The minimal extension of the Standard Model
- the case for Dark Matter -

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- The hierarchy problem
- Dark Matter
- Neutrino physics
- Summary and comments

B.G., J. Wudka, "Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics", arXiv:0902.0628

The hierarchy problem

$$\delta^{(SM)} m_h^2 = \frac{\Lambda^2}{\pi^2 v^2} \left[\frac{3}{2} m_t^2 - \frac{1}{8}\left(6 m_W^2 + 3 m_Z^2\right) - \frac{3}{8} m_h^2\right]$$

The little hierarchy problem:

- $m_h = 130$ GeV $\Rightarrow$ $\delta^{(SM)} m_h^2 \simeq m_h^2$ for $\Lambda \simeq 580$ GeV
- Effective operators $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i$ imply $\Lambda \gtrsim$ few TeV.

Solutions to the little hierarchy problem:

- Increase of the allowed value of the m_h: the inert Higgs model by Barbieri, Hall, Rychkov, arXiv:hep-ph/0603188, (see also Ma) $\Rightarrow$ $m_h \sim 400 - 600$ GeV, (m_h^2 terms in T parameter canceled by $m_{H^\pm}, m_A, m_S$ contributions).
- Suppression of corrections growing with Λ $\Rightarrow$ e.g. supersymmetry

Our goal: to lift up the cutoff to multi TeV range preserving $\delta^{(SM)} m_h^2 \leq m_h^2$.

- Extra gauge singlet φ with $\langle\varphi\rangle = 0$ (to prevent $H \leftrightarrow \varphi$ mixing from $\varphi^2|H|^2$).
- Symmetry $\mathbb{Z}_2$: $\varphi \to -\varphi$ (to eliminate $|H|^2\varphi$ couplings).
- Gauge singlet neutrinos: $\nu_{R\,i}$ for $i = 1, 2, 3$.

$$V(H,\varphi) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \mu_\varphi^2\varphi^2 + \frac{1}{24}\lambda_\varphi\varphi^4 + \lambda_x|H|^2\varphi^2$$

with

$$\langle H\rangle = \frac{v}{\sqrt{2}}, \qquad \langle\varphi\rangle = 0 \qquad \text{for} \qquad \mu_\varphi^2 > 0$$

then

$$m_h^2 = 2\mu_H^2 \qquad \text{and} \qquad m^2 = 2\mu_\varphi^2 + \lambda_x v^2$$

- Positivity (stability) in the limit $h, \varphi \to \infty$: $\qquad \lambda_H\lambda_\varphi > 6\lambda_x^2$
- Unitarity in the limit $s \gg m_h^2, m^2$: $\lambda_H \leq \frac{4\pi}{3}$ (the SM requirement) and $\lambda_\varphi \leq 8\pi$, $\lambda_x < 4\pi$

$$\delta^{(\varphi)} m_h^2 = -\frac{\lambda_x}{8\pi^2}\left[\Lambda^2 - m^2 \log\left(c + \frac{\Lambda^2}{m^2}\right)\right]$$
$$|\delta m_h^2| = \delta^{(SM)} m_h^2 + \delta^{(\varphi)} m_h^2 = D_t m_h^2$$

