADD

- M - fundamental scale in $4 + \delta$ dim (Planck mass in $4 + \delta$)
- M_{Pl} (Planck mass in 4 dim): $M_{Pl}^2 = V_\delta \times M^{2+\delta}$
- SM on a 3-brane (4 d)
- gravity propagates in the bulk $(4+\delta)$

⋆ Warped extra dimensions (RS II): a solution to the hierarchy problem B

Randall & Sundrum, 1999, $D = 4 + 1$ (*"The wood has warped in drying."*)

$$\begin{aligned}\mathcal{S} &= \int d^4x \int_{-\pi}^{+\pi} dy \sqrt{-g}(2M^3R - \Lambda) + \\ &\int d^4x \sqrt{-g_{\rm vis}}(\mathcal{L}_{\rm vis} - V_{\rm vis}) + \int d^4x \sqrt{-g_{\rm hid}}(\mathcal{L}_{\rm hid} - V_{\rm hid})\end{aligned}$$

$$\Downarrow$$

if $\Lambda < 0$ and $V_{\rm hid} = -V_{\rm vis} = 24M^3k$ then

$$g_{MN} = \left(\begin{array}{c|c} e^{-kr_c|y|}\eta_{\mu\nu} & \\ \hline & r_c^2 \end{array}\right)$$

for $\Lambda = -24M^3k^2$

$$S_{\text{eff}} \supset 2 \int d^4x \left[\int_{-\pi}^{+\pi} dy \, M^3 r_c e^{-2kr_c|y|} \right] \sqrt{-\bar{g}} \, \bar{R} + \cdots \quad \text{for} \;\; \bar{g}_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

$$\Downarrow$$

To reproduce GR

$$M_{Pl}^2 = M^2 r_c \int_{-\pi}^{+\pi} dy \, e^{-2kr_c|y|} = \frac{M^3}{k}(1 - e^{-2kr_c\pi})$$

The hierarchy within the RS II:

$$v = e^{-kr_c\pi} v_0 = 246 \text{ GeV}$$

- If $e^{kr_c\pi} \simeq 10^{16}$ ($kr_c\pi \simeq 40$), then v_0 could be $\mathcal{O}(M_{Pl})$

- One can assume that the 5d theory has a single scale $\simeq M_{Pl}$ (so no hierarchy), while the E-W scale is generated through the warping!

$$\mathcal{L} \supset -\frac{1}{M_{Pl}} T^{\mu\nu} h_{\mu\nu}^{(0)} - \frac{1}{\Lambda} T^{\mu\nu} \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$$

for $\Lambda \sim M_{Pl} e^{-kr_c\pi} \sim 1$ TeV with $m_n \sim \Lambda \sim 1$ TeV.

Experimental signature:

