Extra Dimensions

One of fundamental problems of contemporary theory of electroweak interaction is the so called hierarchy problem. The presence of the spin zero (scalar) elementary field in the Lagrangian (so in Nature) lies in its roots. Radiative corrections to the mass of scalars are quadratically divergent, and consequently it is VERY difficult to keep the Higgs boson mass small since the corrections are pushing it to very high values of the order of the cutoff of the UV theory. The most natural value of the UV cutoff (the energy scale at which the Standard Models (SM) breaks down), is the Planck mass $\sim 10^{19}$ GeV, or a GUT scale $\sim 10^{15}$ GeV. On the other hand, the experimental data are saying that the Higgs mass is below 140 GeV. So even if the cutoff is at the GUT scale, the Higgs mass of this order is at least by factor 10^{13} too large - quite a problem! [1] The most celebrated solution to the hierarchy problem is the supersymmetric SM, where, because of cancellations between bosonic and fermionic degrees of freedom the quadratic divergence is replaced by the logarithmic one. Consequently, there is no need for new physics up to energies of the order of 10^{16} GeV, some could say that the solution provided by the supersymmetry is too good as it is natural to expect some new physics below 10^{16} GeV. In fact the quadratic divergence is just a part of the hierarchy problem, the other aspect of which is a failure to explain why the two scales (the electroweak scale $m_{\text{weak}} \sim 10^2$ GeV and the Planck mass $m_{Pl} \sim 10^{19}$ GeV) are so different. We expect that a proper theory of fundamental interactions should explain the ratio of the Planck mass over the electroweak scale, for that aspect of the hierarchy problem the supersymmetry is not a remedy. However there exist another possible way to attack the hierarchy problem, namely to go beyond 4 space time dimensions, i.e. to assume the existence of extra dimensions. There are many separate streams in the activity under a generic name of extra dimensions. To me the most attractive ones are those that explain the hierarchy between the scale of weak interaction and gravity through a simple warping of geometry of the space-time in the extra direction (the Randall-Sundrum model) and those within which the Higgs boson emerges as an extra component of 5-dimensional gauge field (the Higgs-gauge unification).

- Warped Extra Dimensions (Randall-Sundrum-like models):
  Assuming the existence of proper cosmological constants in 5-dimensional

[1] This is suggesting that in fact the SM cutoff is much lower, of the order of TeV.

bulk and 4-dimensional branes (submanifolds of dimension 4), one can show that the 5-dimensional Einstein equations have simple solutions such that the background metric varies exponentially with the 5th coordinate. Closer inspection shows that in such a scenario $m_{\rm weak}/m_{Pl} \propto e^{-k\pi r}$, where r is the distance between the branes and k is a parameter of the metric (that is the solution of the Einstein equation). Since the exponential function is varying rapidly therefore even for modest values of the argument (so, no hierarchy) one can reproduce the desired ratio. Therefore this simple setting can accommodate in a natural manner both the Planck mass and the weak scale within a single theory. This is a great success. This scenario has various phenomenological consequences which could be investigated experimentally at colliders, e.g. at the LHC, see for instance [1], [2]. Some theoretical aspect of warping have been discussed in [3] and [4].

- Higgs-Gauge Unification:
  On the other hand, the problem of quadratic divergences could be solved assuming that the space time is 5-dimensional (as in the Randall-Sundrum setting) and that 4-dimensional Higgs boson is an extra, i.e. 5th, component of a 5-dimensional gauge field. So, there is no need to assume the existence of the Higgs boson at all, since in fact it is a component of a gauge field. In such a case, not only the quadratic divergences disappear, but the Higgs mass is calculable in terms of other parameters of the theory. This is indeed a very attractive scenario. It is worth to mention that both setups (the warping and the gauge-Higgs unification) could be merged within a single theory that explain the mass hierarchy and at the same time eliminates the quadratic divergences. Some aspects of the gauge-Higgs unification were addressed in [5]-[8]. In particular in [8] it was shown that the gauge-Higgs unification provides a new source of CP violation. Note that the SM CP violation is not sufficient to explain explain the absence of animatter, so the existence of the extra sources of CP breaking is a very attractive feature of the theory.

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