Homework problems #8

1. Prove that there is no independent tensor that can be constructed out of first derivatives of $g_{\mu\nu}$.
2. Prove that $R^{\lambda}{}_{\mu\nu\kappa}$ is the only tensor that can be constructed from the metric tensor and its first and second derivatives, that is linear in the second derivatives.
3. Show that the Riemann tensor transforms as a tensor.
4. Show that
$$\frac{D\,T^{\mu}{}_{\nu}}{D\,\tau} = \frac{d\,T^{\mu}{}_{\nu}}{d\,\tau} + \Gamma^{\mu}_{\lambda\rho}\frac{d\,x^{\lambda}}{d\,\tau}T^{\rho}{}_{\nu} - \Gamma^{\sigma}_{\lambda\nu}\frac{d\,x^{\lambda}}{d\,\tau}T^{\mu}{}_{\sigma}$$
transforms as a tensor.
5. Show that
$$\oint x^{\rho}\,dx^{\nu} = \delta a^{\rho}\delta b^{\nu} - \delta b^{\rho}\delta a^{\nu}$$
for the integral along a parallelogram spanned by δa^{μ} and δb^{μ}.
6. Show that the metric
$$g_{tt} = 1, \quad g_{rr} = -1, \quad g_{\theta\theta} = -r^2, \quad g_{\varphi\varphi} = -r^2\sin^2\theta$$
is equivalent to the Minkowski metric.