Homework problems #10

1. Show that the "retarded-potential"

$$h_{\mu\nu}(t,\vec{x}) = 4\pi \int d^3x' \, \frac{S_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}$$

   that is a solution of the graviton equation of motion in the harmonic gauge

$$\Box h_{\mu\nu} = -16\pi G (T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^{\alpha}{}_{\alpha}) \equiv -16\pi G S_{\mu\nu}$$

   indeed satisfies the harmonic gauge condition.

2. Expand the energy-momentum "tensor" of gravitational field

$$t_{\mu\nu} \equiv \frac{1}{8\pi G}\left[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\lambda}{}_{\lambda} - R^{(1)}{}_{\mu\nu} + \frac{1}{2} g_{\mu\nu} R^{(1)\lambda}{}_{\lambda}\right]$$

   up to the second order in powers of the graviton field $h_{\mu\nu}(x)$ and show that in a quasi-Minkowskian coordinate system $t_{\mu\nu} = \mathcal{O}(r^{-4})$ for $r \to \infty$.

3. Show that

$$P^{\lambda} \equiv \int_V \tau^{0\lambda} \, d^3x$$

   is invariant under any coordinate transformation that reduces at infinity to identity.