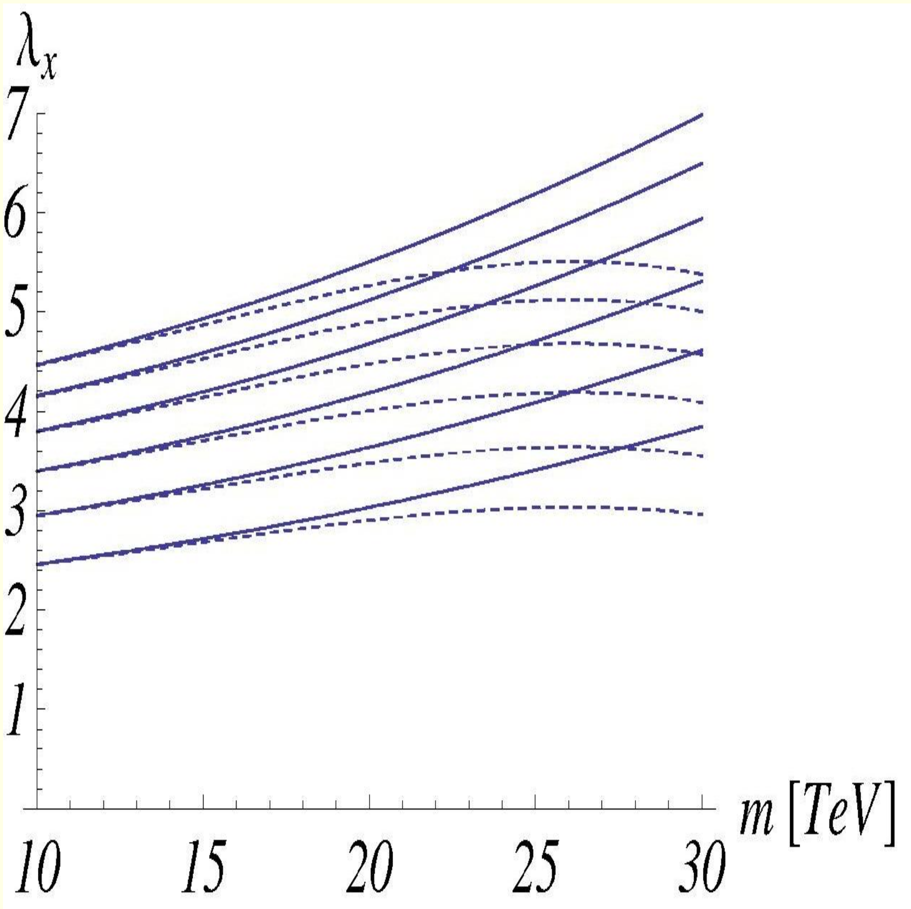

Figure 1: Plot of λ_x as a function of m for $D_t = 1$; we chose $\Lambda = 56$ TeV and $m_h = 130,\ 150,\ 170,\ 190,\ 210,\ 230$ GeV (starting with the uppermost curve); solid (dashed) lines correspond to $c = +1$ ($c = -1$).

Dark Matter

1. V. Silveira and A. Zee, Phys. Lett. B **157**, 191 (1985)
2. J. McDonald, Phys. Rev. D **50**, 3637 (1994)
3. C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B **619**, 709 (2001)
4. H. Davoudiasl, R. Kitano, T. Li and H. Murayama, Phys. Lett. B **609**, 117 (2005)
5. J. J. van der Bij, Phys. Lett. B **636**, 56 (2006)
6. S. Andreas, T. Hambye and M. H. G. Tytgat, JCAP **0810**, 034 (2008)

It is possible to find parameters Λ, λ_x and m such that both the hierarchy is ameliorated to the prescribed level and such that $\Omega_\varphi h^2$ is consistent with Ω_{DM}.

$$\varphi\varphi \to hh, W^+W^-, ZZ \quad \Rightarrow \quad \langle \sigma v \rangle = \frac{1}{8\pi}\frac{\lambda_x^2}{m^2}$$

The Boltzmann equation $\quad \Rightarrow \quad x_f \left(\equiv \frac{m}{T_f}\right) = \ln\left[0.038\frac{m_{Pl} m \langle \sigma v \rangle}{g_\star^{1/2} x_f^{1/2}}\right]$

$$\Omega_\varphi h^2 = 1.06 \cdot 10^9 \frac{x_f}{g_\star^{1/2} m_{Pl} \langle \sigma v \rangle \text{ GeV}}$$

$$x_f \simeq 30 \quad \Rightarrow \quad m \geq x_f T_c \simeq 8 \text{ TeV}$$

$$\Omega_\varphi = \Omega_{DM} \quad \Rightarrow \quad \lambda_x \sim \frac{1}{4} \frac{m}{\text{TeV}}$$

