5-Dimensional Troubles of Gauge-Higgs Unifications

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Plan

- The 5D gauge-Higgs unification, its advantages and difficulties
- Non-standard boundary conditions
- $\sin^2\theta_W$ and ρ
- Conclusions

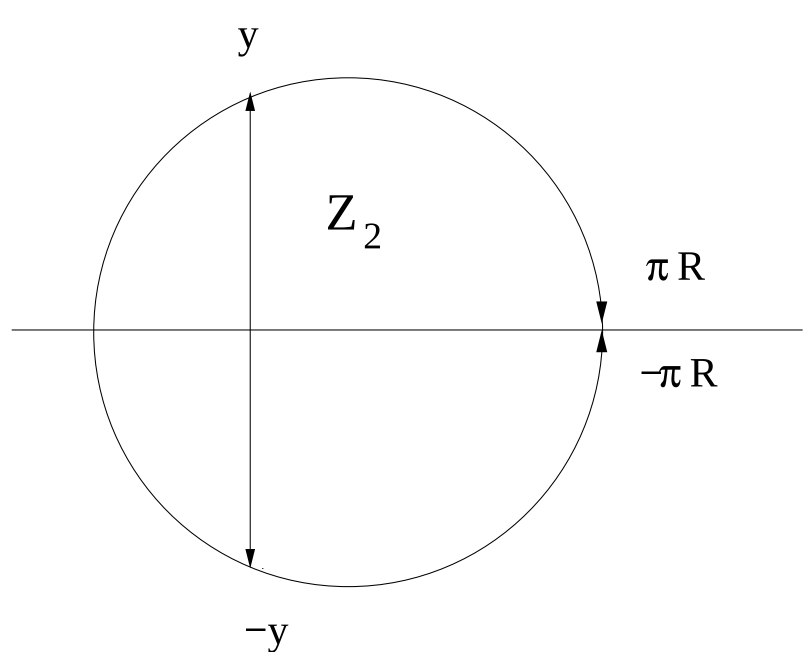
B. Grzadkowski and J. Wudka,

1. "Majorana fermions and CP violation from 5-dimensional theories: A systematic approach", Phys. Rev. D 72, 125012 (2005) [arXiv:hep-ph/0501238]
2. "Light excitations in 5-dimensional gauge theories", Acta Phys. Polon. B 36, 3523 (2005) [arXiv:hep-ph/0511139]
3. "5-Dimensional Difficulties of Gauge-Higgs Unifications," [arXiv:hep-ph/0604225]

- Fairlie, Manton, 1979, The gauge-Higgs unification: the Higgs boson as an extra component of A_N, e.g. in 5D, $H = A_4$.
- No experimental evidence for extra dimensions $\longrightarrow$ compactification and Kaluza-Klein modes in 4D:
  - Compactification on a circle S^1
  - Compactification on the orbifold S^1/Z_2, a circle with identified points, $y \rightarrow -y$

$$\Downarrow$$

$$F(x,y) = \pm F(x,-y)$$



$$F(x,y) = F(x, y+2\pi R)$$

$$\Downarrow$$

$$F(x,y) = \sum_{n=-\infty}^{+\infty} F_n(x) e^{i\frac{n}{R}y} = F_0(x) + \sum_{n=1}^{+\infty} \left\{ F_n^{(+)}(x)\cos(\frac{n}{R}y) + F_n^{(-)}(x)\sin(\frac{n}{R}y) \right\}$$

where $F_n(x)$ are Kaluza-Klein modes.
Equation of motion (momentum along 5th D $\rightarrow$ mass in 4D):

$$(\partial^\mu \partial_\mu + \partial^4 \partial_4) F(x,y) = 0 \;\Rightarrow\; \left[\partial^\mu \partial_\mu + \left(\frac{n}{R}\right)^2\right] F^n(x) = 0 \;\Rightarrow\; m_n = \frac{n}{R}$$

- $n = 0 \rightarrow$ massless modes: gravity & electromagnetism,
- $|n| > 0 \rightarrow$ massive modes: “high” scale physics.

