Theory of elementary particles 2004/05

Homework problems 2

1. Derive the following identities:

   - $$\gamma_5 = \frac{i}{4!}\varepsilon_{\alpha\beta\mu\nu}\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu$$

     for $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$,

   - $$\gamma_\lambda\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\lambda = -2\gamma^\gamma\gamma^\beta\gamma^\alpha$$

   - $$\gamma_\lambda\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma^\lambda = 2(\gamma^\delta\gamma^\alpha\gamma^\beta\gamma^\gamma + \gamma^\gamma\gamma^\beta\gamma^\alpha\gamma^\delta)$$

   - $$\varepsilon^{\alpha\beta\mu\nu}\varepsilon_{\alpha\beta\sigma\tau} = -2(g^\mu_\sigma g^\nu_\tau - g^\mu_\tau g^\nu_\sigma)$$

   - $$\varepsilon^{\alpha\beta\gamma\nu}\varepsilon_{\alpha\beta\gamma\tau} = -6g^\nu_\tau$$

   - $$\varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{\alpha\beta\gamma\delta} = -24$$

   - $$\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta) = 4(g^{\alpha\beta}g^{\gamma\delta} + g^{\alpha\delta}g^{\beta\gamma} - g^{\alpha\gamma}g^{\beta\delta}$$

   - $$\mathrm{Tr}(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta) = -4\varepsilon^{\alpha\beta\gamma\delta}\,.$$

2. Derive the following results for two-particle phase-space integrals

$$\int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta^{(4)}(P-p_1-p_2)f(p_1\cdot p_2) = \frac{\pi}{2}\frac{1}{P^2}w(P^2,m_1^1,m_2^2)f\left[\frac{1}{2}(P^2-m_1^2-m_2^2)\right]$$

$$\int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta^{(4)}(P-p_1-p_2)f(p_1\cdot p_2)p_2^\mu =$$

$$\frac{\pi}{4}\frac{1}{P^4}w(P^2,m_1^1,m_2^2)[P^2+m_2^2-m_1^2]f\left[\frac{1}{2}(P^2-m_1^2-m_2^2)\right]P^\mu$$

$$\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta^{(4)}(P-p_1-p_2)f(p_1\cdot p_2)p_2^\mu p_2^\nu=\frac{\pi}{6}\frac{1}{P^4}w(P^2,m_1^1,m_2^2)$$
$$f\left[\frac{1}{2}(P^2-m_1^2-m_2^2)\right]\cdot\left\{\frac{P^\mu P^\nu}{P^2}[(P^2+m_2^2-m_1^2)^2-P^2m_2^2]-\frac{g^{\mu\nu}}{4}[w(P^2,m_1^1,m_2^2)]^2\right\}$$

where $p_i^2=m_i^2$ for $i=1,2$, and $w(a,b,c)\equiv[a^2+b^2+c^2-2(ab+ac+bc)]^{1/2}$

3. Show that, in the center-of-mass frame, the differential cross-section for

   - the inverse muon decay process
     $$\nu_\mu+e^-\to\nu_e+\mu^-\,,$$
     is given by
     $$\frac{d\sigma}{d\Omega}=\frac{G_F^2E^2}{\pi^2}$$
     if all the lepton masses have been neglected compared with the center-of-mass energy E of the incoming neutrino and if the energy is much smaller than the W boson mass,
   - $$\bar{\nu}_e+e^-\to\bar{\nu}_\mu+\mu^-\,,$$
     is given by
     $$\frac{d\sigma}{d\Omega}=\frac{G_F^2E^2}{4\pi^2}(1-\cos\theta)^2\,,$$
     where θ is the angle between the incoming neutrino and the outgoing muon, see e.g. [1].

4. Let us denote by Φ, the SU(2) Higgs doublet of the Standard Model (SM) and define $\tilde{\Phi}\equiv i\tau_2\Phi^*$. Show that in the scalar potential of the SM one does not need to consider terms $|\Phi^\dagger\tilde{\Phi}|$, $\tilde{\Phi}^\dagger\tilde{\Phi}$, $\Phi^\dagger\Phi|\Phi^\dagger\tilde{\Phi}|$, $\Phi^\dagger\Phi|\tilde{\Phi}^\dagger\Phi|$, $\Phi^\dagger\Phi\tilde{\Phi}^\dagger\tilde{\Phi}$, $|\Phi^\dagger\tilde{\Phi}\Phi^\dagger\tilde{\Phi}|$, $\tilde{\Phi}^\dagger\Phi\Phi^\dagger\tilde{\Phi}$, $|\Phi^\dagger\tilde{\Phi}|\tilde{\Phi}^\dagger\tilde{\Phi}$, $|\tilde{\Phi}^\dagger\Phi|\tilde{\Phi}^\dagger\tilde{\Phi}$, and $\tilde{\Phi}^\dagger\tilde{\Phi}\tilde{\Phi}^\dagger\tilde{\Phi}$, even though they are allowed by the gauge symmetry of the SM.

