Quantum enhanced phase estimation in the presence of loss

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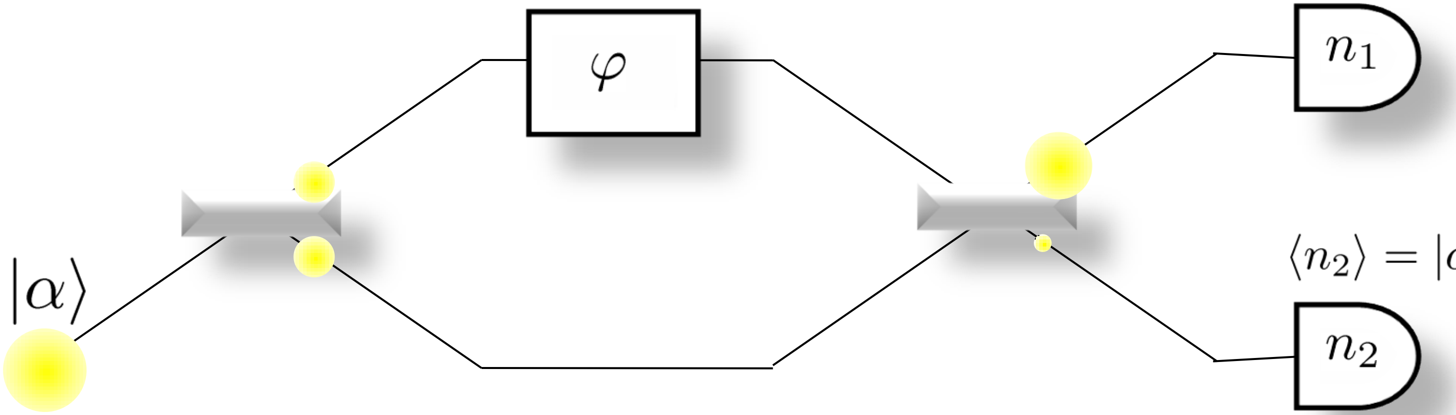
Interferometry

- **Mach-Zehnder Interferometer**

$\langle n_1 \rangle = |\alpha|^2 (1 + \cos\varphi)/2$



$\langle n_2 \rangle = |\alpha|^2 (1 - \cos\varphi)/2$

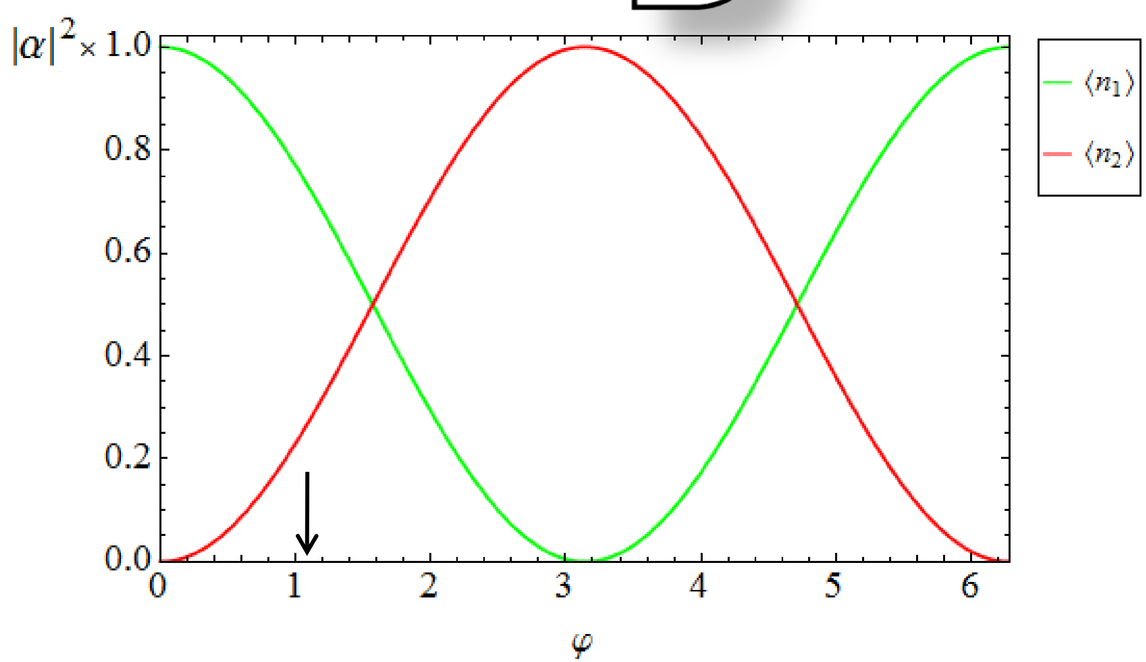
detecting n_1 and n_2

+

knowing theoretical dependence of n_1, n_2 on φ

↓

we can estimate φ



Interferometry

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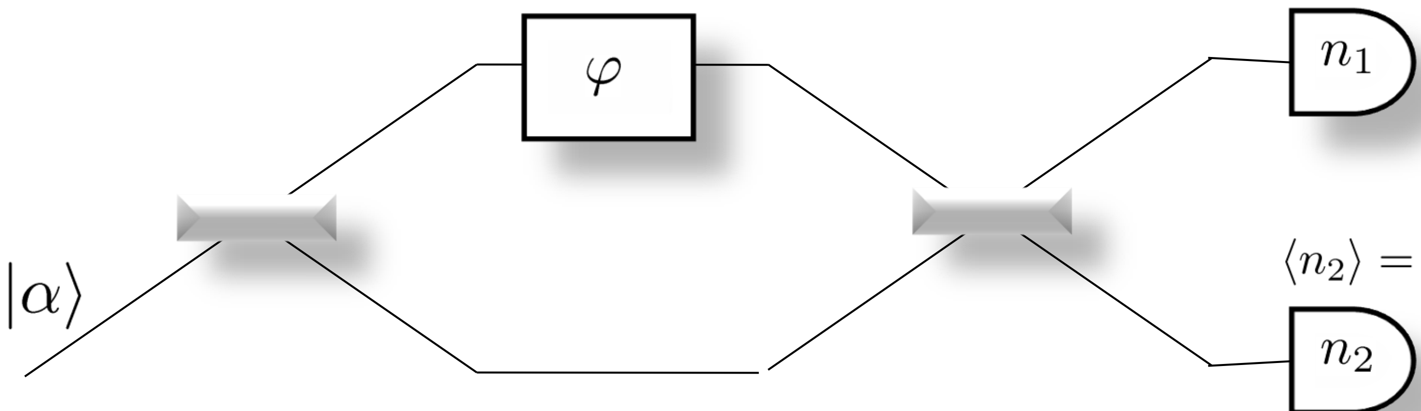