$$\Downarrow$$

$$|\delta m_h^2| = D_t m_h^2 \quad \Rightarrow \quad m = m(\Lambda)$$

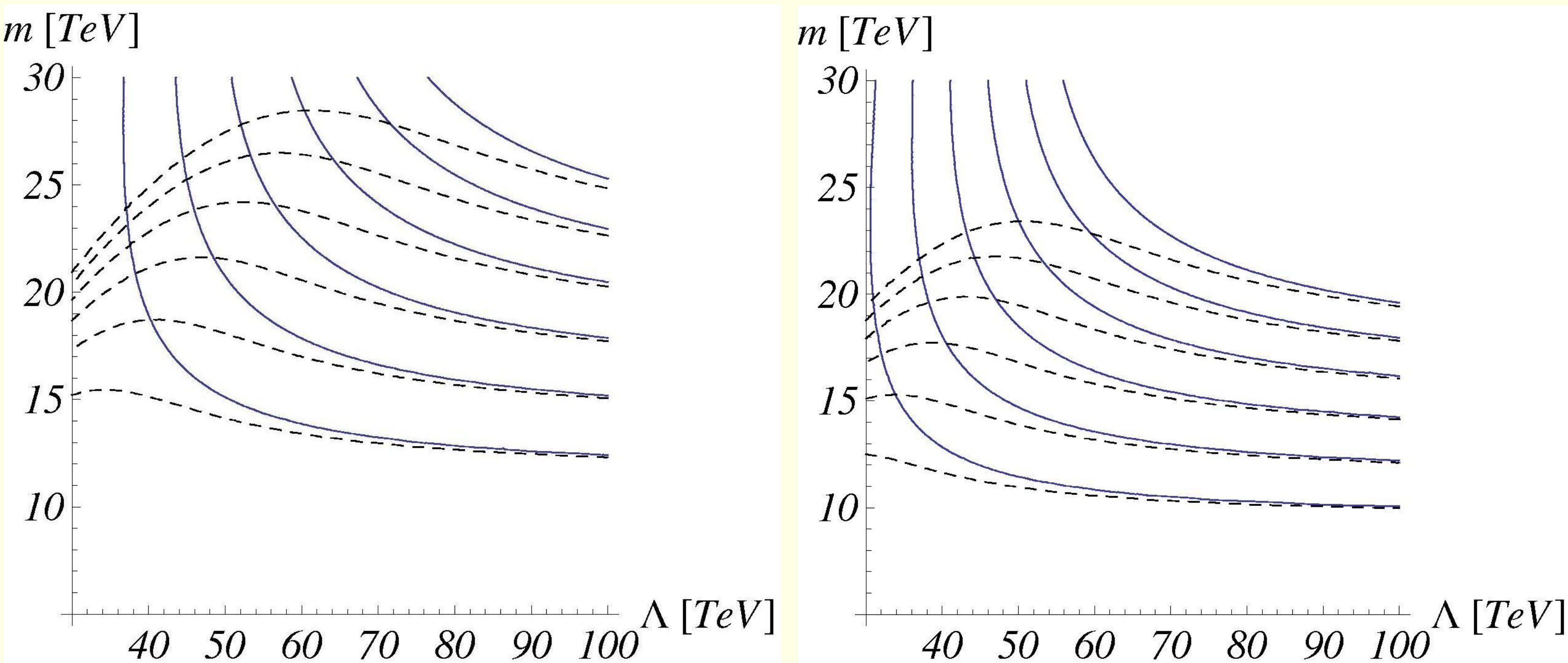

Figure 2: Plot of m as a function of the cutoff Λ when $D_t = 1$ and $\Omega_\varphi = \Omega_{DM}$ at the 3σ level: $\Omega_\varphi h^2 = 0.152$, left plot and $\Omega_\varphi h^2 = 0.104$, right plot; for $m_h = 130, 150, 170, 190, 210, 230$ GeV (starting with the uppermost curve) and for $c = +1$ solid curves and $c = -1$ (dashed curves).

