Homework problems #7

1. Show that
$$T^{\mu\sigma}{}_{\lambda;\rho} = \frac{\partial}{\partial x^\rho} T^{\mu\sigma}{}_{\lambda} + \Gamma^{\mu}_{\rho\nu} T^{\nu\sigma}{}_{\lambda} + \Gamma^{\sigma}_{\rho\nu} T^{\mu\nu}{}_{\lambda} - \Gamma^{\kappa}_{\lambda\rho} T^{\mu\sigma}{}_{\kappa}$$
is a tensor for general coordinate transformations.

2. Prove that the Leibniz rule holds for covariant differentiation of a product of two tensors:
$$(A^{\mu}{}_{\nu} B^{\lambda})_{;\rho} = A^{\mu}{}_{\nu;\rho} B^{\lambda} + A^{\mu}{}_{\nu} B^{\lambda}{}_{;\rho}$$

3. Derive the following identity
$$\mathrm{Tr}\left[M^{-1}(x)\frac{\partial}{\partial x^\lambda} M(x)\right] = \frac{\partial}{\partial x^\lambda} \ln \mathrm{Det}[M(x)]$$