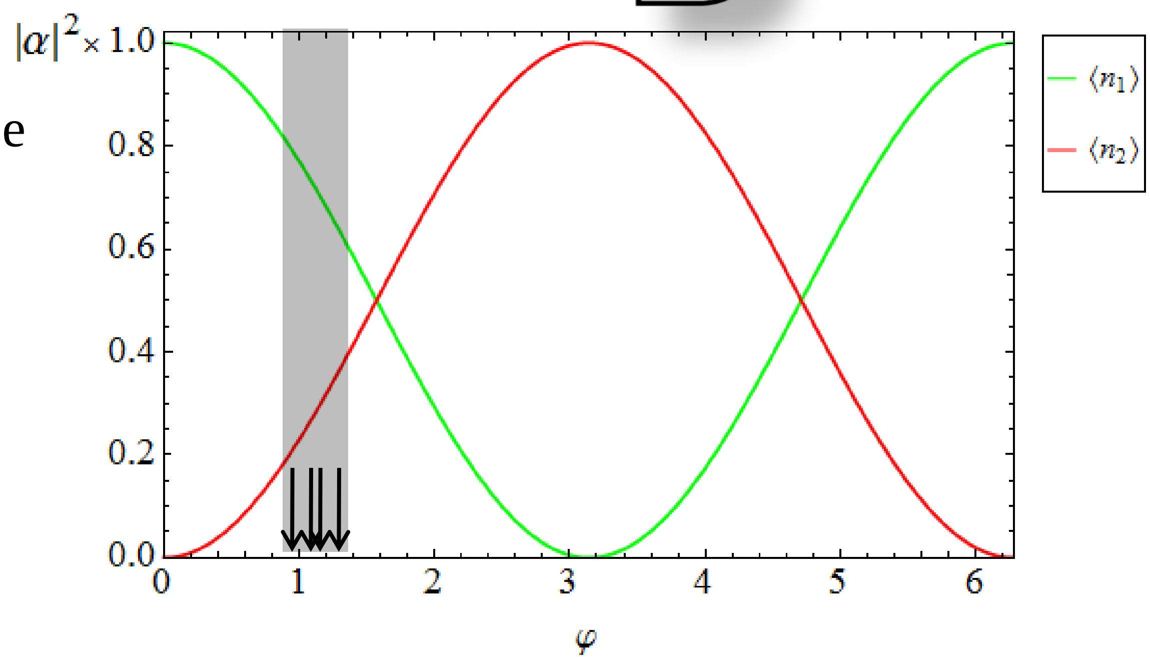
$\langle n_2 \rangle = |\alpha|^2(1 - \cos\varphi)/2$

n_1 and n_2 are subject to shot noise

each measurement yields a bit different φ

$$\delta\varphi \propto \frac{1}{|\alpha|} = \frac{1}{\sqrt{\bar{n}}}$$

Standard scaling



Interferometry

- **Mach-Zehnder Interferometer**

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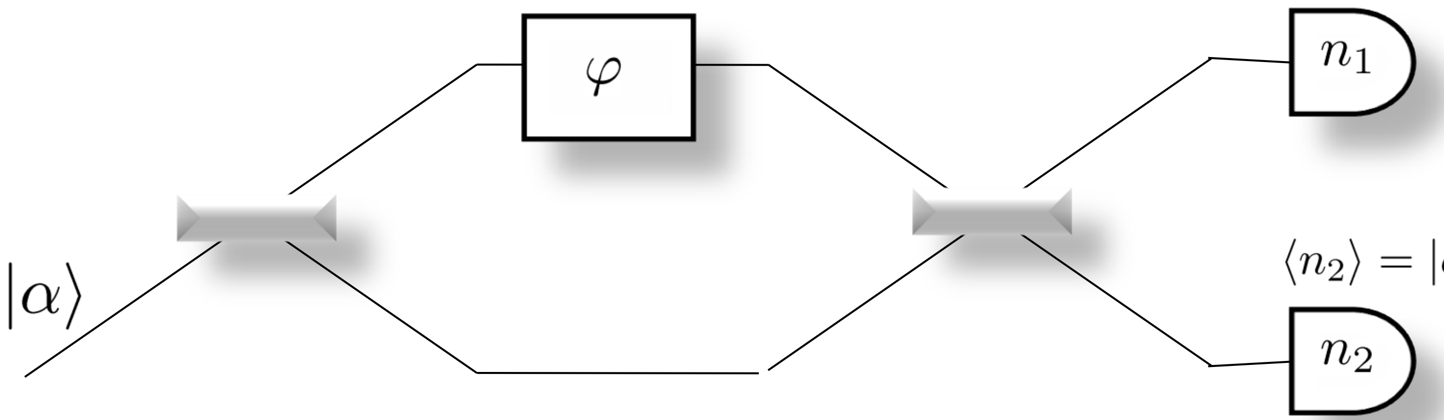
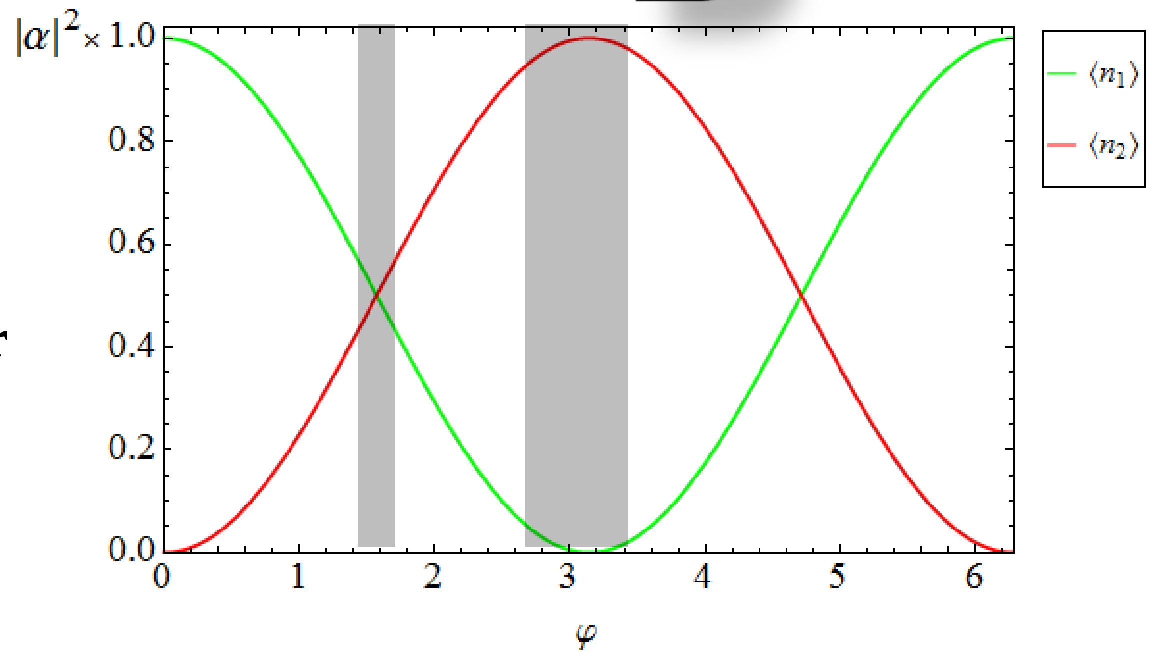


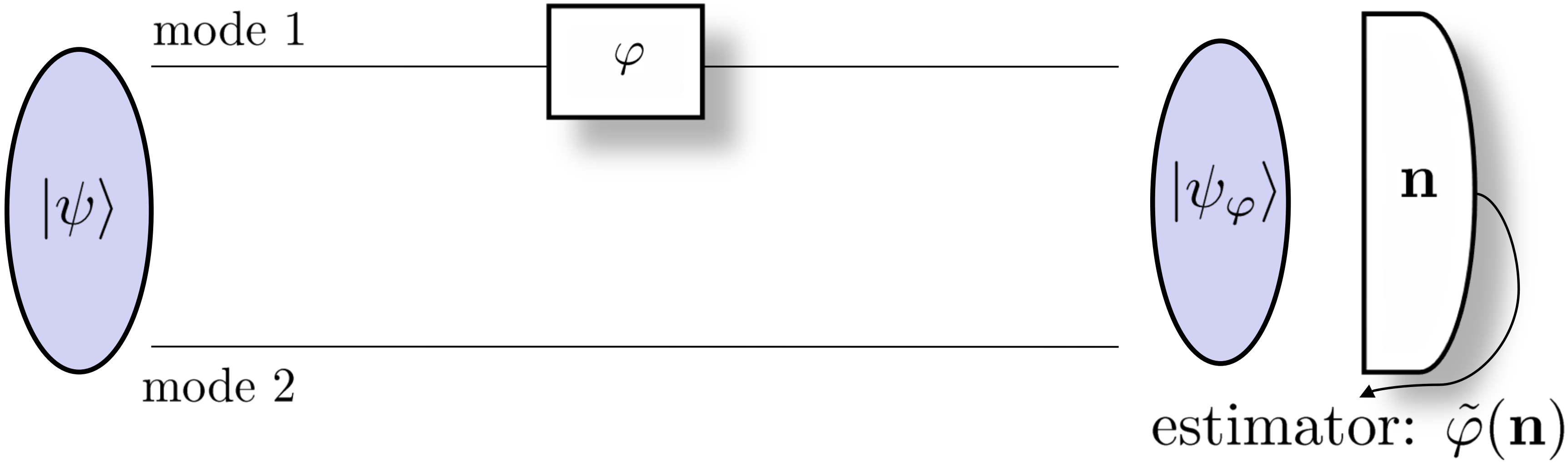
$$\langle n_2 \rangle = |\alpha|^2 (1 - \cos\varphi)/2$$

