Homework problems #4b

1. For polarization tensors $\varepsilon^{(a)}_{\mu\nu}(k)$ of a massive graviton, write down the most general form for $\sum_a \varepsilon^{(a)}_{\mu\nu}(k)\varepsilon^{(a)}_{\lambda\sigma}(k)$ using symmetry repeatedly. For example, it must be invariant under the exchange $\{\mu\nu \leftrightarrow \lambda\sigma\}$. You might end up with something like

$$\begin{aligned}&AG_{\mu\nu}G_{\lambda\sigma} + B(G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) + C(G_{\mu\nu}k_\lambda k_\sigma + k_\mu k_\nu G_{\lambda\sigma})\\ &+D(k_\mu k_\lambda G_{\nu\sigma} + k_\mu k_\sigma G_{\lambda\nu} + k_\nu k_\sigma G_{\mu\lambda} + k_\nu k_\lambda G_{\mu\sigma}) + Ek_\mu k_\nu k_\lambda k_\sigma\end{aligned}$$

where

$$G_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}$$

and $A, \cdots, E$ are unknown constants. Apply $k^\mu \sum_a \varepsilon^{(a)}_{\mu\nu}(k)\varepsilon^{(a)}_{\lambda\sigma}(k) = 0$ and find out what that implies for the constants. Proceeding this way, derive

$$\sum_a \varepsilon^{(a)}_{\mu\nu}(k)\varepsilon^{(a)}_{\lambda\sigma}(k) = (G_{\mu\lambda}G_{\nu\sigma} + G_{\mu\sigma}G_{\nu\lambda}) - \frac{2}{3}G_{\mu\nu}G_{\lambda\sigma}\,.$$

2. Assuming the following interaction Lagrangian beyween the symmetric tensor field $h_{\mu\nu}(x)$ and a source $T^{\mu\nu}(x)$:

$$\mathcal{L}_{\text{int}} = \lambda T^{\mu\nu}(x)h_{\mu\nu}(x)$$

show that the energy of the field configuration created by a stationary source $T_{\mu\nu}(\vec{x})$ could be written as

$$E = \frac{\lambda^2}{2}\int d^3\vec{x}d^4y T^{\mu\nu}(x)D_{\mu\nu\,\sigma\rho}(x-y)T^{\sigma\rho}(y)$$

where $D_{\mu\nu\,\sigma\rho}(x)$ is a propagator of $h_{\mu\nu}(x)$. Find E for $T_{\mu\nu}(x) = m_1\delta_{\mu0}\delta_{\nu0}\delta^{(3)}(\vec{x}) + m_2\delta_{\mu0}\delta_{\nu0}\delta^{(3)}(\vec{x}-\vec{z})$.