Unparticle Physics

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- Scale invariance and conformal transformations
- The scenario – H. Georgi, "Unparticle Physics", Phys. Rev. Lett. **98**, 221601 (2007)
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Scale invariance and conformal transformations

$$x \to x' = sx$$

Is there a corresponding field transformation such that $\mathcal{S} = \int d^4x \mathcal{L}$ is invariant? Assume

$$\phi(x) \to \phi'(x') = s^{-d}\phi(x)$$

where d (the scaling dimension) is to be determined.

- scalars: $\mathcal{S}_\phi = \int d^4x \left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}\phi^4\right\} \to \underbrace{s^4 s^{-2} s^{-2d_\phi}}_{1} \mathcal{S}_\phi \Rightarrow d_\phi = 1$
- fermions: $\mathcal{S}_\psi = \int d^4x \bar{\psi} i\gamma^\mu\partial_\mu\psi \to \underbrace{s^4 s^{-1} s^{-2d_\psi}}_{1} \mathcal{S}_\psi \Rightarrow d_\psi = \frac{3}{2}$
- The scaling dimensions and dimensions coincide.
- Mass terms are not invariant:
  - scalars: $\int d^4x\, m^2\, \phi^2 \to s^4 s^{-2} \int d^4x\, m^2\, \phi^2$
  - fermions: $\int d^4x\, m\, \bar{\psi}\psi \to s^4 s^{-3} \int d^4x\, m\, \bar{\psi}\psi$

Conformal transformations (angle-preserving) are such that

$$\frac{dx^\alpha \, dx_\alpha}{|dx| \, |dx|}$$

remains unchanged, so

$$x^\mu \to x'^\mu = \frac{x^\mu - b^\mu x^2}{1 - 2b \cdot x + b^2 x^2}$$

- conformal invariance $\Rightarrow$ scale invariance
- scale invariance $\Rightarrow$ conformal invariance for all renormalizable field theories of spin ≤ 1

The scenario

$$
\begin{array}{lll}
UV & CFT: \mathcal{BZ}\ \text{fields} \searrow & \swarrow\ SM\ \text{fields} \\
\downarrow & & \\
M_{\mathcal{U}} & & \frac{1}{M_{\mathcal{U}}^k}\mathcal{O}_{\mathcal{BZ}}\mathcal{O}_{\mathsf{SM}} \\
\downarrow & & \\
\Lambda_{\mathcal{U}} & \text{transmutation scale} & \downarrow \\
\downarrow & & \\
IR & \text{fixed point} & c_{\mathcal{U}}\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\mathcal{O}_{\mathcal{U}}\mathcal{O}_{\mathsf{SM}}
\end{array}
$$

where $k = d_{\mathsf{SM}} + d_{\mathcal{BZ}} - 4$. d_{SM} and $d_{\mathcal{BZ}}$ are canonical dimensions of $\mathcal{O}_{\mathsf{SM}}$ and $\mathcal{O}_{\mathcal{BZ}}$, respectively, while $d_{\mathcal{U}}$ is the scaling dimension (the same as the mass dimension in this case) of $\mathcal{O}_{\mathcal{U}}$:

$$\mathcal{O}_{\mathcal{U}}(x) \to \mathcal{O}'_{\mathcal{U}}(x') = s^{-d_{\mathcal{U}}}\mathcal{O}_{\mathcal{U}}(x) \quad \text{with} \quad 1 < d_{\mathcal{U}} < 2 \quad \text{for} \quad x \to x' = sx$$

An example of matching between $\mathcal{O}_{\mathcal{BZ}}$ and $\mathcal{O}_{\mathcal{U}}$:

- $(\bar{q}q)$ in QCD $\quad \Longleftrightarrow \quad M \propto (\bar{q}q)$ mesons in the chiral non-linear model

Anomalous breaking of scale invariance and fixed points

The scale invariance implies the following Ward identity for 1PI Green's function $\Gamma^{(n)}$:

$$\partial^\mu D_\mu = 0 \quad \Rightarrow \quad \left(-\frac{\partial}{\partial t} + D\right)\Gamma^{(n)}(e^t p_1, \ldots, e^t p_{n-1}) = 0$$

where $D = 4 - n d_\phi$ is the canonical dimension of $\Gamma^{(n)}$. The solution reads

$$\Gamma^{(n)}(s p_i) = s^D \Gamma^{(n)}(p_i) \qquad (\star)$$

for $s = e^t$.
However, since the loop expansion requires some sort of regularization, (so some scale must be introduced: μ in the dimensional regularization, or Λ in the cutoff regularization), therefore one can expect that classical scale invariance (i.e. invariance of the Lagrangian) would be broken at the quantum (loops) level.

One can use the RGE to verify if the canonical scaling ($\star$) is satisfied:

$$\left[-\frac{\partial}{\partial t} + \beta(\lambda)\frac{\partial}{\partial\lambda} + (\gamma_m - 1)m\frac{\partial}{\partial m} - n\gamma(\lambda) + D\right]\Gamma^{(n)}(sp_i) = 0$$

for

$$\beta = \mu\frac{d\lambda}{d\mu}, \qquad \gamma = \frac{1}{2}\mu\frac{d}{d\mu}Z_3, \qquad \gamma_m = \frac{\mu}{m}\frac{dm}{d\mu}$$

In a massless, non-interacting theory the scaling is canonical!
In general, there is no scaling in a quantum field theory, even if $m = 0$. The solution of the RGE reads:

$$\Gamma^{(n)}(e^t p_i, \lambda, \mu) = (e^t)^D \Gamma^{(n)}(p_i, \bar{\lambda}(t), \mu)\, e^{-n\int_0^t \gamma[\bar{\lambda}(t')]dt'}$$

- Assume there is an IR fixed point at $\lambda = \lambda_{IR}$: so $\beta(\lambda_R) = 0$. Then
-
$$e^{-n\int_0^t \gamma[\bar{\lambda}(t')]dt'} = e^{-n\gamma[\lambda_{IR}]t}$$

  and
- $\Gamma^{(n)}(sp_i) = s^{D-n\gamma(\lambda_{IR})}\Gamma^{(n)}(p_i)$

Examples of the $\mathcal{BZ}$ sector

- Banks & Zaks (1982): SU(3) YM with n massless fermions in e.g. fundamental representation

$$\beta(g) = -\left(\beta_0 \frac{g^3}{16\pi^2} + \beta_1 \frac{g^5}{(16\pi^2)^2} + 3 \text{ loops} \cdots \right)$$

$$\begin{array}{lll} \beta_0 = 11 - \frac{2}{3}n & \beta_0(n_0) = 0 & n_0 = 16.5 \\ \beta_1 = 102 - \frac{38}{3}n & \beta_1(n_1) = 0 & n_1 \simeq 8.05 \end{array}$$