In general the exact formula for precision depends on φ

The steeper curves, the better precission



Heisenberg scaling - Quantum Cramer-Rao bound



For arbitrary measurement on $|\psi_\varphi\rangle$ and arbitrary estimator $\tilde{\varphi}(\mathbf{n})$, the minimial variance of the estimator is bounded by:

$$\delta\tilde{\varphi} \geq \frac{1}{\sqrt{F}}, \quad F = 4\Delta n_1 = 4[\langle\psi|\hat{n}_1^2|\psi\rangle - \langle\psi|\hat{n}_1|\psi\rangle^2]$$

Given N photons

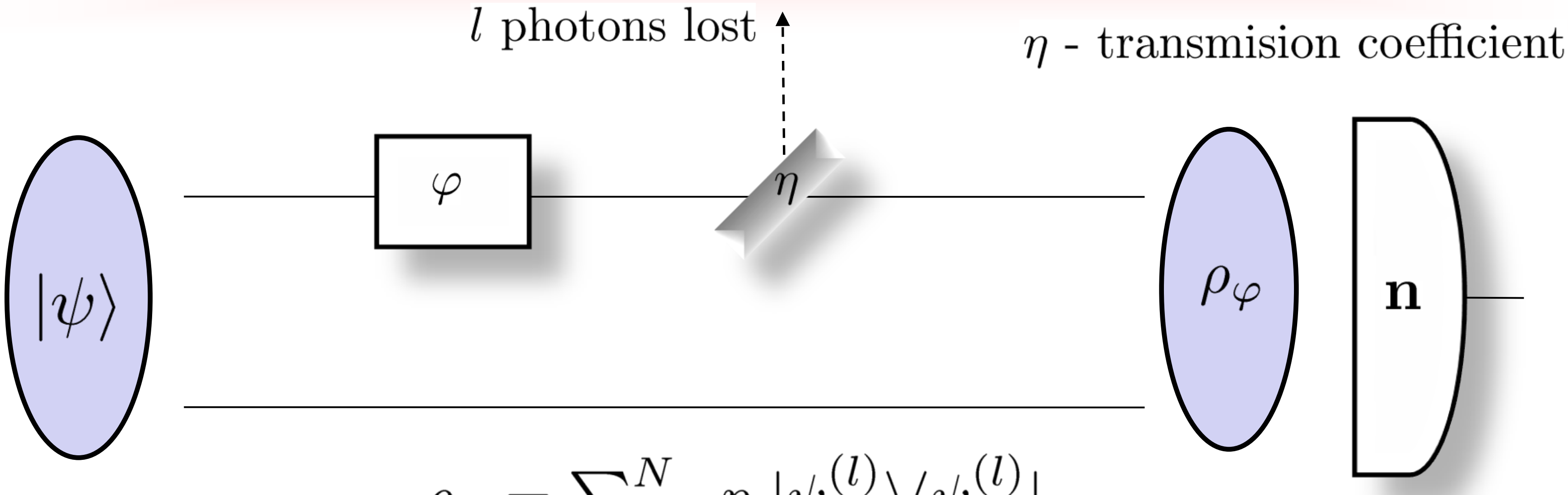
$$\Delta n_1 \leq N^2/4$$

$$\boxed{\delta\tilde{\varphi} \geq \frac{1}{\sqrt{N^2}} = \frac{1}{N}}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle)$$

What are the optimal N photon states if there are losses ?

Lossy interferometry



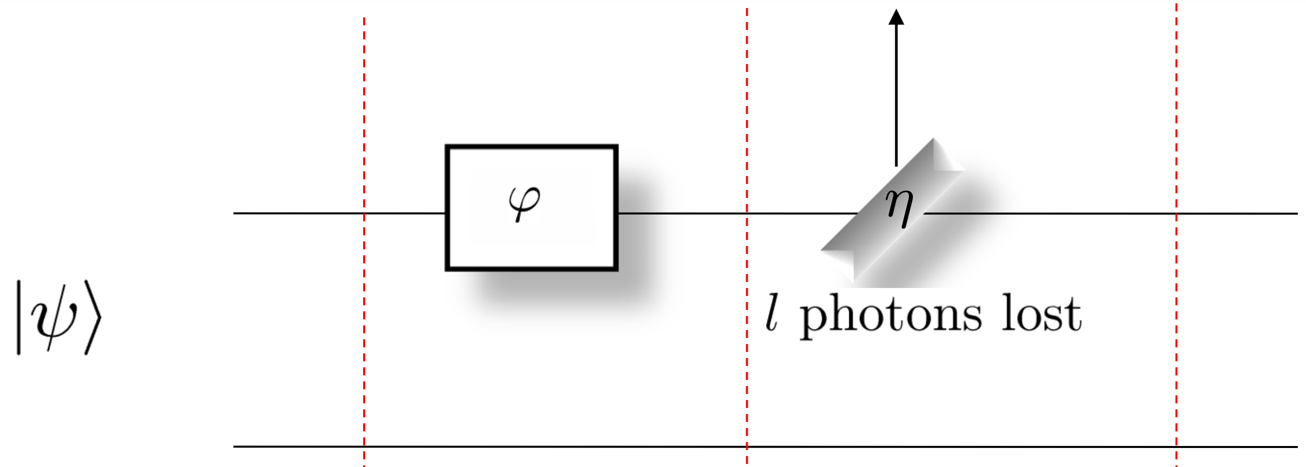
$$\rho_\varphi = \sum_{l=0}^{N} p_l |\psi_\varphi^{(l)}\rangle\langle\psi_\varphi^{(l)}|$$

- **NOON state** $|\psi_\varphi\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle e^{-iN\varphi} + |0,N\rangle)$

- **Even if a single photon is lost**

$|\psi\rangle \longrightarrow |\psi_\varphi^{(1)}\rangle \propto |N-1,0\rangle$ we lose all phase information!

Looking for the optimal state



$$|\psi\rangle = \sum_{n=0}^{N} \alpha_n |n, N-n\rangle$$

$$\rho_\varphi = \sum_{l=0}^{N} p_l |\psi_\varphi^{(l)}\rangle\langle\psi_\varphi^{(l)}|$$

$$|\psi_\varphi^{(l)}\rangle = \sum_{n=0}^{N} e^{-in\varphi} \alpha_n |n, N-n\rangle$$

$$|\psi_\varphi^{(l)}\rangle = \sum_{n=l}^{N} e^{-in\varphi} \alpha_n \sqrt{\binom{n}{l} \eta^{n-l} (1-\eta)^l} |n-l, N-n\rangle$$

Cramer-Rao bound for mixed states

$$|\psi\rangle = \sum_{n=0}^{N} \alpha_n |n, N-n\rangle \longrightarrow \rho_\varphi = \sum_{l=0}^{N} p_l |\psi_\varphi^{(l)}\rangle\langle\psi_\varphi^{(l)}|$$

