Homework problems # 1

1. Show that the Lorentz transformations are the only non-singular coordinate transformations $x \to x'$ that leave $d\tau^2$ invariant (non-singularity means that both $\partial x'^{\alpha}/\partial x^{\beta}$ and $\partial x^{\beta}/\partial x'^{\alpha}$ exists).

2. Show that
$$\eta_{\alpha\beta} = \Lambda_{\alpha}{}^{\gamma}\Lambda_{\beta}{}^{\delta}\eta_{\gamma\delta} \, ,$$
then prove that $\eta_{\alpha\beta}$ is a covariant Lorentz tensor which transforms into itself.

3. Prove that
$$\varepsilon^{\varepsilon\xi\kappa\lambda}\Lambda^{\alpha}{}_{\varepsilon}\Lambda^{\beta}{}_{\xi}\Lambda^{\gamma}{}_{\kappa}\Lambda^{\delta}{}_{\lambda} = \varepsilon^{\alpha\beta\gamma\delta} \, ,$$
then show that $\varepsilon^{\alpha\beta\gamma\delta}$ is a contravariant Lorentz tensor which transforms into itself.