If $n_1 < n < n_0$ (so $\beta_0 > 0$ & $\beta_1 < 0$) then keeping β_0 and β_1 one gets

$$\beta(g_{IR}) = 0 \quad \text{for} \quad \frac{g_{IR}^2}{16\pi^2} = -\frac{33 - 2n}{306 - 38n}$$

Conclusions:

– If $g = g_{IR}$, then the low-energy theory is scale invariant with small anomalous scaling

– For $n \lesssim n_0$, the theory remains perturbative, so the continuous spectrum doesn't emerge.

Correlators and scaling

$$\langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \rho_{\mathcal{U}}(p^2)$$

for $\rho_{\mathcal{U}}(p^2) = (2\pi)^4 \int d\lambda\, \delta^{(4)}(p - p_\lambda)\, |\langle 0|\mathcal{O}_{\mathcal{U}}(0)|\lambda\rangle|^2$.

- Scaling:

$$\mathcal{O}_{\mathcal{U}}(x) \to \mathcal{O}'_{\mathcal{U}}(x') = s^{-d_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}}(x) \quad \text{with} \quad 1 < d_{\mathcal{U}} < 2 \quad \text{for} \quad x \to x' = sx$$

-

$$\rho_{\mathcal{U}}(p^2) = \int d^4x\, e^{ipx} \langle 0|\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle \quad \Rightarrow \quad \rho_{\mathcal{U}}(p^2) = A_{d_{\mathcal{U}}} \theta(p^0)\theta(p^2)(p^2)^\alpha$$

where $A_{d_{\mathcal{U}}}$ is a normalization constant and α is to be determined.

$$\Downarrow$$

$$\alpha = d_{\mathcal{U}} - 2$$

Phase space for unparticles

- In a free scalar quantum field theory

$$\langle 0|\phi(x)\phi(0)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ipx} = \int \underbrace{\frac{d^4p}{(2\pi)^3}\theta(p^0)\delta(p^2-m^2)}_{d\Phi} e^{-ipx} = \int d\Phi e^{-ipx}$$

- For unparticles

$$\langle 0|\mathcal{O}_\mathcal{U}(x)\mathcal{O}_\mathcal{U}(0)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\rho_\mathcal{U}(p^2)e^{-ipx} \quad \text{for} \quad \rho_\mathcal{U}(p^2) = A_{d_\mathcal{U}}\theta(p^0)\theta(p^2)(p^2)^{d_\mathcal{U}-2}$$

$$\Downarrow$$

$$d\Phi_\mathcal{U}(p_\mathcal{U}) = A_{d_\mathcal{U}}\theta(p^0)\theta(p_\mathcal{U}^2)(p_\mathcal{U}^2)^{d_\mathcal{U}-2}\frac{d^4p_\mathcal{U}}{(2\pi)^4}$$

Note that integrating the phase space for n massless particles one gets

$$\int (2\pi)^4\delta^{(4)}\left(p-\sum_{i=1}^n\right)\prod_{i=1}^n \theta(p_i^0)\delta(p_i^2)\frac{d^4p_i}{(2\pi)^3} = A_n\theta(p^0)\theta(p^2)(p^2)^{n-2}$$

for

$$A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n-1)\Gamma(2n)} \quad \Rightarrow \quad A_{d_\mathcal{U}} = A_{n=d_\mathcal{U}}$$

Georgi:

"Unparticle stuff with scale dimension $d_{\mathcal{U}}$ looks like a non-integer number $d_{\mathcal{U}}$ of massless particles."

The limit $n \longrightarrow 1$ reproduces the single particle phase space:

$$\lim_{\epsilon \to 0^+} \frac{\epsilon \theta(x)}{x^{1-\epsilon}} = \delta(x) \quad \text{for} \quad x = p^2 \text{ and } \quad \epsilon \equiv n - 1$$

Unparticle propagator

The Feynman propagator:

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx}\langle 0|T\left\{\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)\right\}|0\rangle = \int_0^\infty \frac{dm^2}{2\pi}\rho_{\mathcal{U}}(m^2)\frac{i}{p^2 - m^2 + i\varepsilon}$$

From the scaling properties $\rho_{\mathcal{U}}(m^2) = A_{d_{\mathcal{U}}}\theta(m^2)\left(m^2\right)^{d_{\mathcal{U}}-2}$, so

$$\Delta_F^{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\sin(\pi d_{\mathcal{U}})}\frac{1}{(-p^2 - i\varepsilon)^{2-d_{\mathcal{U}}}}$$

- Non-trivial phase:

$$\mathbf{Im}\left\{\Delta_F^{\mathcal{U}}(p^2)\right\} = -\frac{A_{d_{\mathcal{U}}}}{2}\theta(p^2)(p^2)^{d_{\mathcal{U}}-2}$$

- Interference with the Z-boson:

$$\Delta_Z(p^2) = \frac{1}{p^2 - m_Z^2 + iM_Z\Gamma_Z}$$

- $\mathbf{Im}\left\{\Delta_F^{\mathcal{U}}(p^2)\right\} \neq 0$ doesn't imply unparticle decay!

