Quantization with (pseudo)-action & angle coherent states

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Abstract

Consider a conservative mechanical system with one-degree of freedom and with observed energy spectrum $E_0, E_1, ...E_n, ...$ (discrete) or $0 \leq \mathcal{E} < \mathcal{E}_M \leq \infty$ (continuous). Inspired by [1], families of corresponding (pseudo-) action & angle coherent states are constructed in view to provide a quantization scheme consistent with these experimental energies. The construction is based on a Bayesian approach [2]: each family corresponds to a choice of probability distributions, $n \mapsto p_n(J)/\mathcal{N}(J)$ (resp. $\mathfrak{J} \mapsto p_{\mathcal{E}}(\mathfrak{J})/\mathcal{N}(\mathfrak{J})$)(prior), $J \mapsto p_n(J)$ (resp. $\mathcal{E} \mapsto p_{\mathcal{E}}(\mathfrak{J})$) (posterior) such that

$$0 < \mathcal{N}(J) \stackrel{\text{def}}{=} \sum_n p_n(J) < \infty, \qquad \int_{R_J} d\tilde{J} E(J)\, p_n(J) = E_n + \text{const},$$

resp.

$$0 < \mathcal{N}(\mathfrak{J}) \stackrel{\text{def}}{=} \int_0^{\tilde{\mathcal{E}}_M} d\tilde{\mathcal{E}}\, p_{\mathcal{E}}(\mathfrak{J}) < \infty, \quad \int_{\mathscr{R}_{\mathfrak{J}}} d\tilde{\mathfrak{J}}\, E(\mathfrak{J})\, p_{\mathcal{E}}(\mathfrak{J}) = \mathcal{E} + \text{const} \;,$$

where $E(J)$ (resp. $E(\mathfrak{J})$) is the classical energy as a function of the (pseudo-) action variable J (resp. $\mathfrak{J}$), and tilded quantities have no physical dimension.

The formalism can be viewed as a natural extension of the Bohr-Sommerfeld rule and an alternative to the canonical quantization. In particular, it yields in the discrete case a satisfying angle operator and dynamically stable coherent states in both discrete or continuous cases. The respective semi-classical behaviors of different families of such states are compared.

References

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[3] V. Bagrov, J.P. Gazeau, D. Gitman, and A. Levine, *Coherent states and related quantization for unbounded motions*, *in preparation*