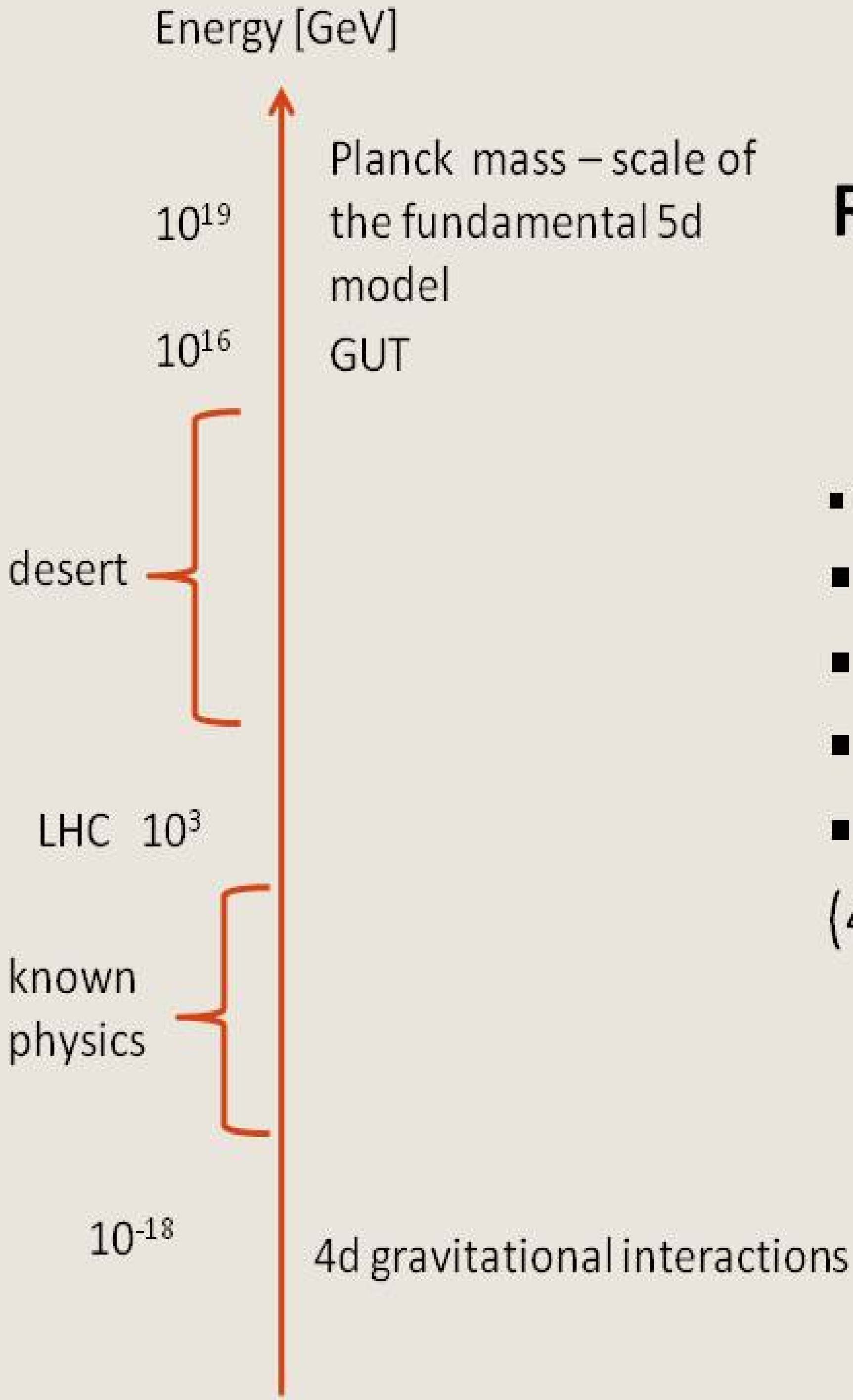
- virtual graviton-exchange effects, e.g. the Drell-Yan production $q+\bar{q} \to h_n^\star \to l^+l^-$: from

$$\mathcal{L} \supset -\frac{1}{M_{Pl}} T^{\mu\nu} h_{\mu\nu}^{(0)} - \frac{1}{\Lambda} T^{\mu\nu} \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}$$

LHC-sensitivity to Λ (95% CL for exclusion)

process	$\sqrt{s}$ TeV	$\mathcal{L}$ fb^{-1}	Λ TeV
$pp \to l^+l^-$	14	100	7.5
$pp \to \gamma\gamma$	14	100	7.1

Hierarchy Problem B in Randall-Sundrum model II



- ~M_{Pl} - fundamental scale in 4 + 1
- $v = e^{-40} M_{Pl}$
- SM on 3-brane (4 d)
- UV physics on the other 3-brane
- gravity propagates in the bulk (4+δ)

⋆ The gauge-Higgs unification: a solution to the hierarchy problem A

(Manton, 1979 & Hosotani, 1983)

The strategy for $D = 4+1$:

- SM Higgs in the fundamental representation of $SU(2)$. For A_5 (adjoint) to have iso-doublet components at least $G = SU(3)_w$ is required:

$$A_M^a \to \left(\begin{array}{c|c} A_M^a & A_M^{\hat{a}} \\ \hline A_M^{\hat{a}} & A_M^a \end{array}\right)$$

- The initial gauge group G broken to $SU(2)_L \times U(1)_Y$ by the Scherk-Schwarz mechanism.
  Periodicity:

$$A_M(x, y + 2\pi R) = T A_M(x,y) T^\dagger$$

  Orbifold boundary conditions:

$$A_\mu(x,-y) = +P A_\mu(x,y) P^\dagger \qquad A_5(x,-y) = -P A_5(x,y) P^\dagger\,,$$

  where T and P are elements of a global symmetry group (e.g. gauge).
  For $SU(3) \to SU(2)_L \times U(1)_Y$: $P = T = \exp(i\pi\lambda_3) = \mathrm{diag}(-1,-1,1)$

- $SU(2)_L \times U(1)_y \to U(1)_{\mathrm{EM}}$ by $\langle A_5^{(0)} \rangle$ through 1-loop effective potential (the Hosotani mechanism).