$$\mathcal{L}_{4D} = \int \mathcal{L}_{5D} \;\; dy$$

The strategy for 5D:

- SM Higgs in the fundamental representation of $SU(2)$. For A_4 (adjoint) to have iso-doublet components at least $G = SU(3)_w$ is required (a chance for unification with $SU(3)_c$) :

$$SU(3): \qquad A_M \equiv A_M^a T_a = A_M^a \frac{\lambda_a}{2} = \left(\begin{array}{c|c} W^\pm, W_3 & H \\ \hline H^\dagger & B \end{array} \right)$$

- The initial gauge group G broken to $SU(2)_L \times U(1)_Y$ by the Scherk-Schwarz mechanism .
Periodicity:

$$A_M(x, y + 2\pi R) = T A_M(x, y) T^\dagger$$

Orbifold boundary conditions:

$$A_\mu(x, -y) = +P A_\mu(x, y) P^\dagger \qquad A_4(x, -y) = -P A_4(x, y) P^\dagger ,$$

where T and P are elements of a global symmetry group (e.g. gauge).
For $SU(3) \to SU(2)_L \times U(1)_Y$:

- $SU(2)_L \times U(1)_y \to U(1)_{\rm EM}$ by $\langle A_4^{(0)} \rangle$ through 1-loop effective potential (the Hosotani mechanism). Higgs boson interactions are predicted by the theory.

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For $SU(3) \to SU(2)_L \times U(1)_Y$: $P = T = \exp(i\pi\lambda_3) = \mathrm{diag}(-1, -1, 1)$

- $SU(2)_L \times U(1)_y \to U(1)_{\mathrm{EM}}$ by $\langle A_4^{(0)} \rangle$ through 1-loop effective potential (the Hosotani mechanism). Higgs boson interactions are predicted by the theory.

Solution to the hierarchy problem ($m_H^2 \propto \Lambda^2$):

- The 5D gauge invariance and locality protects the Higgs mass from large quantum corrections $\rightarrow$ in particular, no quadratic divergences appear.
- The Higgs boson mass is calculable and finite (1- and 2-loop confirmed).

Non-standard boundary conditions

Motivation:
Generalization of the mechanism for CP violation found for 5D $U(1)$ compactified on S^1 (BG & J.Wudka PRL93:211603,2004, hep-ph/0401232) for the orbifold S^1/Z_2

$$A_4^{(0)} \quad \text{must exist !}$$

$$\Downarrow$$

$$A_4(y+L) = A_4(y) \quad \text{and} \quad A_4(-y) = A_4(y)$$

$$\Downarrow$$

$$A_\mu(-y) = -A_\mu(y) \quad \text{since} \quad F_{4\mu} = \partial_4 A_\mu - \partial_\mu A_4$$

$$A_\mu(-y) = -A_\mu(y) \quad \text{and} \quad A_4(-y) = A_4(y)$$

Assume the standard fermionic orbifold transformation:

$$\psi(y) \to \psi(-y) = e^{i\beta}\gamma_5\psi(y)$$

Then

$$\bar{\psi}\gamma^N[\partial_N + ig_5 q A_N]\psi \to \bar{\psi}\gamma^N[\partial_N + ig_5(-q)A_N]\psi$$

Non-standard boundary conditions

We need to switch the sign of the charge q under orbifolding, so it suggests to adopt the charge conjugation ($\Psi^C = C(\bar{\psi})^T$):

The periodicity BC

$$A_\mu(y+L) = A_\mu(y)\,,\, A_4(y+L) = A_4(y)\,,\, \psi(y+L) = e^{i\alpha}\psi(y)$$

The orbifold BC

$$A_\mu(-y) = -A_\mu(y)\,,\, A_4(-y) = A_4(y)\,,\, \psi(-y) = e^{i\beta}\gamma_5\psi^C(y)$$

- $\mathcal{L}(\psi, A_N)$ is invariant.
- The consistency conditions are satisfied.
- Majorana zero modes seem to be conceivable.
- The 4D gauge field zero modes are disallowed by the BC, no 4D QED.