- **Subspaces with different *l* are orthogonal**

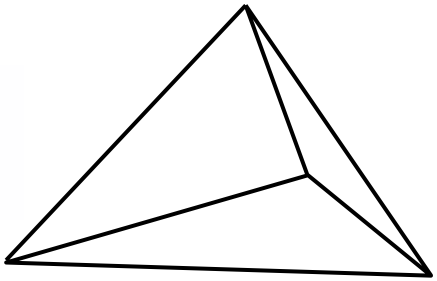
$$\delta\tilde{\varphi} \geq \frac{1}{\sqrt{F}} \qquad F = \sum_{l=0}^{N} p_l F_l$$

$$F_l = 4(\Delta n_1)_l = 4(\langle\psi^{(l)}|\hat{n}_1^2|\psi^{(l)}\rangle - (\langle\psi^{(l)}|\hat{n}_1|\psi^{(l)}\rangle)^2)$$

$$F = F(x_0, \ldots, x_N) \qquad x_n = |\alpha_n|^2$$

concave function

- **Problem: maximization of a concave function over a convex set**

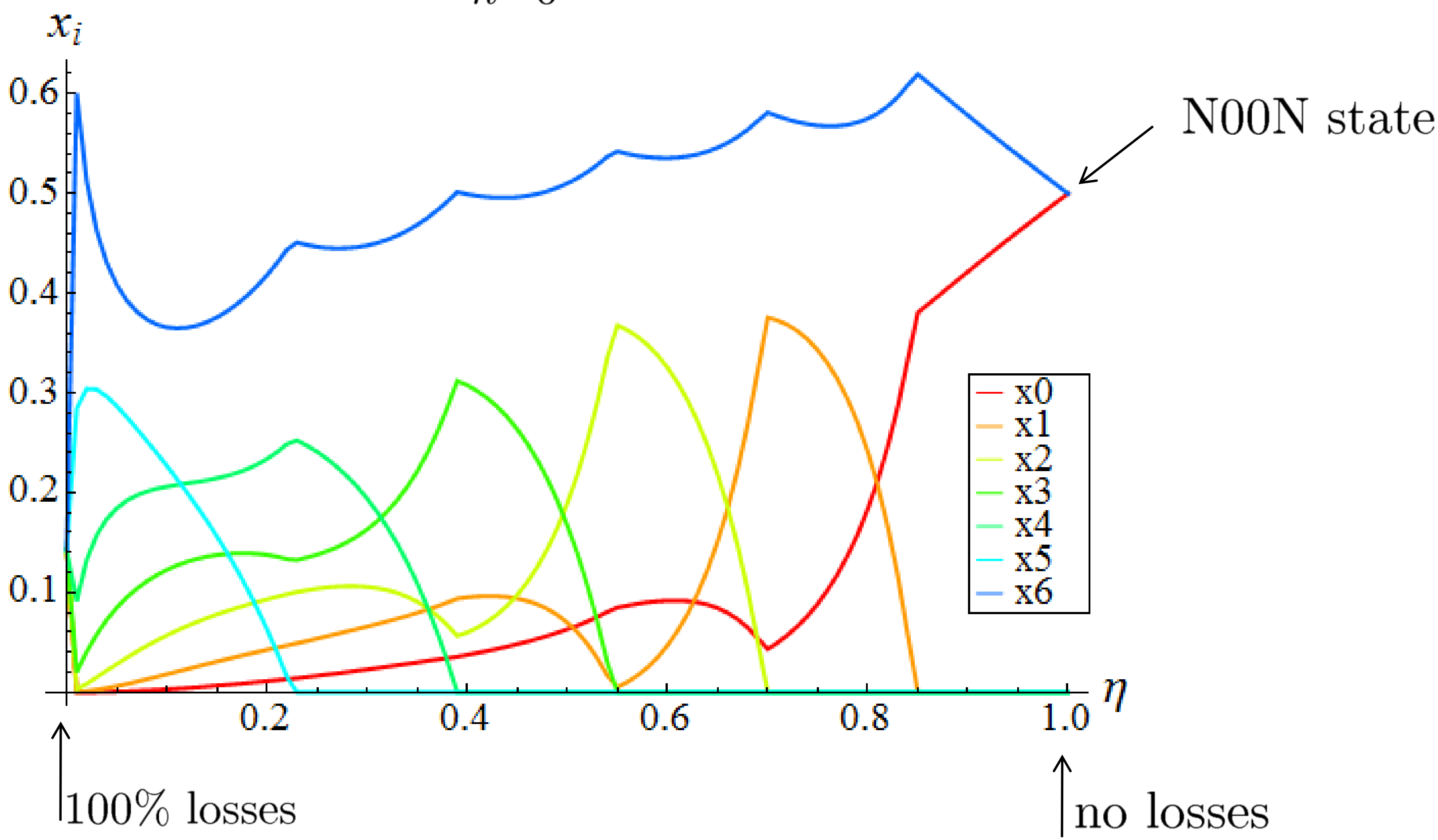


$$x_n \geq 0$$

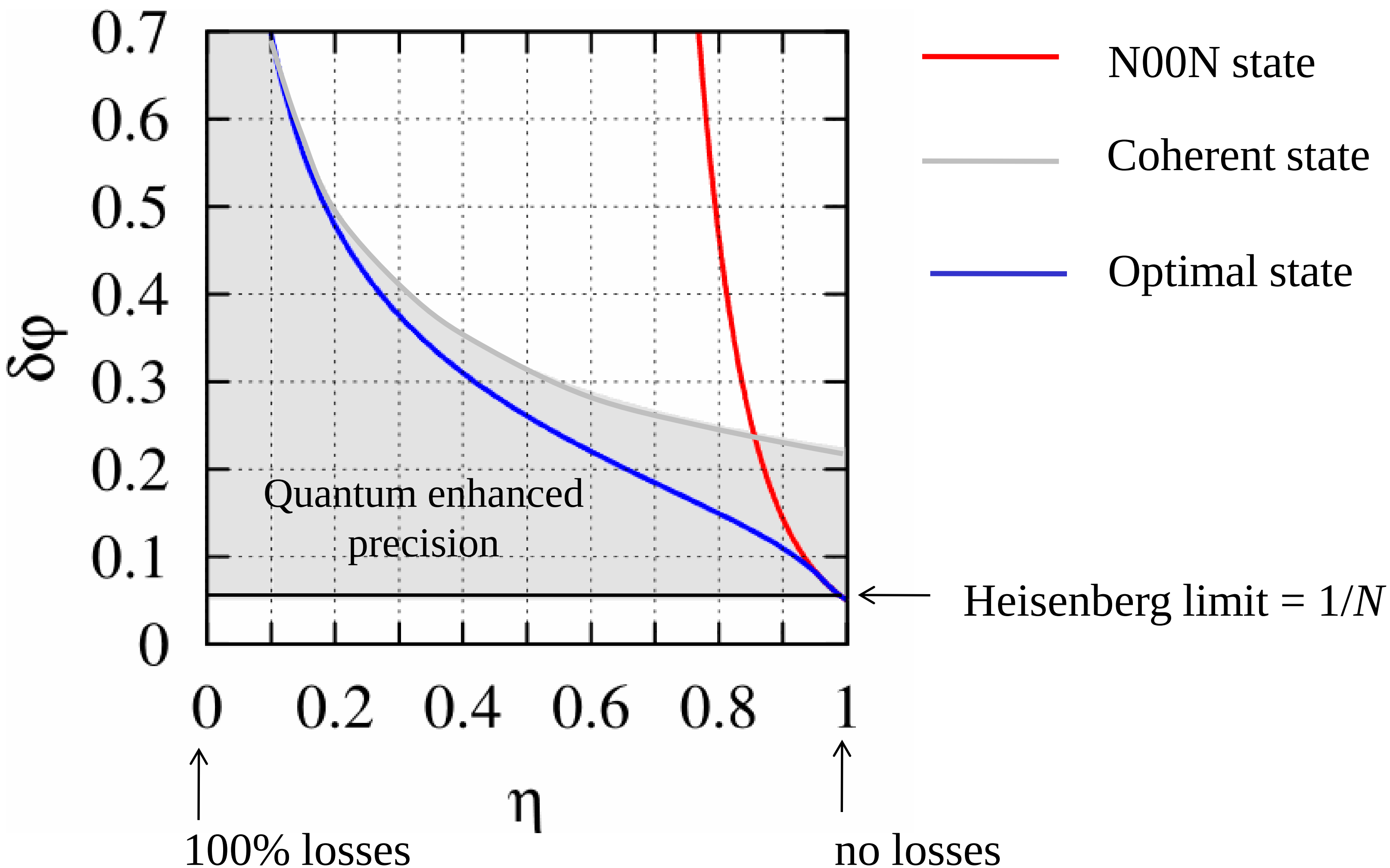
$$\sum_{n=0}^{N} x_n = 1$$

Optimal *N*=6 photon state

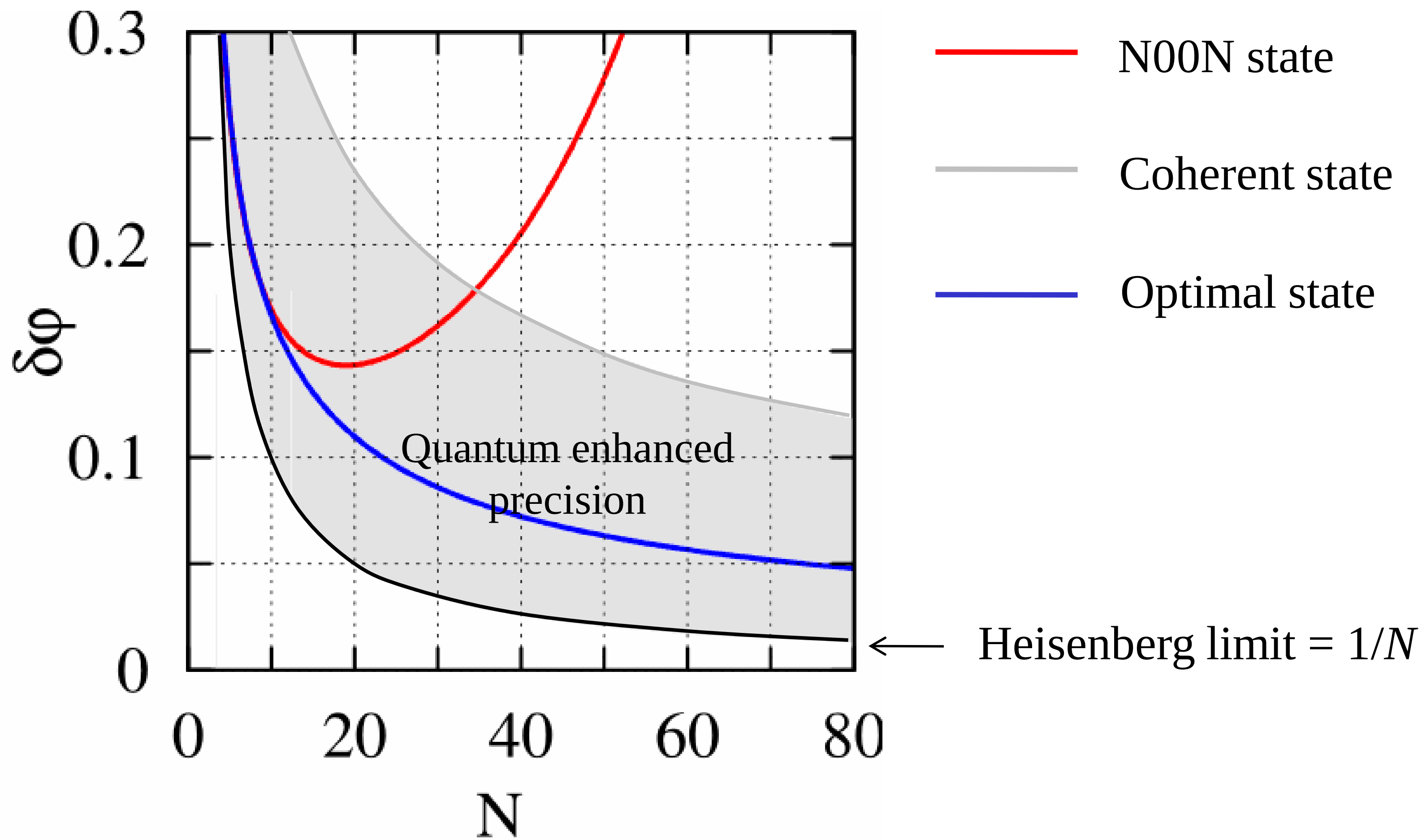
$$|\psi\rangle = \sum_{n=0}^{N} \sqrt{x_n} |n, N-n\rangle$$



Optimal vs N00N states, *N*=20



Scaling for large *N*, $\eta = 0.9$

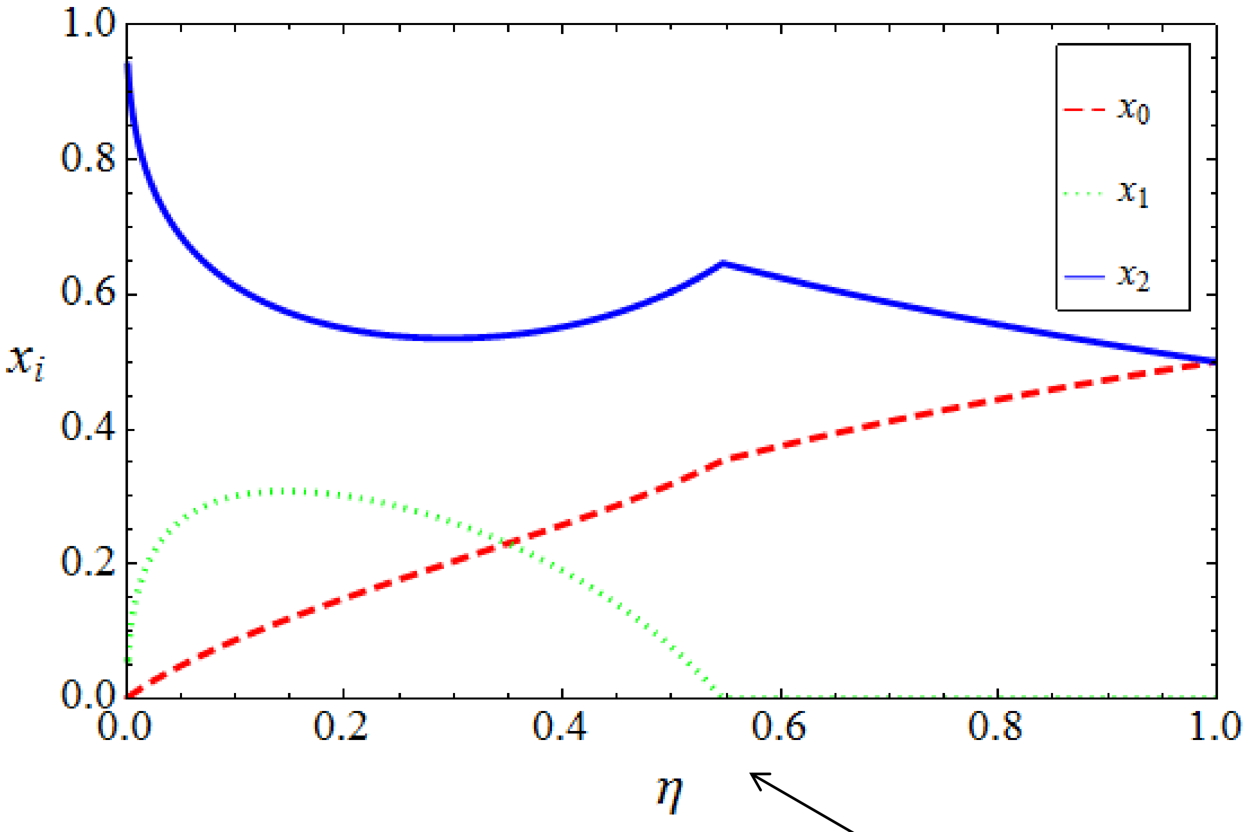


For arbitrary small losses, if *N* is large enough, the scaling becomes $\delta\varphi \propto \frac{1}{\sqrt{N}}$