Advantages and difficulties:

- Solution to the hierarchy problem ($m_h^2 \propto \Lambda^2$) for $U(1)$ gauge symmetry:

$$A_\mu(x,y) \to A_\mu(x,y) + \partial_\mu \lambda(x,y) \qquad \text{and} \qquad A_5(x,y) \to A_5(x,y) + \partial_y \lambda(x,y)$$

  forbids a mass term for A_5, however vanishing of the mass for the zero mode of A_5 is not protected: $A_5^{(0)}(x) \to A_5^{(0)}(x)$, so after the compactification $m_{A_5}^2$ could be generated in the perturbation expansion:

$$m_{A_5}^2 \propto \frac{1}{L^2}$$

  No hierarchy problem: the Higgs boson mass is calculable and finite.

- Chance for an extra source of CP violation: 5D QED compactified on a circle spontaneously breaks CP if at least two fermions are present:
  B.G. and J. Wudka, "CP violation from five-dimensional QED", Phys.Rev.Lett.93:211603,2004

- Troubles with $\rho \equiv m_W^2/(m_Z^2 \cos^2\theta) = 1$:
  B. G. and J. Wudka, "5-Dimensional Difficulties of Gauge-Higgs Unifications", Phys.Rev.Lett.97:211602,2006

- The minimal realistic model: $SO(5) \times U(1) \Rightarrow$ non-universality in t, b couplings.

⋆ Naïve approach to the hierarchy problem

$$m_h^2 = m_h^{(B)\,2} + \delta^{(SM)} m_h^2 + \cdots$$

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2}\left[\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2\right]$$

$$m_h = 130 \text{ GeV} \quad \Rightarrow \quad \delta^{(SM)} m_h^2 \simeq m_h^2 \qquad \text{for} \qquad \Lambda \simeq 580 \text{ GeV}$$

- The Veltman condition
  No Λ^2 terms at the 1-loop level:

$$\frac{3}{2}m_t^2 - \frac{1}{8}\left(6m_W^2 + 3m_Z^2\right) - \frac{3}{8}m_h^2 = 0 \qquad \Longrightarrow \qquad m_h \simeq 310 \text{ GeV}$$

