Warszawa, 19 April 2005 r.

Theory of elementary particles – homework problems

1. For a two-to-two process, $a+b \to c+d$, show that the Mandelstam variables defined as $s = (p_a+p_b)^2$, $t = (p_a-p_c)^2$, $u = (p_a-p_d)^2$, satisfy $s+t+u = m_a^2+m_b^2+m_c^2+m_d^2$.

2. Express the matrix elements squared for processes (a) $e^+e^- \to \mu^+\mu^-$, (b) $e^-\mu^- \to e^-\mu^-$ in terms of Mandelstam variables. Find the "crossing symmetry", i.e. how the matrix element squared for (b) can be obtained from (a). Relate this symmetry to the change of 4-momenta when going from process (a) to (b).

3. Calculate $d\sigma/d\Omega$ for $e^+e^- \to \mu^+\mu^-$ neglecting the electron mass, but keeping the finite mass of the muon. Discuss the energy dependence of the total cross-section near the threshold, i.e. for E_{CM} just above $4m_\mu^2$.

4. Calculate $d\sigma/d\Omega\, dE'$ for the deep-inelastic scattering of electron on a scalar particle (for example on a pion π^+) in the one-photon exchange approximation. The Feynman rule for the vertex with photon coupling to a scalar particle is given by $-ie(p+p')^\mu$, where p (p') is the 4-momentum of the scalar particle entering (leaving) the vertex.

5. Calculate $d\sigma/d\Omega$ and σ for $e^+e^- \to \pi^+\pi^-$ keeping $m_\pi \neq 0$. Compare the pion angular distribution and the energy dependence of the total cross section near threshold with the case $e^+e^- \to \mu^+\mu^-$.

6. Applying the technique used in the class, calculate $d\sigma/dx_1\, dx_2$ for $e^+e^- \to q\bar{q}S$, where S is a scalar particle that couples to the quark line with the following Feynman rule: ig_s. Compare to $e^+e^- \to q\bar{q}g$.

7. Calculate the first-order correction to the photon propagator from the loop with a scalar particle S. Notice that two diagrams are necessary to get the gauge invariant matrix element. The two vertices are $SS\gamma$ with the Feynman rule as in Problem 4, and the other $SS\gamma\gamma$ with $2ie^2g^{\mu\nu}$. Compare the result with the fermionic correction.

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