High entangling power in a non-chaotic regime

the study of coupled kicked tops

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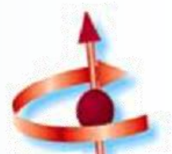
Abstract

Entangling properties of the system of coupled kicked tops are studied. Initial product states are evolved in order to find out how they are being entangled, depending on the chaoticity of dynamics. Two different ensembles of initial product states are considered: product states of independent spin-coherent states and product states of random states. In these two cases initial entanglement growth rate is analysed as well as asymptotic entanglement. It appears that the choice of either of these ensembles results in significantly different averaged entanglement behaviour. In the case of ensemble of random product states, regular dynamics yields both higher entanglement growth rate and asymptotic entanglement, as compared to the chaotic one. Additionally lower bound on averaged asymptotic entanglement is derived, expressed in terms of the eigenvector entanglement.

1. Single Kicked Top

- Kicked top is a periodically kicked system, with the total spin j conserved



j – total spin $\quad$ $\boldsymbol{H}$ - Hilbert space $\quad$ $\dim \boldsymbol{H} = 2j+1$ $\quad$ $|-j\rangle, \ldots, |j\rangle \in \boldsymbol{H}$

- Hamiltonian:

$$\mathrm{H}(t) = \underbrace{\tfrac{\pi}{2} J_y}_{\text{free rotation}} + \underbrace{\tfrac{k}{2j} J_z^2 \sum_{n=-\infty}^{+\infty} \delta(n-t)}_{\text{kicks}}$$

(kick strength: k)

One period evolution:

$$U = e^{-i\frac{k}{2j}J_z^2} e^{-i\frac{\pi}{2}J_y}$$

2. Spin-coherent states

- Spin-coherent states are the most classical spin states. They satisfy minimal uncertainty with respect to angular momentum components:

$$\Delta J^2 = \Delta J_x^2 + \Delta J_y^2 + \Delta J_z^2$$ - is minimal for spin-coherent states

- Spin-coherent states are obtained as rotations of $|j\rangle$ state

$$|\theta,\varphi\rangle = R(\theta,\varphi)|j\rangle \qquad R(\theta,\varphi) = \exp\left(-i\theta\left(J_x \sin\varphi - J_y \cos\varphi\right)\right)$$

3. Classical limit for the Kicked Top

- When $j \to \infty$ spin-coherent states became classical states with well defined angular momentum components.

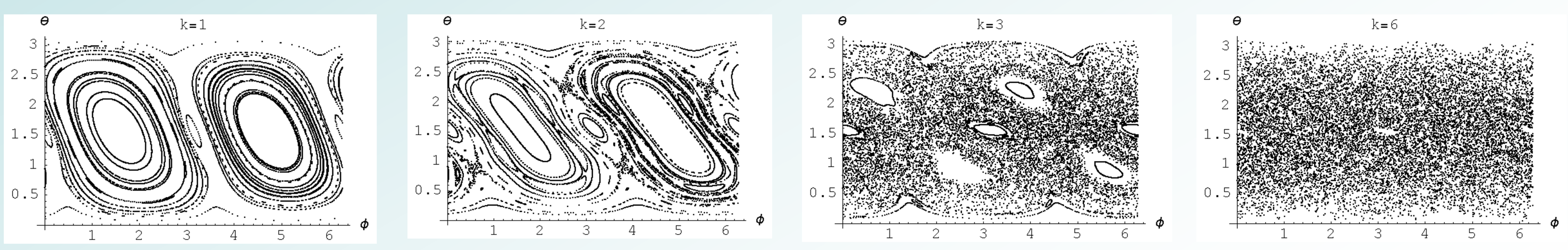
- Direction of the angular momentum of the top evolves according to the map:

$$\begin{cases} \tilde{X} = Z\cos(kX) + Y\sin(kX) \\ \tilde{Y} = -Z\sin(kX) + Y\cos(kX) \\ \tilde{Z} = -X \end{cases} \qquad X^2 + Y^2 + Z^2 = 1$$

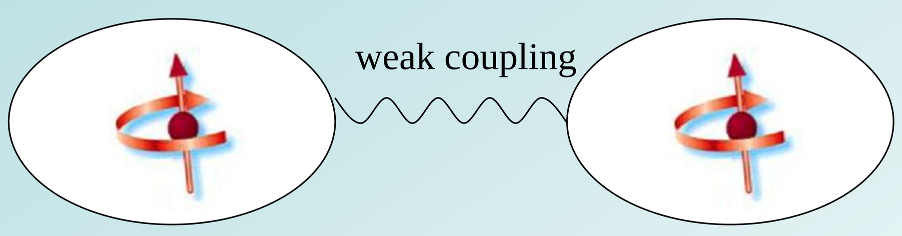
discrete dynamics on a sphere

- Depending on the value of k parameter, classical dynamics is either regular or chaotic:

k=1 $\quad$ *k*=2 $\quad$ *k*=3 $\quad$ *k*=6



4. Coupled kicked tops



- System consisting of two weakly coupled kicked tops, each with the same chaoticity parameter k.