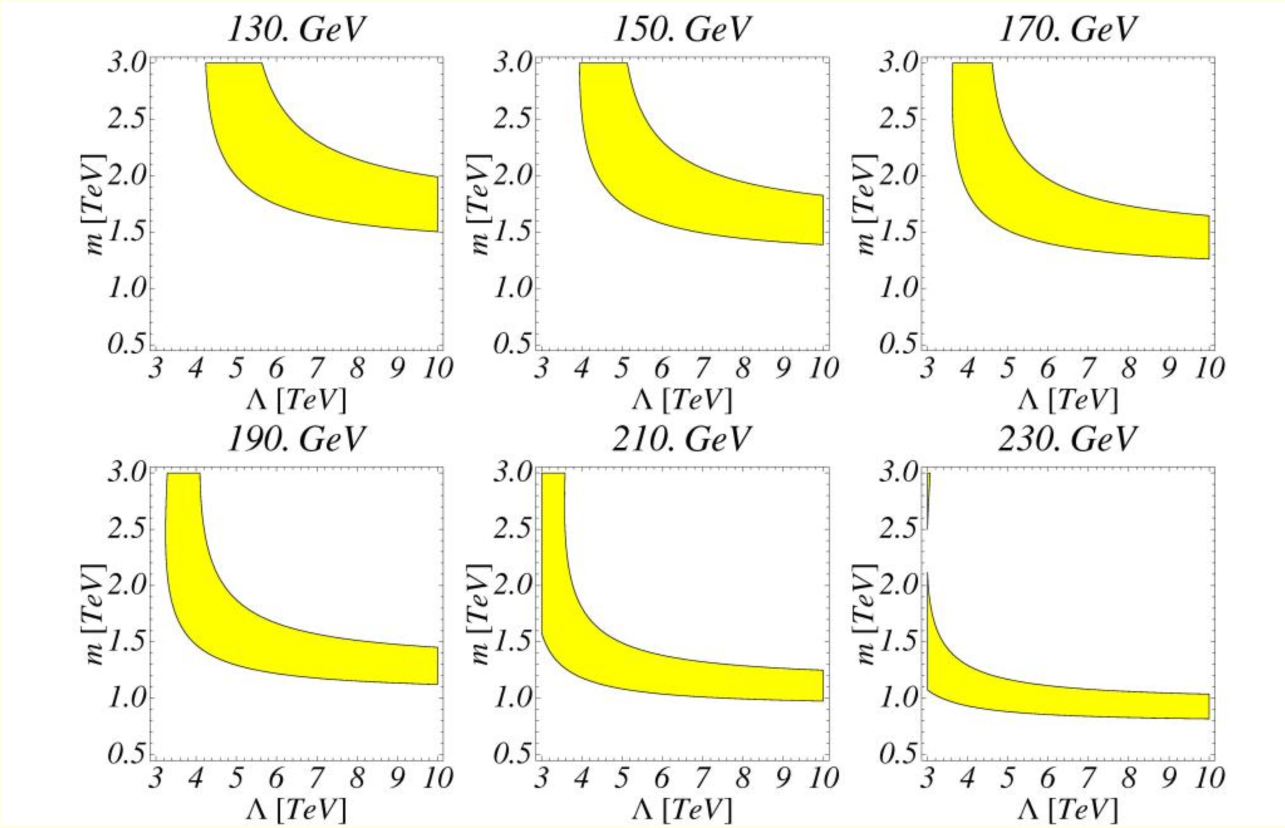
- SM + singlets ⇒ less divergence + DM

  B.G. and J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", Phys.Rev.Lett.103:091802,2009

$$V(H,\varphi_i) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\varphi^2\sum_{i=1}^{N_\varphi}\varphi_i^2 + \frac{\lambda_\varphi}{24}\sum_{i=1}^{N_\varphi}\varphi_i^4 + \lambda_x|H|^2\sum_{i=1}^{N_\varphi}\varphi_i^2$$

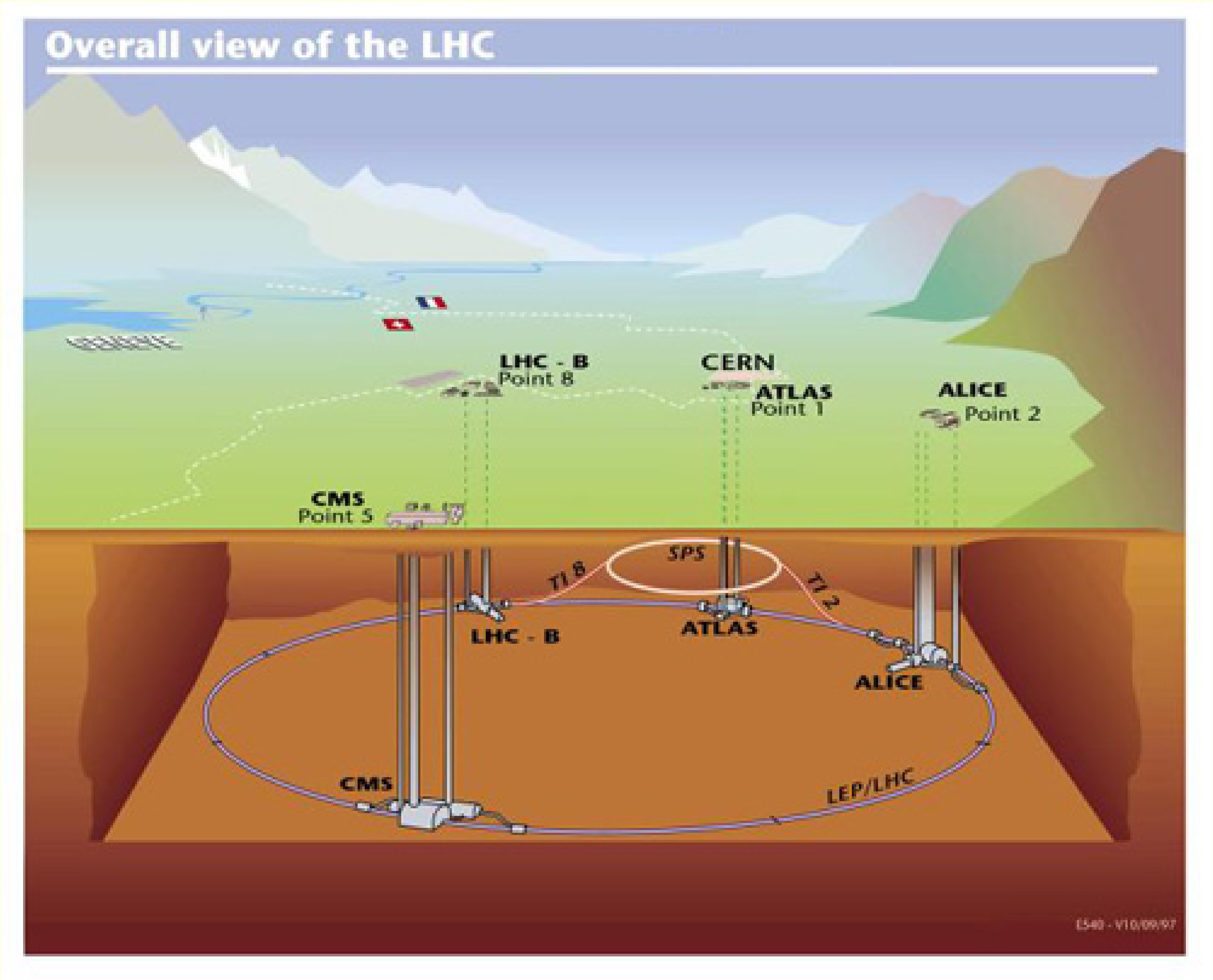
with $\langle H \rangle = \frac{v}{\sqrt{2}}$, $\quad \langle \varphi_i \rangle = 0 \quad$ for $\quad \mu_\varphi^2 > 0$