Experimental lossy phase estimation using the optimal N=2 states

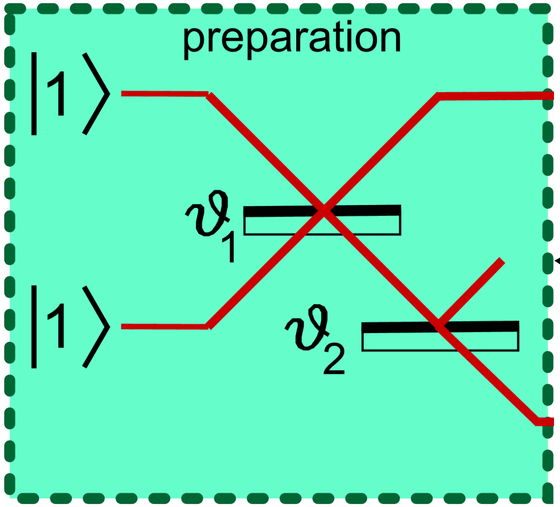
Optimal *N*=2 state

$$|\psi\rangle = \sqrt{x_0}|0,2\rangle + \sqrt{x_1}|1,1\rangle + \sqrt{x_2}|2,0\rangle$$



$\eta_{th} = (1 + 2\sqrt{2})/7 \approx 0.5469$

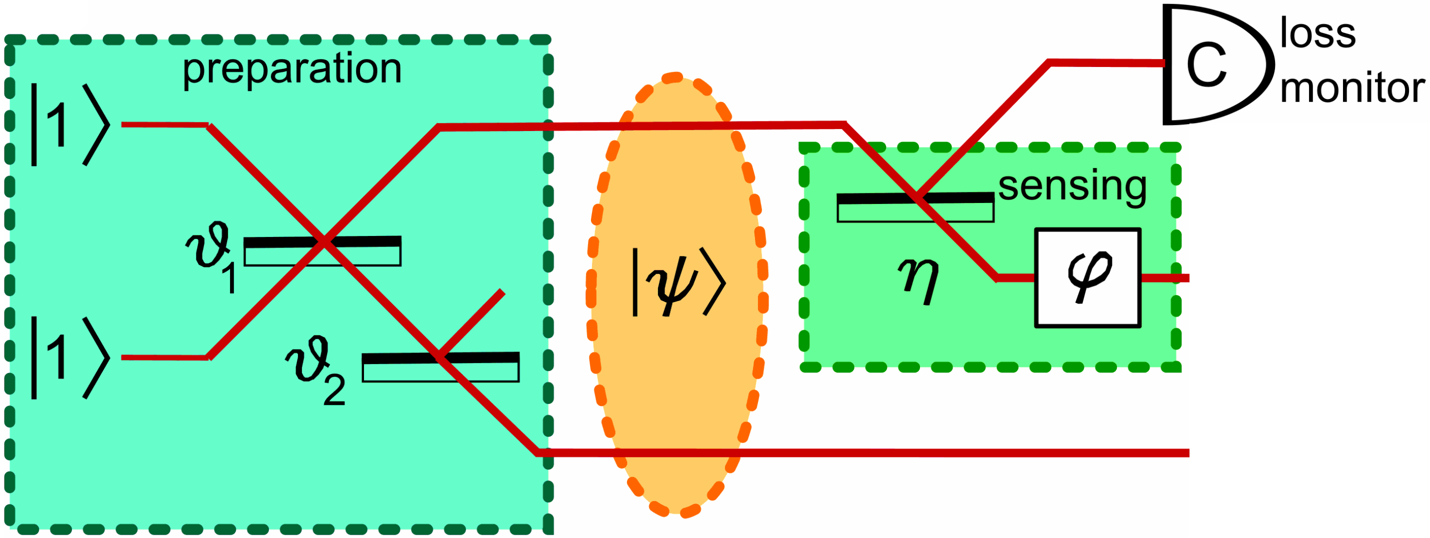
Linear optical scheme - preparation



provided no photon goes here

$$|\psi\rangle = \sqrt{x_0}|0,2\rangle + \sqrt{x_1}|1,1\rangle + \sqrt{x_2}|2,0\rangle$$

Linear optical scheme - sensing

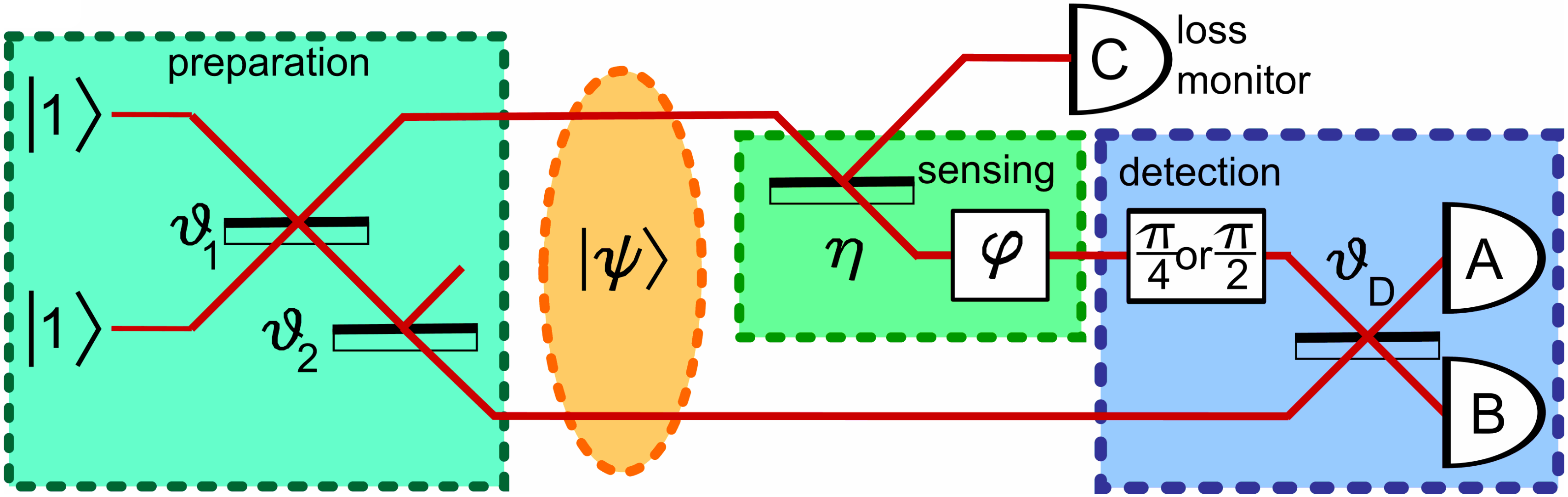


no photon lost $\longrightarrow$ $|\psi_\varphi^{(0)}\rangle \propto \sqrt{x_0}|0,2\rangle + \sqrt{x_1}\sqrt{\eta}e^{-i\varphi}|1,1\rangle + \sqrt{x_2}\eta e^{-2i\varphi}|2,0\rangle$

one photon lost $\longrightarrow$ $|\psi_\varphi^{(1)}\rangle \propto \sqrt{x_1}\sqrt{1-\eta}|0,1\rangle + \sqrt{x_2}\sqrt{2\eta(1-\eta)}e^{-i\varphi}|1,0\rangle$

two photons lost $\longrightarrow$ $|\psi_\varphi^{(2)}\rangle \propto \sqrt{x_2}(1-\eta)|0,0\rangle$ No phase information