An example: $t \to u\,\mathcal{O}_{\mathcal{U}}$

$$\mathcal{L}_{\rm int} = i\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\,\bar{u}\gamma_\mu(1-\gamma_5)t\,\partial^\mu \mathcal{O}_{\mathcal{U}} + \text{H.c.} \qquad \text{for} \quad \lambda = c_{\mathcal{U}}\left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{\mathcal{BZ}}}$$

The differential decay rate

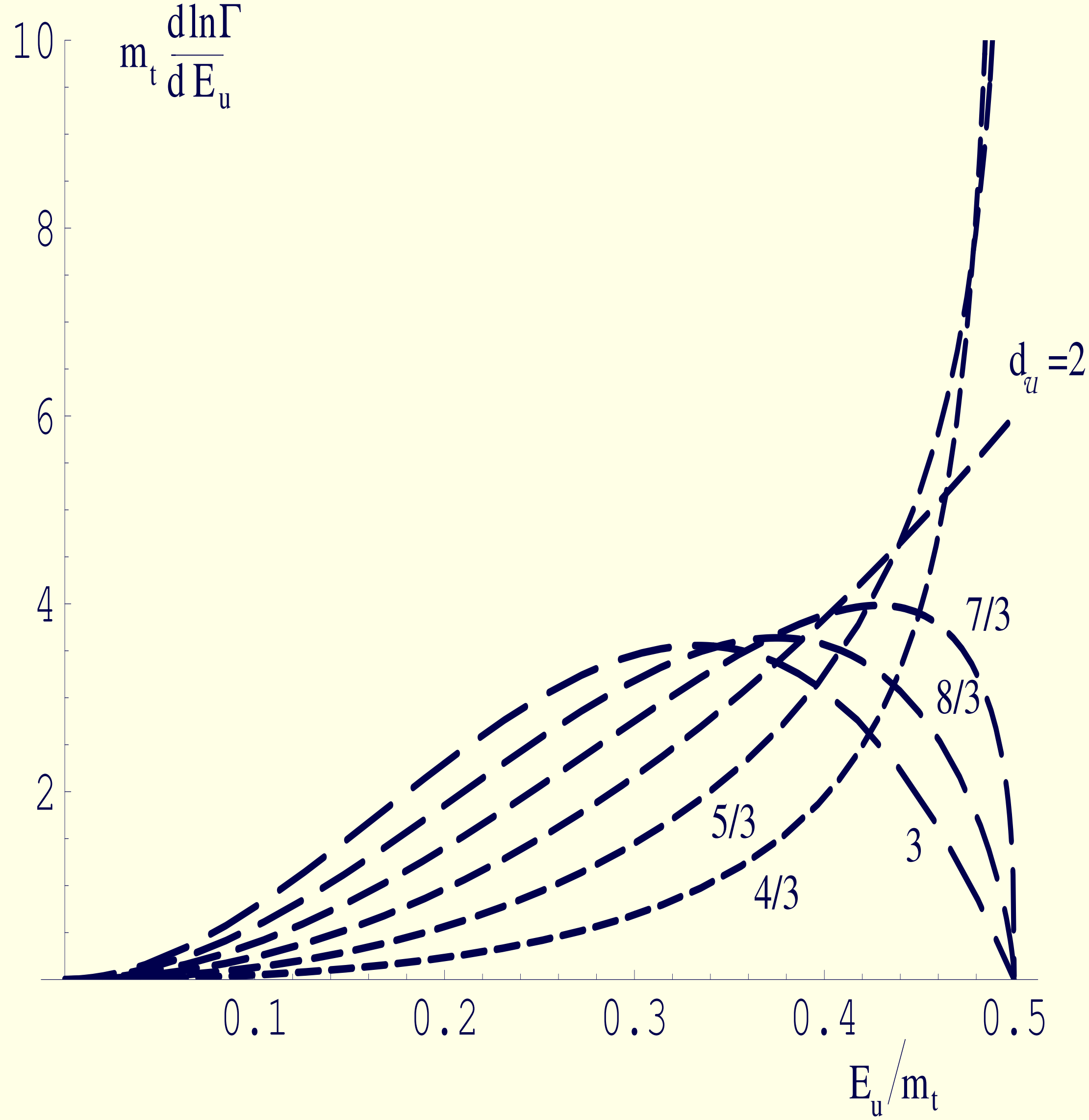
$$d\Gamma = \frac{|\mathcal{M}|^2}{2m_t}\,d\Phi_{\mathcal{U}}\,d\Phi_u$$

$$\Downarrow$$

$$\frac{d\Gamma}{dE_u} = \frac{A_{d_{\mathcal{U}}} m_t^2 E_u^2 |\lambda|^2}{2\pi^2\Lambda_{\mathcal{U}}^{2d_{\mathcal{U}}}}\frac{\theta(m_t - 2E_u)}{(m_t^2 - 2m_t E_u)^{2-d_{\mathcal{U}}}}$$

$$\Downarrow$$

$$m_t\frac{1}{\Gamma}\frac{d\Gamma}{dE_u} = 4d_{\mathcal{U}}(d_{\mathcal{U}}^2 - 1)\left(1 - 2\frac{E_u}{m_t}\right)^{d_{\mathcal{U}}-2}\left(\frac{E_u}{m_t}\right)^2$$


$m_t \frac{d\ln\Gamma}{dE_u}$
10
8
6
4
2
0.1
0.2
0.3
0.4
0.5
E_u/m_t
$d_u=2$
7/3
8/3
5/3
3
4/3

Couplings of unparticles to the SM

Assumptions:

- $\mathcal{O}_{\mathcal{U}}$ in neutral under the SM gauge group
- $\dim(\mathcal{O}_{\mathsf{SM}}) \leq 4$

$$\mathcal{L}_{\rm int} = c_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \mathcal{O}_{\mathcal{U}} \mathcal{O}_{\mathsf{SM}} \propto \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^k \Lambda_{\mathcal{U}}^{4-d_{\mathsf{SM}}-d_{\mathcal{U}}} \mathcal{O}_{\mathcal{U}} \mathcal{O}_{\mathsf{SM}} \quad \text{for} \quad k = d_{\mathsf{SM}} + d_{\mathcal{BZ}} - 4$$

- Scalar unparticles $\mathcal{O}_{\mathcal{U}}$: $\propto \Lambda_{\mathcal{U}}^{2-d_{\mathcal{U}}} H^\dagger H \mathcal{O}_{\mathcal{U}}$ for $d_{\mathsf{SM}} = 2$
- Spinor unparticles $O_{\mathcal{U}}^s$: $\propto \Lambda_{\mathcal{U}}^{5/2-d_{\mathcal{U}}} \bar{\nu}_R O_{\mathcal{U}}^s$ for $d_{\mathsf{SM}} = 3/2$

Deconstruction of unparticles

Källen-Lehman representation of the Feynman propagator:

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx} \langle 0 | T\{\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)\} | 0 \rangle = \int_0^\infty \frac{dm^2}{2\pi} \rho(m^2) \frac{i}{p^2 - m^2 + i\varepsilon}$$

with $\rho_{\mathcal{U}}(m^2) = A_{d_{\mathcal{U}}} \theta(m^2)(m^2)^{d_{\mathcal{U}}-2}$. Deconstruction (Stephanov'07):

$$\mathcal{O}_{\mathcal{U}} \to \sum_{n=0}^{\infty} F_n \varphi_n \quad \text{with} \quad m_n^2 = \Delta^2 n$$