Non-standard boundary conditions

Periodicity

$$\Psi(y+L) = \Gamma\Psi(y) + \Upsilon^*\Psi^c(y)$$
$$A_N(y+L) = \begin{cases} +U_1^\dagger A_N(y) U_1 & (P1) \\ -U_2^\dagger A_N^T(y) U_2 & (P2) \end{cases},$$

where $U_{1,2}$ are global elements of the gauge group.

$$\chi \equiv \begin{pmatrix} \Psi \\ -\Psi^c \end{pmatrix} \qquad \mathcal{A} \equiv \begin{pmatrix} \Gamma & -\Upsilon^* \\ \Upsilon & \Gamma^* \end{pmatrix} \qquad \tau^a \equiv \begin{pmatrix} T_a & 0 \\ 0 & -T_a^* \end{pmatrix}$$

The periodicity conditions:

$$A_N^a(y+L) = \mathbb{V}_{ab} A_N^b(y); \qquad \chi(y+L) = \mathcal{A}\chi(y)$$

Requiring invariance of the kinetic term $\bar{\Psi} i\gamma^N D_N \Psi$ gives the following conditions on the acceptable BC:

$$\mathcal{A}\tau_a\mathcal{A}^\dagger = \mathbb{V}_{ba}\tau_b, \quad \mathcal{A}^\dagger\mathcal{A} = \mathbb{1}$$

Non-standard boundary conditions

Orbifold Parity

$$\begin{array}{rcl} \chi(-y) & = & \gamma_5 \mathcal{B}^* \chi(y) \\ A_N(-y) & = & \left\{ \begin{array}{ll} (-1)^{s_N} \tilde{U}_1^\dagger A_N(y) \tilde{U}_1 & (R1) \\ (-1)^{1-s_N} \tilde{U}_2^\dagger A_N^T(y) \tilde{U}_2 & (R2) \end{array} \right. \end{array} ,$$

where $s_N = \delta_{N,4}$, $\tilde{U}_{1,2}$ are global gauge transformations and

$$\mathcal{B} \equiv \left(\begin{array}{cc} -\tilde{\Gamma} & \tilde{\Upsilon}^* \\ \tilde{\Upsilon} & \tilde{\Gamma}^* \end{array} \right)$$

The boundary conditions:

$$A_N^a(-y) = (-1)^{\delta_{N,4}} \tilde{\mathbb{V}}_{ab} A_N^b(y); \qquad \chi(-y) = -\gamma_5 \mathcal{B} \chi(y)$$

Requiring now the invariance of $\mathcal{L}$ under the twist implies

$$\mathcal{B} \tau_a \mathcal{B}^\dagger = \tilde{\mathbb{V}}_{ba} \tau_b, \quad \mathcal{B}^\dagger \mathcal{B} = \mathbb{1}$$

General solutions for $\mathcal{A}$ and $\mathcal{B}$ could be found in terms of U_i and $\tilde{U}_i$. (BG & J.Wudka, PRD72:125012,2005, hep-ph/0501238)

Non-standard boundary conditions

Non-standard boundary conditions allow for:

- bare fermion masses (relevant for the Hosotani-type CP violation)
- Majorana fermionic KK modes
- spontaneous CP violation through $\langle A_4^{(0)} \rangle \neq 0$

$\sin^2\theta_W$ *and* ρ

Assumptions:

- 5D space-time
- The high-energy gauge group is a Lie group
- The low-energy group (massless vector bosons) is $SU(2) \times U(1)$
- Bulk gauge field, no brane kinetic terms, . . .