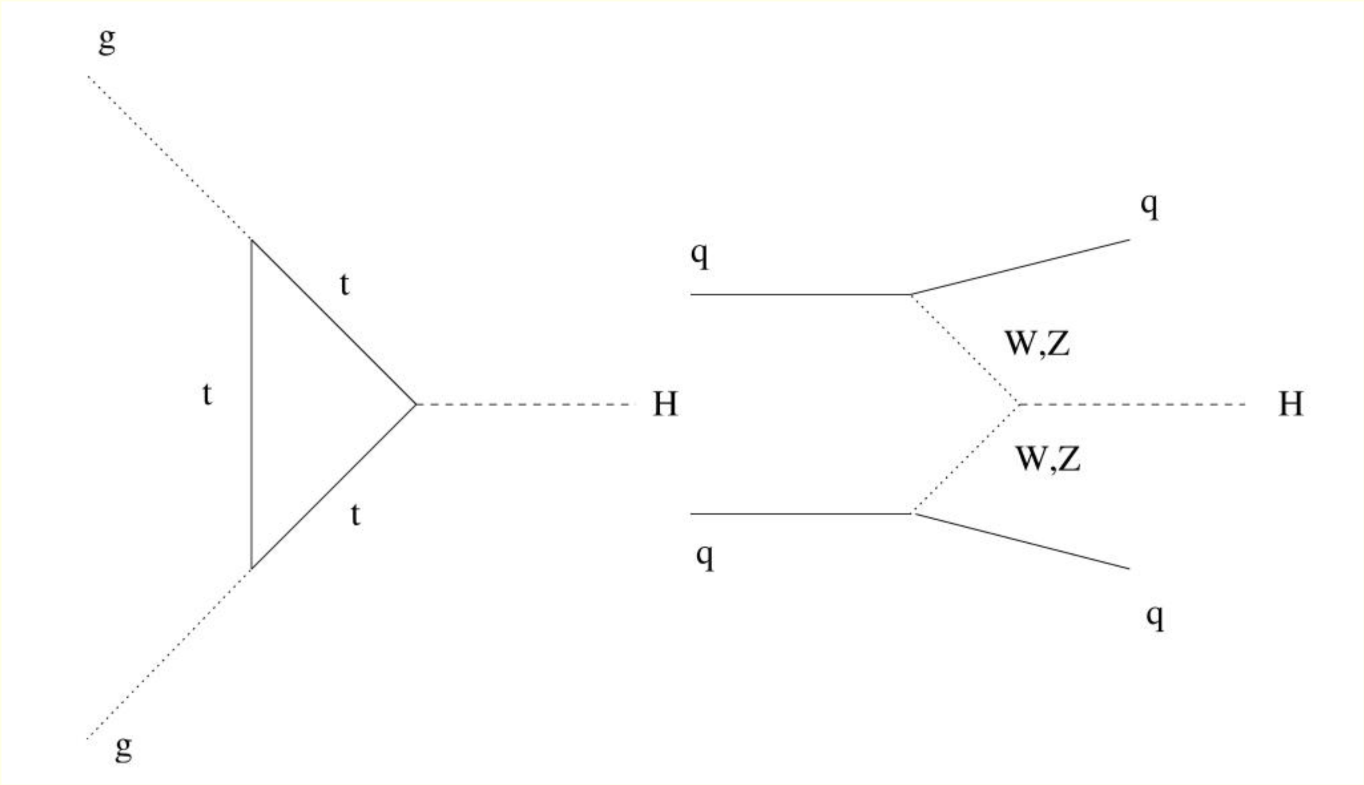
$$\delta^{(\varphi)} m_h^2 = -N_\varphi \frac{\lambda_x}{8\pi^2} \left[\Lambda^2 - m^2 \ln \left(1 + \frac{\Lambda^2}{m^2} \right) \right]$$