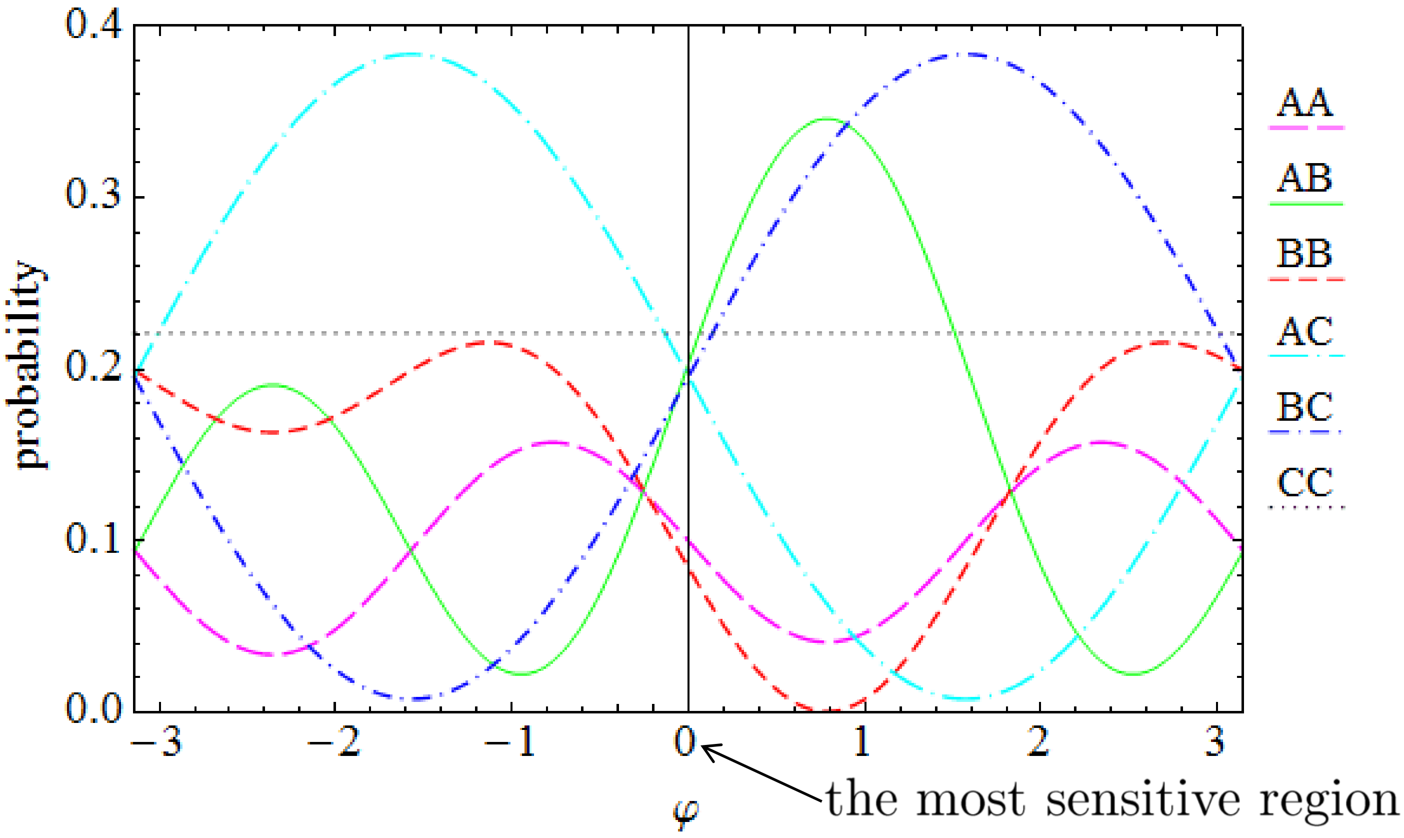
Linear optical scheme - detection



Optimal measurement saturating the Cramer-Rao bound

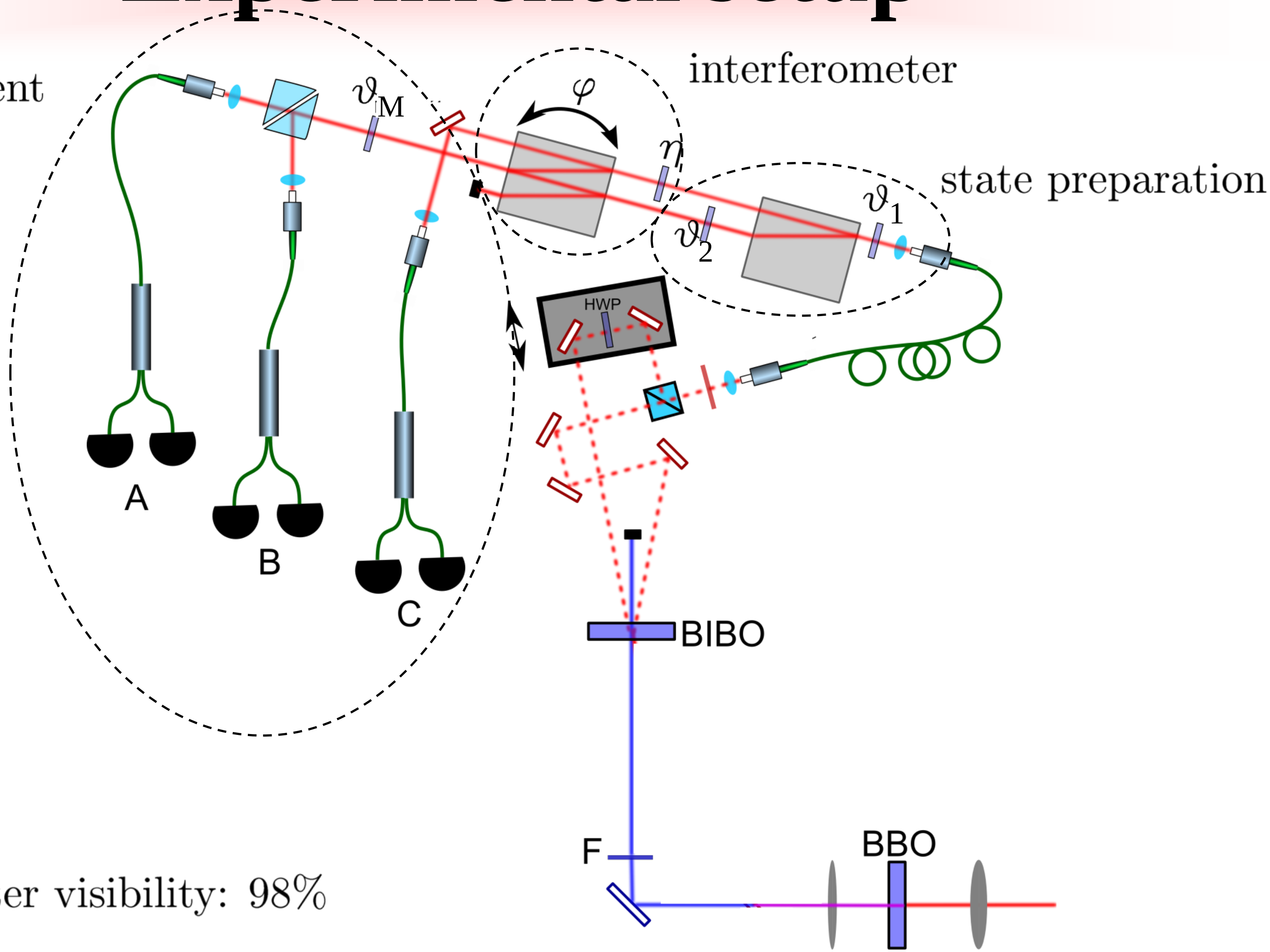
Theoretical coincidence probabilites

- η =0.361, the optimal state



- Registering given number of coincidences, we perform Max-Likelihood estimation of φ