$E_{\boldsymbol{\alpha}}$ and H_i are the root and Cartan generators of Lie algebra of the full theory

$$\text{tr}(H_i H_j) = \delta_{ij}, \ \text{tr}(E_{-\boldsymbol{\beta}} E_{\boldsymbol{\alpha}}) = \delta_{\boldsymbol{\alpha},\boldsymbol{\beta}}$$

Then the generators for the SM $SU(2)$ sub-algebra must be of the form

$$J_0 = \frac{1}{|\boldsymbol{\alpha}|^2} \boldsymbol{\alpha} \cdot \mathbf{H} \qquad J_+ = \frac{\sqrt{2}\eta}{|\boldsymbol{\alpha}|} E_{\boldsymbol{\alpha}} \qquad J_- = (J_+)^\dagger$$

The SM hypercharge generator generates a $U(1)$ subgroup and is of the form

$$Y = \hat{\mathbf{y}} \cdot \mathbf{H}$$

Since the SM group is a product of $SU(2)$ and $U(1)$ therefore: $\hat{\mathbf{y}} \cdot \boldsymbol{\alpha} = 0$

$\sin^2\theta_W$ *and* ρ

Light scalar modes that can contribute to the mass matrix of light $SU(2)\times U(1)$ vector bosons are associated with root generators $E_{\boldsymbol{\beta}}$. Denote the scalar state which is an eigenvector of J_0 with the eigenvalue I that belongs to a multiplet of isospin $I_{\max}(I_{\max}+1)$ by

$$|I\rangle = \sum_{\boldsymbol{\beta}} v_{\boldsymbol{\beta}} \left|E_{\boldsymbol{\beta}}\right\rangle$$

then $J_0 \left|I\right\rangle = I \left|I\right\rangle$ implies

$$\boldsymbol{\alpha}\cdot\boldsymbol{\beta} = |\boldsymbol{\alpha}|^2 I$$

One can show that a single root-vector $\boldsymbol{\beta}$ can contribute to the sum in $|I\rangle = \sum_{\boldsymbol{\beta}} v_{\boldsymbol{\beta}} \left|E_{\boldsymbol{\beta}}\right\rangle$ and that

$$\hat{\mathbf{y}} = \frac{\boldsymbol{\beta}-(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{\beta})\hat{\boldsymbol{\alpha}}}{|\boldsymbol{\beta}-(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{\beta})\hat{\boldsymbol{\alpha}}|}$$

$\sin^2\theta_W$ *and* ρ

The (canonically normalized) electroweak bosons correspond to the zero modes of the gauge fields associated with the generators $\hat{\boldsymbol{\alpha}} \cdot \mathbf{H}$, $E_{\pm\boldsymbol{\alpha}}$, $\hat{\mathbf{y}} \cdot \mathbf{H}$; ($W^0, W^{\pm}$ and B).

$$\begin{aligned} A_\mu &= W_\mu^+ E_{\boldsymbol{\alpha}} + W_\mu^- E_{-\boldsymbol{\alpha}} + W_\mu^0 \hat{\boldsymbol{\alpha}} \cdot \mathbf{H} + B_\mu \hat{\mathbf{y}} \cdot \mathbf{H} + \cdots \\ A_4 &= \phi E_{\boldsymbol{\beta}} + \phi^\dagger E_{-\boldsymbol{\beta}} \end{aligned}$$

Assume that $|I\rangle$ is a member of a multiplet with maximum isospin $I_{\max}$ and it is the component that gets the $\langle H \rangle$ v/$\sqrt{2}$, then the vector boson mass-terms (tr$F_{4\mu}^2$) in the Lagrangian are

$$\mathcal{L}_{\text{mass}} = \frac{v^2}{2} \left\{ |\boldsymbol{\alpha}|^2 [I_{\max}(I_{\max}+1) - I^2] W^+ \cdot W^- + (\hat{\boldsymbol{\alpha}} \cdot \boldsymbol{\beta} W^0 + \hat{\mathbf{y}} \cdot \boldsymbol{\beta} B)^2 \right\}$$

which implies that the electroweak mixing angle is given by

$$\sin^2\theta_W = 1 - (\hat{\boldsymbol{\alpha}} \cdot \hat{\boldsymbol{\beta}})^2$$

and the ρ parameter equals

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{I_{\max}(I_{\max}+1)}{2I^2} - \frac{1}{2}$$

$\sin^2\theta_W$ *and* ρ

$$\rho = 1 \qquad \Longrightarrow \qquad (I, I_{\max}) = \left(\frac{1}{2}, \frac{1}{2}\right),\ (2,3),\ \left(\frac{15}{2}, \frac{25}{2}\right), \cdots$$

However, the following condition must be satisfied

$$\frac{1}{2}|I| = (\hat{\boldsymbol{\alpha}} \cdot \hat{\boldsymbol{\beta}})^2 = \frac{m}{4}$$

for $m = 0, 1, 2, 3, 4$. Therefore only $m = 1$ and 4 are allowed, then $\sin^2\theta_W = \frac{3}{4}, 0$!