Then

$$i\Delta_F^{\mathcal{U}}(p^2) = \int d^4x e^{ipx} \langle 0 | T\{\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)\} | 0 \rangle = \sum_{n=0}^{\infty} \frac{iF_n^2}{p^2 - m_n^2 + i\varepsilon}$$

if $F_n^2 = \frac{A_{d_{\mathcal{U}}}}{2\pi} \Delta^2 (m_n^2)^{d_{\mathcal{U}}-2}$ then

$$i\frac{A_{d_{\mathcal{U}}}}{2\pi} \sum_{n=0}^{\infty} \frac{(m_n^2)^{d_{\mathcal{U}}-2}}{p^2 - m_n^2 + i\varepsilon} \Delta^2 \underset{\Delta\to 0}{\longrightarrow} i\frac{A_{d_{\mathcal{U}}}}{2\pi} \int \frac{(m^2)^{d_{\mathcal{U}}-2} dm^2}{p^2 - m^2 + i\varepsilon} = \int \frac{dm^2}{2\pi} \rho(m^2) \frac{i}{p^2 - m^2 + i\varepsilon}$$

Let's focus on the non-trivial phase:

$$\mathbf{Im}\left\{\sum_{n=0}^{\infty}\frac{F_n^2}{p^2-m_n^2+i\varepsilon}\right\} = -\sum_n F_n^2\pi\delta(p^2-m_n^2) \underset{\Delta\to 0}{\longrightarrow} -\frac{A_{d_\mathcal{U}}}{2}\theta(p^2)(p^2)^{d_\mathcal{U}-2}$$

So, each peak becomes lower as $F_n^2 \sim \Delta^2 \to 0$, but their density increases.

- Each mode φ_n breaks the scale invariance.
- In the limit

$$\lim_{N\to\infty}\sum_{n=0}^{N}$$

the scale invariance is recovered.

The deconstruction for $t \to u\mathcal{O}_\mathcal{U}$ decay

$$i\frac{\lambda}{\Lambda_\mathcal{U}^{d_\mathcal{U}}}\,\bar{u}\gamma_\mu(1-\gamma_5)t\,\partial^\mu\mathcal{O}_\mathcal{U} \longrightarrow i\frac{\lambda}{\Lambda_\mathcal{U}^{d_\mathcal{U}}}\,\bar{u}\gamma_\mu(1-\gamma_5)t\,\sum_{n=0}^{\infty}F_n\partial^\mu\varphi_n$$

$$\Downarrow$$

$$\Gamma(t\to u\varphi_n) = \frac{\lambda^2}{\Lambda_\mathcal{U}^{2d_\mathcal{U}}}\frac{m_t E_u^2}{2\pi}F_n^2 \quad \text{with} \quad E_u = \frac{m_t^2 - m_n^2}{2m_t} \ \text{and} \ F_n^2 = \frac{A_{d_\mathcal{U}}}{2\pi}\Delta^2(m_n^2)^{d_\mathcal{U}-2}$$

Number of states $|\varphi_n\rangle$ in the interval $(E_u, E_u + dE_u)$: $dN = dE_u\frac{2m_t}{\Delta^2}$

$$\Downarrow$$

$$\frac{d\Gamma}{dE_u} = \frac{2m_t}{\Delta^2}\Gamma(t\to u+\varphi_n) = \frac{\lambda^2}{\Lambda_\mathcal{U}^{2d_\mathcal{U}}}A_{d_\mathcal{U}}\frac{m_t^2}{2\pi^2}\,E_u^2(m_t^2 - 2m_tE_u)^{(d_\mathcal{U}-2)}\theta(m_t - 2E_u)$$

The same as the Georgi's result!

Spontaneous symmetry breaking with unparticles and Higgs boson physics

(Delgado, Espinosa, Quiros'07)

$$UV: \qquad \frac{1}{M_{\mathcal{U}}^{d_{\mathcal{BZ}}-2}}|H|^2\,\mathcal{O}_{\mathcal{BZ}}$$

$$\Downarrow$$

$$IR: \quad c_{\mathcal{U}}\left(\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\mathcal{BZ}}-2}}|H|^2\right)\mathcal{O}_{\mathcal{U}} \equiv \kappa_{\mathcal{U}}|H|^2\,\mathcal{O}_{\mathcal{U}}$$

Deconstruction ($\mathcal{O}_{\mathcal{U}} \to \sum_n F_n\varphi_n$, $m_n^2 = \Delta^2 n$) $\Rightarrow$

$$V_{\rm tot} = m^2|H|^2 + \lambda|H|^4 + \delta\,V$$

for

$$\delta\,V = \frac{1}{2}\sum_{n=0}^{\infty} m_n^2\varphi_n^2 + \kappa_{\mathcal{U}}|H|^2\sum_{n=0}^{\infty}F_n\varphi_n$$

$$\langle\varphi_n\rangle = -\frac{\kappa_{\mathcal{U}}v^2F_n}{m_n^2} \quad \text{for} \quad \langle|H|^2\rangle = v^2, \qquad F_n^2 = \frac{A_{d_{\mathcal{U}}}}{2\pi}\Delta^2(m_n^2)^{d_{\mathcal{U}}-2}$$

So,

$$\langle\mathcal{O}_{\mathcal{U}}\rangle = \sum_{n=0}^{\infty}F_n\langle\varphi_n\rangle \longrightarrow -\kappa_{\mathcal{U}}v^2\frac{A_{d_{\mathcal{U}}}}{2\pi}\int_0^{\infty}\frac{dm^2}{(m^2)^{3-d_{\mathcal{U}}}} = -\infty$$

- The IR divergence!

- A possible regularization $\delta\, V' = \zeta |H|^2 \sum \varphi_n^2$ is not scale invariant.