Neutrino physics

$$\mathcal{L}_Y = -\bar{L} Y_l H l_R - \bar{L} Y_\nu \tilde{H} \nu_R - \frac{1}{2}\overline{(\nu_R)^c} M \nu_R - \varphi \overline{(\nu_R)^c} Y_\varphi \nu_R + \text{H.c.}$$

$$\mathbb{Z}_2: \qquad H \to H,\ \varphi \to -\varphi,\ L \to S_L L,\ l_R \to S_{l_R} l_R,\ \nu_R \to S_{\nu_R} \nu_R$$

The symmetry conditions ($S_i S_i^\dagger = S_i^\dagger S_i = \mathbb{1}$):

$$S_L^\dagger Y_l S_{l_R} = Y_l, \quad S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu, \quad S_{\nu_R}^T M S_{\nu_R} = +M, \quad S_{\nu_R}^T Y_\varphi S_{\nu_R} = -Y_\varphi$$

The implications of the symmetry:

$$S_{\nu_R}^T M S_{\nu_R} = +M \quad \Rightarrow \quad S_{\nu_R} = \pm \mathbb{1}, \qquad S_{\nu_R} = \pm\,\text{diag}(1, 1, -1)$$

$$S_{\nu_R} = \pm \mathbb{1} \quad \Rightarrow \quad Y_\varphi = 0 \ \text{ or } \ S_{\nu_R} = \pm\,\mathsf{diag}(1,1,-1) \quad \Rightarrow \quad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

$$S_L^\dagger Y_l S_{l_R} = Y_l \quad \Rightarrow \quad S_L = S_{l_R} = \mathsf{diag}(s_1, s_2, s_3), \quad |s_i| = 1$$

$$S_L^\dagger Y_\nu S_{\nu_R} = Y_\nu \quad \Rightarrow \qquad \text{10 Dirac neutrino mass textures}$$

For instance the solution corresponding to $s_{1,2,3} = \pm 1$:

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h & 0 \end{pmatrix}$$

$$\mathcal{L}_m = -(\bar{n} M_n n + \bar{N} M_N N)$$

with the see-saw mechanism explaining $M_n \ll M_N$:

$$M_N \sim M \qquad \text{and} \qquad M_n \sim M_D \frac{1}{M} M_D^T$$

where

$$\nu_L = n_L + M_D \frac{1}{M} N_L \qquad \text{and} \qquad \nu_R = N_R - \frac{1}{M} M_D^T n_R$$

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h & 0 \end{pmatrix} \quad \Rightarrow \quad M_D = Y_\nu \frac{v}{\sqrt{2}} \quad \Rightarrow \quad M_n$$

To compare our results with the data, we use the following approximate lepton mixing matrix that corresponds to $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \arcsin(1/\sqrt{3})$:

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$

Writing the diagonal light neutrino mass matrix as

$$m_{\text{light}} = \mathsf{diag}(m_1, m_2, m_3)$$

we find

$$M_n = U m_{\text{light}} U^T = \frac{1}{3}\begin{pmatrix} 2m_1 + m_2 & -m_1 + m_2 & -m_1 + m_2 \\ -m_1 + m_2 & \frac{1}{2}(m_1 + 2m_2 + 3m_3) & \frac{1}{2}(m_1 + 2m_2 - 3m_3) \\ -m_1 + m_2 & \frac{1}{2}(m_1 + 2m_2 - 3m_3) & \frac{1}{2}(m_1 + 2m_2 + 3m_3) \end{pmatrix}$$

$$\Downarrow$$

$$m_1 = -4v^2\left(\frac{a^2 - d^2}{M_1} + \frac{b^2 - e^2}{M_1}\right) \qquad \text{and} \qquad m_2 = -4v^2\left(\frac{2d^2 - a^2}{M_1} + \frac{2e^2 - b^2}{M_1}\right)$$

for $d = g$, $h = e$, $(a + 2d)(a - d) = 0$, $(b + 2e)(b - e) = 0$ with $m_3 \ll m_{1,2}$.

Does $Y_\varphi \neq 0$ imply $\varphi \to n_i n_j$ decays?

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ d & e & 0 \\ g & h & 0 \end{pmatrix},\ Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix} \quad \Rightarrow \quad \varphi \to N_a n_3,\ N_a N_3,\ (a = 1, 2)$$

that can be kinematically forbidden by requiring $M_i > m$.