- 2HDM + singlets ⇒ less divergence + DM + CPV

CERN (Geneva), pp collider, 7 TeV + 7 TeV ($\sqrt{s} = 14$ TeV), $\mathcal{L}_{tot} \sim 10 - 100$ fb^{-1}/year, ($\mathcal{L} \sim 10^{33} - 10^{34}$ cm^{-2} s^{-1}), detectors: ATLAS, CMS, LHCb, ALICE





- Expected number of Higgs-boson events: $10^3 - 10^6$ per $\mathcal{L}_{tot} = 10$ fb^{-1} (1 year at the beginning) for $115 < m_h < 1000$ GeV.

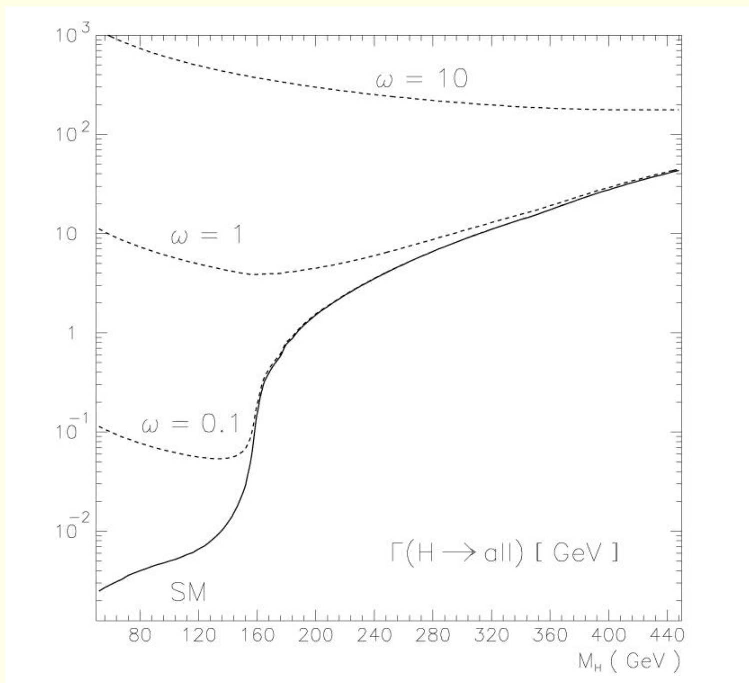
- Combining the capabilities of ATLAS and CMS only $\mathcal{L}_{tot} \sim 10$ fb^{-1} (1 year of operation at low luminosity) is needed for the SM Higgs boson discovery in the mass range $115 < m_h < 1000$ GeV at the 5σ level.

The most pessimistic scenarios:

- SM Higgs boson observed with 115 GeV $\leq m_h \lesssim 160$ GeV (production rates and couplings (BR) as in the SM).