Since the scaling invariance is anyway violated by the vacuum expectation value $\neq 0$ through $|H|^2 \mathcal{O}_{\mathcal{U}}$ so we adopt

$$\delta\, V' = \zeta |H|^2 \sum_n \varphi_n^2$$

as the IR regulator. Then

$$v_n = \langle \varphi_n \rangle = -\frac{\kappa_{\mathcal{U}} v^2}{m_n^2 + \zeta v^2} F_n$$

The minimization for H reads:

$$m^2 + \lambda v^2 + \kappa_{\mathcal{U}} \sum_n F_n v_n + \zeta \sum_n v_n^2 = 0$$

Inserting v_n one gets in the continuum limit ($\Delta \to 0$):

$$m^2 + \lambda v^2 - \lambda_{\mathcal{U}} (\mu_{\mathcal{U}}^2)^{2-d_{\mathcal{U}}} v^{2(d_{\mathcal{U}}-1)} = 0$$

$$\text{for} \quad \lambda_{\mathcal{U}} \equiv \frac{d_{\mathcal{U}}}{4}\zeta^{d_{\mathcal{U}}-2}\Gamma(d_{\mathcal{U}}-1)\Gamma(2-d_{\mathcal{U}}) \quad \text{and} \quad (\mu_{\mathcal{U}}^2)^{2-d_{\mathcal{U}}} \equiv \kappa_{\mathcal{U}}^2 \frac{A_{d_{\mathcal{U}}}}{2\pi}$$

$$V_{\text{eff}} = m^2|H|^2 + \lambda|H|^4 - \frac{2^{d_{\mathcal{U}}-1}}{d_{\mathcal{U}}}\lambda_{\mathcal{U}}(\mu_{\mathcal{U}}^2)^{2-d_{\mathcal{U}}}|H|^{2d_{\mathcal{U}}}$$

Even if $m^2 = 0$ one can get the vacuum expectation value $\neq 0$ ($\Lambda_{\mathcal{U}}$ provides the scale):

$$v^2 = \left(\frac{\lambda_{\mathcal{U}}}{\lambda}\right)^{\frac{1}{2-d_{\mathcal{U}}}} \mu_{\mathcal{U}}^2 \quad \text{for} \quad \mu_{\mathcal{U}}^2 = \left(\frac{A_{d_{\mathcal{U}}}}{2\pi}\right)^{\frac{1}{2-d_{\mathcal{U}}}} \left(\frac{\Lambda_{\mathcal{U}}^2}{M_{\mathcal{U}}^2}\right)^{\frac{d_{\mathcal{BZ}}-2}{2-d_{\mathcal{U}}}} \Lambda_{\mathcal{U}}^2$$

UnCosmology

B. G. and Jose Wudka, "UnCosmology," arXiv:0809.0977

• The equation of state for unparticles

The trace anomaly of the energy momentum tensor for a gauge theory with massless fermions:

$$\theta_\mu{}^\mu = \frac{\beta}{2g} N \left[F_a^{\mu\nu} F_{a\ \mu\nu} \right] \tag{1}$$

where β denotes the beta function and N stands for the normal product.

Non-trivial IR fixed point at $g = g_\star$, so in the IR we assume

$$\beta = \delta(g - g_\star), \quad \delta > 0$$

in which case the running coupling reads

$$g(\mu) = g_\star + c\mu^\delta; \qquad \beta[g(\mu)] = \delta c \mu^\delta$$

where c is an integration constant and μ is the renormalization scale.

From the thermal average of (1) choosing the renormalization scale $\mu = T$ and using $\langle \theta_\mu{}^\mu \rangle = \rho_\mathcal{U} - 3p_\mathcal{U}$, we get

$$\rho_\mathcal{U} - 3p_\mathcal{U} = \frac{\beta}{2g_\star} \langle N\left[F_a^{\mu\nu} F_{a\ \mu\nu}\right] \rangle = AT^{4+\gamma}$$

$$\rho_\mathcal{U} - 3p_\mathcal{U} = AT^{4+\gamma}$$

$$\Downarrow$$

$$\rho_\mathcal{U} = \sigma T^4 + A\left(1 + \frac{3}{\gamma}\right) T^{4+\gamma} \quad \text{and} \quad p_\mathcal{U} = \sigma\frac{T^4}{3} + \frac{A}{\gamma} T^{4+\gamma}$$

where σ is an integration constant.

$$\Downarrow$$

$$\boxed{p_\mathcal{U} = \tfrac{1}{3}\rho_\mathcal{U}\left(1 - B\rho_\mathcal{U}^{\gamma/4}\right) \quad \text{for} \quad B \equiv \frac{A}{\sigma^{1+\gamma/4}}}$$

One can expect that $A \propto \Lambda_\mathcal{U}^{-\gamma}$, therefore we obtain

$$\boxed{\rho_{\mathsf{NP}} = \frac{\pi^2}{30} T^4 \times \begin{cases} g_{\mathsf{IR}} + f\left(\frac{T}{\Lambda_\mathcal{U}}\right)^\gamma & \text{for} \quad T \lesssim \Lambda_\mathcal{U} \\ g_{\mathcal{BZ}} & \text{for} \quad T \gtrsim \Lambda_\mathcal{U} \end{cases}}$$

where $g_{\mathcal{BZ}} = 2(n_c^2 - 1 + \frac{7}{8} n_c n_f)$ for $SU(n_c)$ with n_f flavours in the $\mathcal{BZ}$ sector.

- From the continuity at $T = \Lambda_{\mathcal{U}}$, the constant f could be determined: $f = g_{\mathcal{BZ}} - g_{\mathsf{IR}}$.

- We will assume $g_{\mathcal{BZ}} \sim g_{\mathsf{IR}}$.

$$\boxed{\rho_{\mathsf{NP}} = \frac{\pi^2}{30} T^4 \times \begin{cases} g_{\mathsf{IR}} + f \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{\gamma} & \text{for} \quad T \lesssim \Lambda_{\mathcal{U}} \\ g_{\mathcal{BZ}} & \text{for} \quad T \gtrsim \Lambda_{\mathcal{U}} \end{cases}}$$

Deconstruction (Stephanov'07):

$$\mathcal{O}_{\mathcal{U}} \to \sum_{n=0}^{\infty} F_n \varphi_n \quad \text{with} \quad m_n^2 = \Delta^2 n$$

The above result fits the following guess for the effective number of degrees of freedom:

$$g_{\mathcal{U}}(T) \propto \frac{\int_0^{T^2} dM^2 \rho(M^2) \theta(\Lambda_{\mathcal{U}}^2 - M^2)}{\int_0^{\Lambda_{\mathcal{U}}^2} dM^2 \rho(M^2)}$$

where $\rho(M^2) \propto (M^2)^{(d_{\mathcal{U}}-2)}$. Then

$$g_{\mathcal{U}}(T) \propto \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{2(d_{\mathcal{U}}-1)}$$

$\Longrightarrow$ In the presence of just one unparticle operator one can argue that $\gamma = 2(d_{\mathcal{U}} - 1)$.

