

Homework problems #9

1. Derive the following expression for the Riemann tensor

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2} \left[\frac{\partial^2 g_{\lambda\nu}}{\partial x^\kappa \partial x^\mu} - \frac{\partial^2 g_{\mu\nu}}{\partial x^\kappa \partial x^\lambda} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^\nu \partial x^\mu} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^\nu \partial x^\lambda} \right] + g_{\eta\sigma} \left(\Gamma_{\nu\lambda}^\eta \Gamma_{\mu\kappa}^\sigma - \Gamma_{\kappa\lambda}^\eta \Gamma_{\mu\nu}^\sigma \right)$$

2. Show the following properties of the Riemann tensor:

- $R_{\lambda\mu\nu\kappa} + R_{\lambda\nu\kappa\mu} + R_{\lambda\kappa\mu\nu} = 0$
- $R_{\lambda\mu\nu\kappa;\eta} + R_{\lambda\mu\kappa\eta;\nu} + R_{\lambda\mu\eta\nu;\kappa} = 0$