

Homework problems #12

1. Show that the following Lagrangian

$$\mathcal{L} = \sqrt{g} \left(\frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) \quad (1)$$

is invariant under the local scale (the Weyl transformation)

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) \quad \phi(x) \rightarrow \phi'(x) = \Omega^{-1}(x) \phi(x).$$

2. Find in N-dimensional space-time an analog of (1) that is also invariant under the Weyl transformation.