• Freeze-out and thaw-in

♣ *Brief history of the Universe in the presence of unparticles (no mass-gap).*

- $T \gg M_{\mathcal{U}}$: the $\mathcal{BZ}$ sector in form of massless particles (no unparticles yet), thermal equilibrium with the SM is maintained (assumption), so $T = T_{\mathcal{BZ}} = T_{\mathsf{SM}}$

- $T \lesssim M_{\mathcal{U}}$:
  - The $\mathcal{BZ}$ sector starts to decouple, as the average energy is no longer sufficient to create mediators.
  - However, the thermal equilibrium may still be maintained ($T = T_{\mathcal{BZ}} = T_{\mathsf{SM}}$) depending on the strength of effective couplings between the SM and the extra sector (which at higher temperature, $T \gtrsim \Lambda_{\mathcal{U}}$, is made of the $\mathcal{BZ}$ matter, while below $\Lambda_{\mathcal{U}}$ of unparticles).

Let's denote by T_f the decoupling temperature at which

$$\Gamma(SM \leftrightarrow NP) \simeq H$$

where H is the Hubble parameter

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho_{\mathsf{tot}}(T) \qquad \text{for} \qquad \rho_{\mathsf{tot}} = \rho_{\mathsf{SM}} + \rho_{\mathsf{NP}}$$

There are 2 interesting cases:

- $M_{\mathcal{U}} > T_f > \Lambda_{\mathcal{U}}$:
  - T_f is determined by the condition

$$\Gamma(SM \leftrightarrow \mathcal{BZ}) \simeq H$$

  - For $T > T_f$ the SM and the $\mathcal{BZ}$ sectors evolve in thermal equilibrium, but even for $T < T_f$ their temperatures remain equal ($T = T_{\mathcal{BZ}} = T_{\mathsf{SM}}$) since $\Lambda_{\mathcal{U}} > v$.

- $\Lambda_{\mathcal{U}} > T_f$:
  - Till $T = \Lambda_{\mathcal{U}}$ the SM and unparticles still have the same temperature.
  - For $\Lambda_{\mathcal{U}} \gtrsim T \gtrsim T_f$ still the equilibrium is maintained (assumption, in general this depends on $d_{\mathcal{U}}$). The decoupling temperature T_f must be now determined by

$$\Gamma(SM \leftrightarrow \mathcal{O}_{\mathcal{U}}) \simeq H$$

  - Till $T \sim v$ temperatures of SM and unparticles remain equal, at $T \sim v$ they split.

$\Longrightarrow$ The unparticle cosmic background should be there.

♣ *The Banks-Zaks phase.*

$$\mathcal{L}_{\mathcal{BZ}} = \frac{1}{M_{\mathcal{U}}} \left(H^\dagger H\right) \left(\bar{q}_{\mathcal{BZ}} q_{\mathcal{BZ}}\right)$$

Then

$$\Gamma_{\mathcal{BZ}} \propto \frac{T^3}{M_{\mathcal{U}}^2} \quad \text{and} \quad H \propto \frac{T^2}{M_{Pl}} \quad \Longrightarrow \quad \text{decoupling for} \quad T \lesssim T_{f-\mathcal{BZ}}$$

♣ *The unparticle phase.*

$$\mathcal{L}_{\mathcal{U}} = \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \mathcal{O}_{\mathcal{U}} \mathcal{O}_{\mathsf{SM}} \quad \text{for} \quad k = d_{\mathsf{SM}} + d_{\mathcal{BZ}} - 4$$

The most relevant operators for scalar unparticles are

$$\mathcal{L}_s = \frac{\Lambda_{\mathcal{U}}^{3-d_{\mathcal{U}}}}{M_{\mathcal{U}}} \left(H^\dagger H\right) \mathcal{O}_{\mathcal{U}}, \quad \mathcal{L}_f = \frac{\Lambda_{\mathcal{U}}^{3-d_{\mathcal{U}}}}{M_{\mathcal{U}}^3} \left(\bar{\ell} H e\right) \mathcal{O}_{\mathcal{U}}, \quad \mathcal{L}_v = \frac{\Lambda_{\mathcal{U}}^{3-d_{\mathcal{U}}}}{M_{\mathcal{U}}^3} \left(B_{\mu\nu} B^{\mu\nu}\right) \mathcal{O}_{\mathcal{U}}$$

$$\mathcal{L}_s \quad \Longrightarrow \quad \Gamma_{\mathcal{U}} \propto \frac{\Lambda_{\mathcal{U}}^3}{M_{\mathcal{U}}^2} \left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{2d_{\mathcal{U}}-3} \quad \text{and} \quad H \propto \frac{T^2}{M_{Pl}} \quad \Longrightarrow \quad T_{f-\mathcal{U}}$$

$$\frac{\Gamma_{\mathcal{U}}}{H} \propto T^{2d_{\mathcal{U}}-5} \quad \Longrightarrow \quad \begin{cases} d_{\mathcal{U}} > \frac{5}{2} & \text{decoupling for} \quad T < T_{f-\mathcal{U}} \quad \text{freeze-out} \\ d_{\mathcal{U}} < \frac{5}{2} & \text{decoupling for} \quad T > T_{f-\mathcal{U}} \quad \text{thaw-in} \end{cases}$$