Does φ explain the PAMELA data?

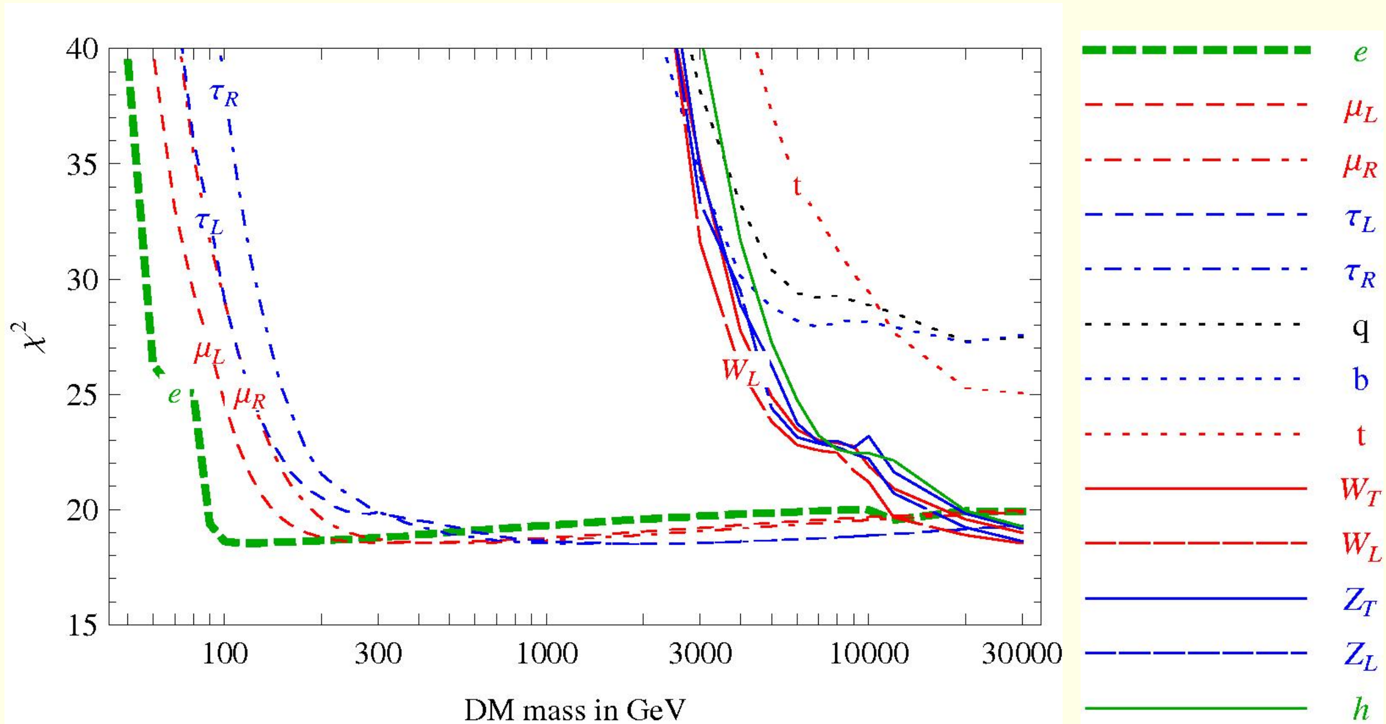

Figure 3: Combined fit of different DM annihilation channels to the PAMELA positron and PAMELA anti-proton data, from Cirelli, Kadastik, Raidal and Strumia, arXiv:0809.2409.

Summary and comments

- The addition of a real scalar singlet φ to the SM may ameliorate the little hierarchy problem (by lifting the cutoff Λ to $50 - 100$ TeV range).

- It also provides a realistic candidate for DM.

- Since $m \gtrsim 10$ TeV therefore φ can properly describe the PAMELA results both for e^+ and $\bar{p}$.

- The $\mathbb{Z}_2$ symmetry implies a realistic texture for the neutrino mass matrix.

- φ cannot be assumed to be responsible neither for inflation nor for dark energy.