5. Adopting the unitary gauge, derive all purely bosonic interactions of the SM. The result should be expressed in terms of fields which are mass-eigenstates.

6. Show that in the SM, it is possible to perform a gauge transformation such that any constant value of the scalar field which corresponds to the minimal energy (in the classical sense) could be transformed to

$$\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} ,$$

where v is a real number.

7. Derive transformation properties under CP for the following bilinear forms:
   - $\bar{\psi}_1 \sigma_{\mu\nu} \psi_2$
   - $\bar{\psi}_1 \gamma_5 \sigma_{\mu\nu} \psi_2$
   - $\bar{\psi}_1 \gamma_\mu \partial_\nu \psi_2$
   - $\bar{\psi}_1 \gamma_5 \gamma_\mu \partial_\nu \psi_2$

8. Check transformations under C, P and CP for the following interactions
   - $F_{\mu\nu} \bar{\psi}_1 \sigma^{\mu\nu} \psi_2$
   - $\tilde{F}_{\mu\nu} \bar{\psi}_1 \sigma^{\mu\nu} \psi_2$

   where $\tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$.

9. In the SM, derive the transformation properties of W_μ, $W_\mu^\dagger$, Z_μ and A_μ.

10. Derive the following Firz identity:

$$\begin{aligned} [\gamma_\beta(1+\lambda_2\gamma_5)]_{\lambda\rho}\,[\gamma^\beta(1+\lambda_4\gamma_5)]_{\mu\sigma} &= \\ (1-\lambda_2\lambda_4)[1+\lambda_2\gamma_5]_{\lambda\sigma}\,[1+\lambda_4\gamma_5]_{\mu\rho} &- \frac{1}{2}(1+\lambda_2\lambda_4)[\gamma_\beta(1+\lambda_2\gamma_5)]_{\lambda\sigma}\,[\gamma^\beta(1+\lambda_4\gamma_5)]_{\mu\rho} \,, \end{aligned}$$

where $\lambda_{2,4} = \pm 1$.

11. Assuming the following form of interactions between the leptonic current $J_\alpha^{(l)\,\dagger} = \bar{\psi}_l \gamma_\alpha (1-\gamma_5)\psi_{\nu_l}$ and the hadronic one $J^{(h)\,\alpha}$,

$$\mathcal{L}_{\rm int} = -\frac{G_F}{\sqrt{2}} J_\alpha^{(l)\,\dagger} J^{(h)\,\alpha}$$

calculate

$$\frac{\Gamma(\pi^- \to e^- + \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu)}$$

Use the fact that the hadronic matrix element $\langle 0|J^{(h)\,\alpha}|\pi^-\rangle$ could be parameterized as

$$\langle 0|J^{(h)\,\alpha}|\pi^-\rangle = \frac{\cos\theta_C f_\pi}{(2E_\pi)^{1/2}} q^\alpha$$

where f_π is the pion decay constant, θ_C is the Cabibbo angle, and q is the momentum of the pion. Discuss the result taking into account $V - A$ structure of the interactions, see e.g. [2].

12. In the SM, calculate the ratio

$$R \equiv \frac{\sigma(\nu_\mu + e^- \to \nu_\mu + e^-)}{\sigma(\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-)}$$

and discuss how R could be adopted to determine the Weinberg angle θ_W, see e.g. [2].

13. In the SM calculate the following Higgs-boson decay widths

- $\Gamma(H \to \bar{f}f)$, $f = t, b$
- $\Gamma(H \to ZZ)$
- $\Gamma(H \to W^+W^-)$

For each decay plot the corresponding branching ratio in the range of the Higgs boson mass between 115 GeV and 250 GeV. Discuss the result in the context of possible discovery of the Higgs boson. See [3].

14. In the SM calculate $\sigma(e^+e^- \to HZ$ and estimate the center-of-mass energy that corresponds for the maximal cross-section, see [3].

References

[1] F. Mandl, G. Shaw, *Quantum Field Theory.*

[2] E. Commins, P. Bucksbaum, *Weak interactions of leptons and quarks.*

[3] J. Gunion, H. Haber, G. Kane, S. Dawson, *The Higgs Hunters Guide.*

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