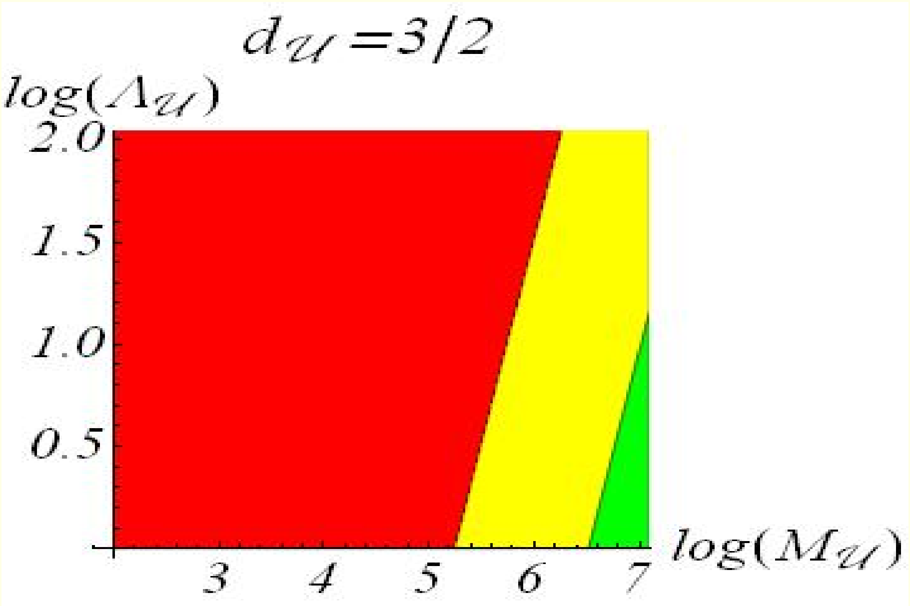


Figure 1: Regions of $(M_{\mathcal{U}}, \Lambda_{\mathcal{U}})$ for various scenarios of decoupling for $d_{\mathcal{U}} = 3/2$.

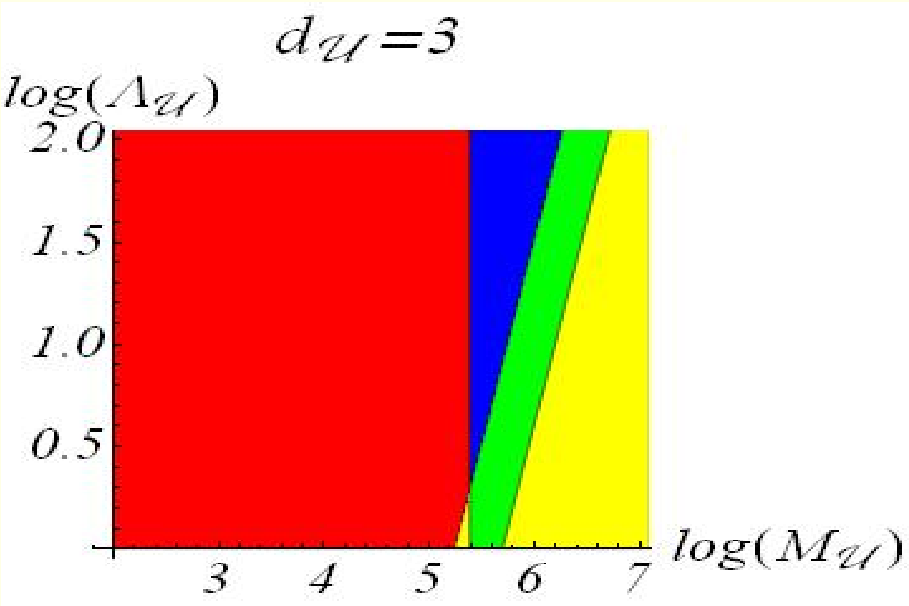


Figure 2: Regions of $(M_{\mathcal{U}}, \Lambda_{\mathcal{U}})$ for various scenarios of decoupling for $d_{\mathcal{U}} = 3$.

	decoupling in $\mathcal{BZ}$ phase	decoupling in the unparticle phase
green	+	+
blue	−	+
yellow	+	−
red	−	−

• BBN constraints

Big-Bang Nucleosynthesis $\Longrightarrow \Delta N_\nu = -0.37^{+0.10}_{-0.11} \Longrightarrow$ upper limit for g_{IR}

- Assume freeze-out above the EW scale ($T_f > v = 246$ GeV, $\mathcal{L} \propto H^\dagger H \mathcal{O}_\mathcal{U}$)

$$\rho_\mathcal{U}(T_{BBN}) = g_{\mathsf{IR}} \frac{\pi^2}{30} T_{BBN}^4 \left[\frac{g_{SM}^{(s)}(T_{BBN})}{g_{SM}^{(s)}(T = v)}\right]^{4/3} \quad \text{and} \quad \rho_\mathcal{U}(T_{BBN}) = \frac{7}{4}\Delta N_\nu \frac{\pi^2}{30} T_{BBN}^4$$

$$\Downarrow$$

$$g_{\mathsf{IR}} \lesssim 4.3 \ \text{ at } \ 4\sigma$$

To be compared with e.g. $g_{\mathcal{B}\mathcal{Z}} = 2(n_c^2 - 1 + \frac{7}{8} n_c n_f)$, for $n_c = 3$ and $n_f = 10$, $g_{\mathcal{B}\mathcal{Z}} \simeq 60$.

- Assume freeze-out below T_{BBN} ($T_{\mathsf{f}\text{-}\mathcal{U}} < T_{BBN}$, $\mathcal{L} \propto B_{\mu\nu} B^{\mu\nu} \mathcal{O}_\mathcal{U}$)

$$g_{\mathsf{IR}} = \frac{7}{4}\Delta N_\nu \quad \Longrightarrow \quad g_{\mathsf{IR}} \lesssim 0.05 \ \text{ at } \ 4\sigma$$

Summary

- Intensive activity on unparticles (~ 200 citations of the first Georgi's paper)
- Unparticles could be deconstructed
- Troubles with IR divergences
- Cosmological consequences
  - Rough arguments for the equation of state for unparticles: $p_{\mathcal{U}} = \frac{1}{3}\rho_{\mathcal{U}}\left[1 - B\rho_{\mathcal{U}}^{\delta/4}\right]$
  - Rough arguments for the energy density for unparticles "derived":

$$\rho_{\mathsf{NP}} = \frac{\pi^2}{30}T^4 \times \begin{cases} \left[g_{\mathsf{IR}} + (g_{\mathcal{BZ}} - g_{\mathsf{IR}})\left(\frac{T}{\Lambda_{\mathcal{U}}}\right)^{\delta}\right] & \text{for} \quad T \lesssim \Lambda_{\mathcal{U}} \\ g_{\mathcal{BZ}} & \text{for} \quad T \gtrsim \Lambda_{\mathcal{U}} \end{cases}$$

  - Unparticles in equilibrium: freeze-out and thaw-in.
  - BNN bounds on the number of degrees of freedom for unparticles.
- HEIDI

Experimental constraints

From A. Freitas and D. Wyler, "Astro Unparticle Physics", arXiv:0708.4339.