- Hamiltonian: $\quad \mathrm{H} = \mathrm{H}_1 + \mathrm{H}_2 + \mathrm{H}_{\text{int}}$

$$\underbrace{\mathrm{H}_i(t) = \tfrac{\pi}{2} J_{y_i} + \tfrac{k}{2j} J_{z_i}^2 \sum_{n=-\infty}^{+\infty} \delta(n-t)}_{\text{single kicked top Hamiltonian}} \qquad \underbrace{\mathrm{H}_{\text{int}}(t) = \tfrac{\varepsilon}{j} J_{z_1} J_{z_2} \sum_{n=-\infty}^{+\infty} \delta(n-t)}_{\text{weak interaction } (\varepsilon \text{ is small})}$$

- One period evolution operator:

$$U = \underbrace{U_1 \otimes U_2}_{\text{separate evolution of each of the tops}} \underbrace{U_{\text{int}}}_{\text{entangling part}}$$

$$U_1 = e^{-i\frac{k}{2j}J_{z_1}^2} e^{-i\frac{\pi}{2}J_{y_1}} \qquad U_2 = e^{-i\frac{k}{2j}J_{z_2}^2} e^{-i\frac{\pi}{2}J_{y_2}}$$

$$U_{\text{int}} = e^{-i\frac{\varepsilon}{j}J_{z_1}J_{z_2}}$$

5. Entanglement production

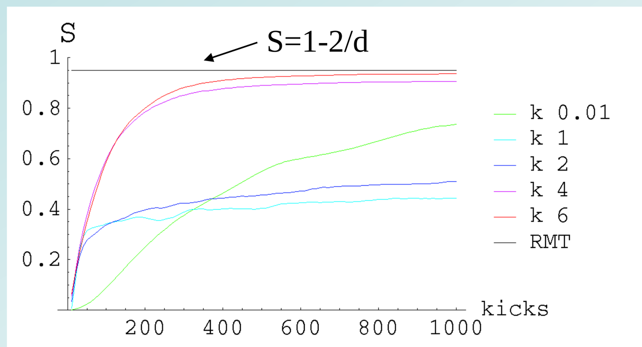
- Linear entropy is used as a measure of entanglement:

$$S(|\psi\rangle) = 1 - \mathrm{Tr}(\rho_1^2) \qquad \rho_1 = \mathrm{Tr}_2|\psi\rangle\langle\psi|$$ - reduced density matrix

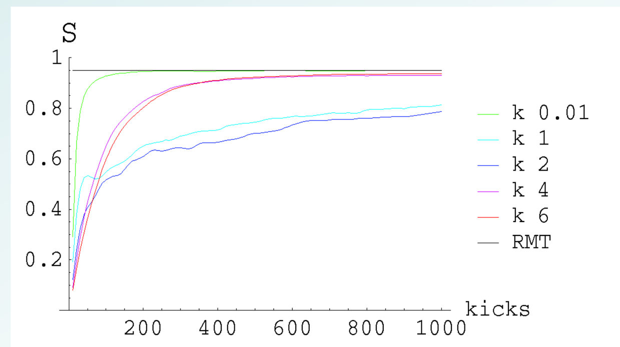
- Evolution of entanglement, averaged over two different ensembles of initial product states:

(j=20, ε=0.01)

spin-coherent states



random product states

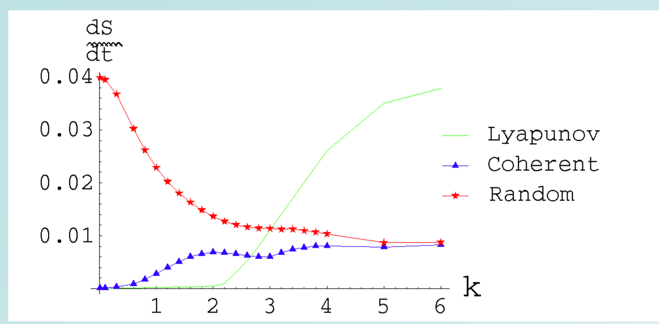


- chaos helps in achieving high asymptotic entanglement
- very regular dynamics has extremely low initial entanglement growth rate.