Conclusions

- The non-standard BCs offer
  - bare fermion masses (relevant for the Hosotani-type CP violation)
  - Majorana fermionic KK modes
  - CP violation on S^1/Z_2 (as well as for S^1)
- If
  - 5D space-time
  - The high-energy gauge group is a Lie group
  - The low-energy group (massless vector bosons) is $SU(2) \times U(1)$
  - Only bulk gauge field, no brane kinetic terms, . . .

  then there is no way to satisfy $\rho = 1$ and $\sin^2\theta_W \simeq \frac{1}{4}$.

Light Spectrum

The light gauge bosons will be denoted by $A_\mu^{\hat a}$ and the light fermions by $\chi^{(0)}$, the light modes associated with $A_{N=4}$ behave as 4-dimensional scalars and will be denoted by $\phi_{\hat r} = A^{\hat r}_{N=4}$. Using the $y-$independence of these modes we find

$$
\begin{aligned}
A_\mu^{\hat a} &= \mathbb{V}_{\hat a \hat b} A_\mu^{\hat b} = \tilde{\mathbb{V}}_{\hat a \hat b} A_\mu^{\hat b}, \\
\phi^{\hat r} &= \mathbb{V}_{\hat r \hat s} \phi^{\hat s} = -\tilde{\mathbb{V}}_{\hat r \hat s} \phi^{\hat s}, \\
\chi^{(0)} &= \mathcal{A} \chi^{(0)} = -\gamma_5 \mathcal{B} \chi^{(0)}.
\end{aligned}
$$

Light particles are associated with $+1$ eigenvalues of two matrices: $+\mathbb{V}$ and $+\tilde{\mathbb{V}}$ for the gauge bosons; $+\mathbb{V}$ and $-\tilde{\mathbb{V}}$ for the scalars; and $\mathcal{A}$ and $-\gamma_5 \mathcal{B}$ for the fermions.
The appropriate basis:

$$
\mathcal{B} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad \tilde{\mathbb{V}} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}
$$

Non-standard boundary conditions

Example: single $U(1)$ fermion with ($P1-R2$) BC

$$P1: \qquad \Psi(y+L) = e^{i\alpha}\Psi(y) \qquad \Rightarrow \qquad \Psi(y) = \frac{1}{\sqrt{L}}\sum_{n=-\infty}^{\infty}\psi_n e^{i(2\pi n+\alpha)y/L}$$

$$R2: \qquad \Psi(-y) = \gamma_5\psi^C(y) \qquad \Rightarrow \qquad \psi_n = \gamma_5\psi_n^c$$

K-K modes are 4D Majorana fermions ($C_4 = \gamma_0\gamma_2,\ C_5 = \gamma_1\gamma_3 \Rightarrow \gamma_5 C_5 = -iC_4$).

$$\psi_n = \begin{pmatrix} -i\sigma_2\varphi_n^* \\ \varphi_n \end{pmatrix} \qquad (P1-R2)$$

Bare mass fermion term in the Lagrangian is allowed as under the orbifold twist transformation, $\psi \to \gamma_5\psi^c$, the 5D fermion mass term is invariant:

$$\bar{\psi}\psi \to -\bar{\psi}^c\psi^c = \bar{\psi}\psi\,.$$

In addition, the kinetic term also generates a mass term:

$$\int_0^L dy\bar{\Psi}\left(i\gamma^4\partial_4 - M\right)\Psi = \sum_{n=-\infty}^{\infty}\left[2M\varphi_n^\dagger\varphi_n - \frac{(2\pi n+\alpha)}{L}\left(\varphi_n^T\sigma_2\varphi_n + \text{H.c.}\right)\right] \qquad (P1{-}R2)$$