$$
\begin{aligned}
\mathcal{L}_{\mathcal{U}ff} \quad = \quad & \frac{c_{\mathrm{V}}}{M_{\mathrm{Z}}^{d_{\mathcal{U}}-1}}\,\bar{f}\gamma_\mu f\,O_{\mathcal{U}}^{\mu} + \frac{c_{\mathrm{A}}}{M_{\mathrm{Z}}^{d_{\mathcal{U}}-1}}\,\bar{f}\gamma_\mu\gamma_5 f\,O_{\mathcal{U}}^{\mu} + \frac{c_{\mathrm{S1}}}{M_{\mathrm{Z}}^{d_{\mathcal{U}}}}\,\bar{f}\not{D} f\,O_{\mathcal{U}} + \frac{c_{\mathrm{S2}}}{M_{\mathrm{Z}}^{d_{\mathcal{U}}}}\,\bar{f}\gamma_\mu f\,\partial^\mu O_{\mathcal{U}} \\
& + \frac{c_{\mathrm{P1}}}{M_{\mathrm{Z}}^{d_{\mathcal{U}}}}\,\bar{f}\not{D}\gamma_5 f\,O_{\mathcal{U}} + \frac{c_{\mathrm{P2}}}{M_{\mathrm{Z}}^{d_{\mathcal{U}}}}\,\bar{f}\gamma_\mu\gamma_5 f\,\partial^\mu O_{\mathcal{U}}.
\end{aligned}
$$

Here the coefficients have been scaled to a common mass, chosen as the Z-boson mass M_{Z}, so that the only unknown quantities are the dimensionless coupling constants c_i.

Coupling	c_V				c_A			
$d_{\mathcal{U}}$	1	4/3	5/3	2	1	4/3	5/3	2
5th force	$7 \cdot 10^{-24}$	$1.4 \cdot 10^{-15}$	$1.8 \cdot 10^{-10}$	$2 \cdot 10^{-5}$	$4 \cdot 10^{-24}$	$8 \cdot 10^{-16}$	$1 \cdot 10^{-10}$	$1.1 \cdot 10^{-5}$
Star cooling	$5 \cdot 10^{-15}$	$2.5 \cdot 10^{-12}$	$1 \cdot 10^{-9}$	$3.5 \cdot 10^{-7}$	$6.3 \cdot 10^{-15}$	$2 \cdot 10^{-12}$	$7.3 \cdot 10^{-10}$	$3 \cdot 10^{-7}$
SN 1987A	$1 \cdot 10^{-9}$	$3.5 \cdot 10^{-8}$	$1 \cdot 10^{-6}$	$3 \cdot 10^{-5}$	$2 \cdot 10^{-11}$	$5.5 \cdot 10^{-10}$	$1.5 \cdot 10^{-8}$	$4.1 \cdot 10^{-7}$
LEP	0.005	0.045	0.04	0.01	0.1	0.045	0.04	0.008
Tevatron		0.4	0.05					
ILC	$1.6 \cdot 10^{-4}$	$1.4 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$3.2 \cdot 10^{-4}$	$3.2 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$2.5 \cdot 10^{-4}$
LHC		0.25	0.02					
Precision	1	0.2	0.025		1	0.15	0.01	
Quarkonia		0.01	0.1	0.45				
Positronium		0.25				$2 \cdot 10^{-13}$	$2 \cdot 10^{-8}$	0.03

Coupling	c_{S1}				c_{P1}, $2c_{P2}$			
$d_{\mathcal{U}}$	1	4/3	5/3	2	1	4/3	5/3	2
5th force	$6.5 \cdot 10^{-22}$	$1.2 \cdot 10^{-13}$	$1.6 \cdot 10^{-8}$	$1.7 \cdot 10^{-3}$	—	—	—	—
Star cooling	$1.3 \cdot 10^{-9}$	$7 \cdot 10^{-7}$	$3 \cdot 10^{-4}$	0.13	$4 \cdot 10^{-8}$	$1.1 \cdot 10^{-5}$	$3.3 \cdot 10^{-3}$	1
SN 1987A	$8 \cdot 10^{-8}$	$2.4 \cdot 10^{-6}$	$6.6 \cdot 10^{-5}$	$2 \cdot 10^{-3}$	$5.5 \cdot 10^{-8}$	$1.3 \cdot 10^{-6}$	$3.5 \cdot 10^{-5}$	$9 \cdot 10^{-4}$
LEP	> 1	> 1	> 1	> 1	> 1	> 1	> 1	> 1
ILC	> 1	> 1	> 1	> 1	> 1	> 1	> 1	> 1

From A. Freitas and D. Wyler, "Astro Unparticle Physics", arXiv:0708.4339 [hep-ph].

HEIDI

Jochum van der Bij and S. Dilcher:

1. J. J. van der Bij and S. Dilcher, "HEIDI and the unparticle," Phys. Lett. B **655**, 183 (2007) [arXiv:0707.1817 [hep-ph]].
2. J. J. van der Bij and S. Dilcher, "A higher dimensional explanation of the excess of Higgs-like events at CERN LEP," Phys. Lett. B **638**, 234 (2006) [arXiv:hep-ph/0605008].
3. J. J. van der Bij, "The minimal non-minimal standard model," Phys. Lett. B **636**, 56 (2006) [arXiv:hep-ph/0603082].

The model:

- Extra-dimensional (δ) scalars neutral under the SM gauge group

$$\phi(x,y) = \frac{1}{\sqrt{2}L^{\delta/2}} \sum_{\vec{k}} \phi_{\vec{k}}(x) e^{i\frac{2\pi}{L}\vec{k}\vec{y}}$$

- Extra terms in the scalar potential

$$V(H,\phi) = \cdots - \frac{\lambda_1}{8}(2f_1\phi - |H|^2)$$

Similarities:

- The continuous mass spectrum e.g. for $s \to \infty$: $\rho(s) \sim s^{-3+\delta/2}$

Differences

- In HEIDI only scalars, while unparticles could have any spin

- Van der Bij and Dilcher don't assume scale invariance of the extra sector

- In HEIDI interactions between the SM and the extra scalars assumed to be renormalizable

- Van der Bij and Dilcher claim that only for $0 < \delta < 1$ there is no tachyons in the scalar spectrum, so the potential is stable $(1 < d_{\mathcal{U}} < 2)$