- asymptotic entanglement is very high both for regular and chaotic dynamics.
- initial entanglement growth is the highest for very regular dynamics.

6. Initial entanglement growth

- Initial entanglement growth rate for two ensembles of initial product states and Lyapunov exponent of the single kicked top classical dynamics vs. chaoticity parameter k:



- chaos enhances initial entanglement growth rate for spin-coherent states.
- chaos diminishes entanglement growth rate for random product states
- short time *entangling power* (which corresponds to averaging over random states) is the highest for very regular dynamics!

- Explanation:

Perturbative formula by Fujisaki et. al. (PRE 66, 045201 (2002)), relates initial entanglement growth rate to the time correlation function of J_z operator of a single top, evolved under non-coupled dynamics:

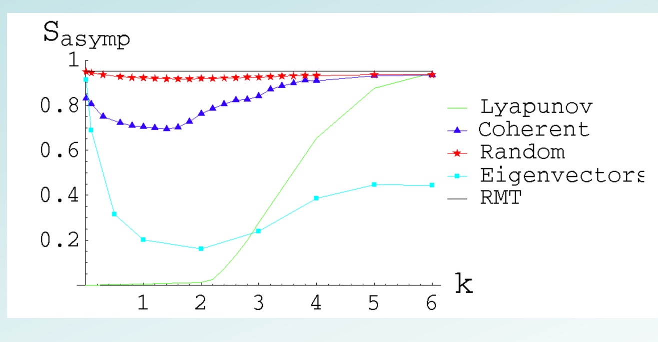
$$S(t) = 2\varepsilon^2 j^2 \sum_{t_1=1}^{t}\sum_{t_2=1}^{t} C^2(t_1,t_2) \qquad C(t_1,t_2) = \frac{\langle J_z(t_1)J_z(t_2)\rangle - \langle J_z(t_1)\rangle\langle J_z(t_2)\rangle}{j^2}$$

operator J_z in the Heisenberg picture, evolved according to single kicked top dynamics

- **spin-coherent states**, have very low dispersion of J_z operator. Chaotic evolution quickly increases dispersion of J_z operator and thus increases $C(t,t)$ and eventually increases initial entanglement growth rate. In the case of spin-coherent states chaos helps initial entanglement growth.
- **random states**, have already large dispersion of J_z operator, so chaotic dynamics doesn't help. Additionally chaotic dynamics quickly destroys correlations $C(t_1,t_2)$ (for t1≠t2). Consequently chaos lowers initial entanglement growth rate in the case of initial random product states.

7.Asymptotic entanglement

- Asymptotic entanglement for two ensembles of initial product states, Lyapunov exponent of the single kicked top classical dynamics and averaged entanglement of eigenvectors of evolution operator U vs. chaoticity parameter k:



- asymptotic entanglement is very high for both chaotic and very regular dynamics
- asymptotic entanglement in the case of averaging over random states shows very weak dependence on chaoticity parameter k.
- High entanglement of eigenvectors in the case of very regular dynamics is the reason for high asymptotic entanglement of evolved states (see below)

8. Entanglement of eigenvectors

- Entanglement of eigenvectors of a transformation does not give full information on the entangling properties of the transformation. Nevertheless it gives a lower bound on the asymptotic entanglement of evolved states

$$\underbrace{\bar{S}_{asymp}}_{\text{Asymptotic entanglement averaged over initial product states}} \geq 2\underbrace{\bar{S}_{eigen}}_{\text{Mean entanglement of eigenvectors}} - 1$$

Conclusions

- Chaos does not help in initial entanglement growth rate, when, averaging is performed over random product states – entangling power is lower for chaotic dynamics
- Chaos may help in enhancing initial entanglement growth, when evolving low-uncertainty states – for example spin-coherent states.
- Chaos assures high asymptotic entanglement, though high asymptotic entanglement can also be reached in the case of regular dynamics due to high entanglement of eigenvectors

Reference: Rafał Demkowicz-Dobrzański, Marek Kuś *„Global entangling properties of the coupled kicked tops"* quant-ph/0403232