- No Higgs boson observed:
  - New physics (strongly interacting Higgs boson sector) to be observed in $W_L W_L \to W_L W_L$.
  - N extra singlets of $SU(2) \times U(1)$ (Binoth & Van der Bij) $O(N)$ model with $\vec{\varphi}$:

$$\mathcal{L}_{\rm int} \;\; \propto \;\; \omega \vec{\varphi}^2 \, |H|^2$$



- Curvature-Higgs mixing (G. Giudice, R. Rattazzi and J. Wells, Nucl.Phys.B595:250,2001, D. Dominici, B. G. and J. F. Gunion, M. Toharia,"The Scalar sector of the Randall-Sundrum model", Nucl.Phys.B671:243,2003) and large extra dimensions (ADD)

$$\mathcal{S} = \frac{M^{2+\delta}}{2}\int d^4x d^\delta y \sqrt{-g}R + \int d^4x \sqrt{-g_{\rm ind}}\mathcal{L}_{SM} - \xi \int d^4x \sqrt{-g_{\rm ind}}R(g_{\rm ind})|H|^2$$

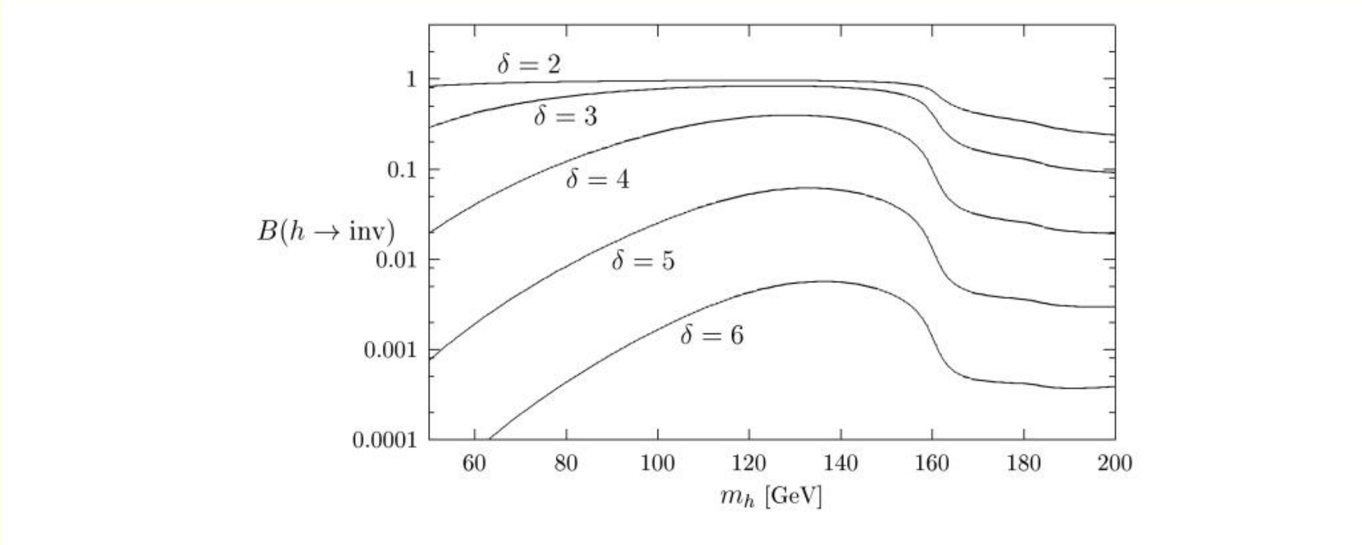
KK expansion:

$$g_{AB} = \eta_{AB} + h_{AB} \qquad \text{for} \qquad h_{AB} = \sum_{n_1=-\infty}^{+\infty} \cdots \sum_{n_\delta=-\infty}^{+\infty} \frac{h_{AB}^{(n)}(x)}{V_\delta^1/2} e^{in^j y_j/L}$$

$$\Downarrow$$

– There are scalar modes in $h_{AB}^{(n)}$: $\qquad H^{(n)}$
– Higgs-curvature mixing $\Longrightarrow \mathcal{L}_{\rm mix} = -m_{\rm mix}^2 h \sum_{\vec{n}} H^{(n)}$ for $m_{\rm mix}^2 \propto \xi$

For $\xi = 1$ and $M = 2$ TeV, $B(h \to \text{inv}) \equiv \frac{\Gamma(h \to \text{inv})}{\Gamma(h \to \text{all})}$



Optimistic scenarios:

- non-standard Higgs bosons
- superparticles
- 4th generation of quarks and leptons
- Z', W_R, $\cdots$
- KK resonances
- radion, . . .

Summary (15 : 28)

- SM drawbacks:
  - the hierarchy problem
  - lack of dark matter
  - lack of dark energy
  - baryogenesis (?)
  - the strong CP problem
- Alternatives:
  - The Randall-Sundrum model (warped extra dimensions)
  - The gauge-Higgs unification (warped?)
  - Large extra dimensions (ADD)
  - Naïve strategies
  - ...