Homework problems #12

1. Derive the following relation

$$R = \Omega^2 R' - \Omega^2 \left[2(N-1)\Box' f - (N-1)(N-2) g'^{\mu\nu} f_\mu f_\nu\right]$$

for

$$g'_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) \qquad \text{and} \qquad f_\mu \equiv \partial_\mu \ln \Omega$$

The primes indicate that the corresponding quantity is calculated for $g'_{\mu\nu}(x)$ and N denotes the dimension of the space-time.

2. Show that the following Lagrangian

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{12}\phi^2 R + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right) \tag{1}$$

is invariant under the local scale (the Weyl transformation)

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) \qquad \phi(x) \to \phi'(x) = \Omega^{-1}(x)\phi(x)\,.$$

3. Find in N-dimensional space-time an analog of (1) that is also invariant under the Weyl transformation.