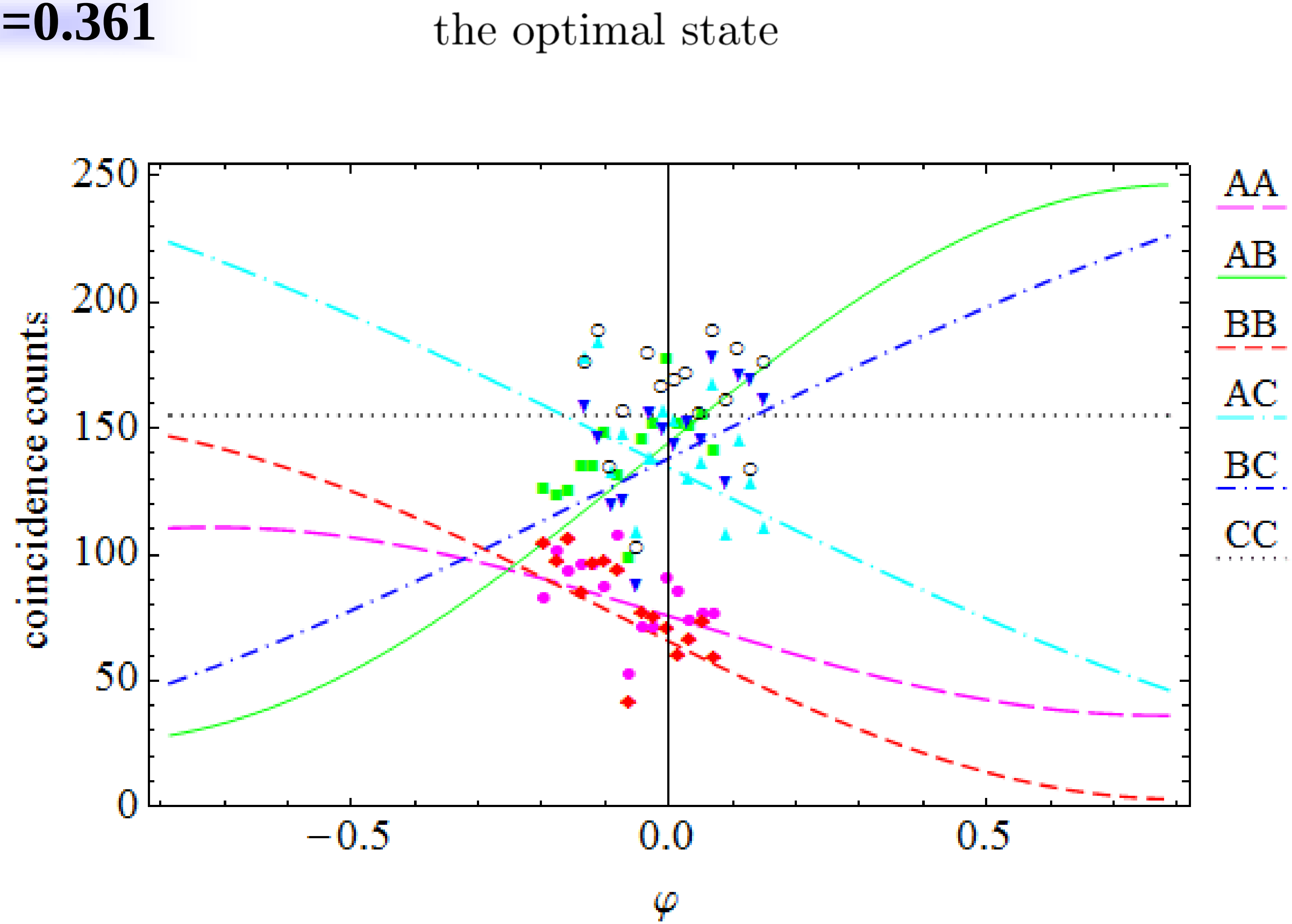
Experimental setup



inteferometer visibility: 98%

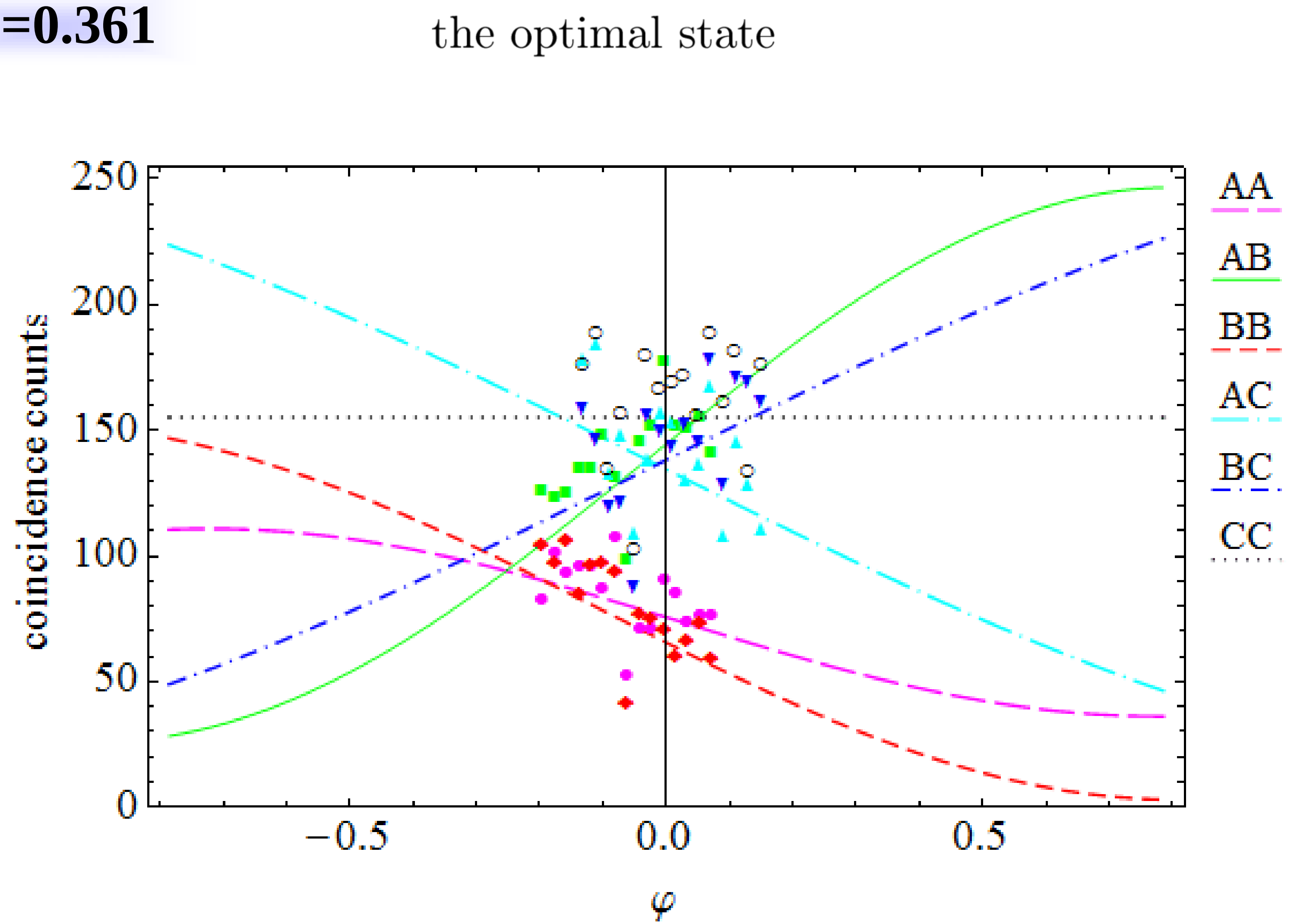
Experimental results

- η =0.361



Experimental results

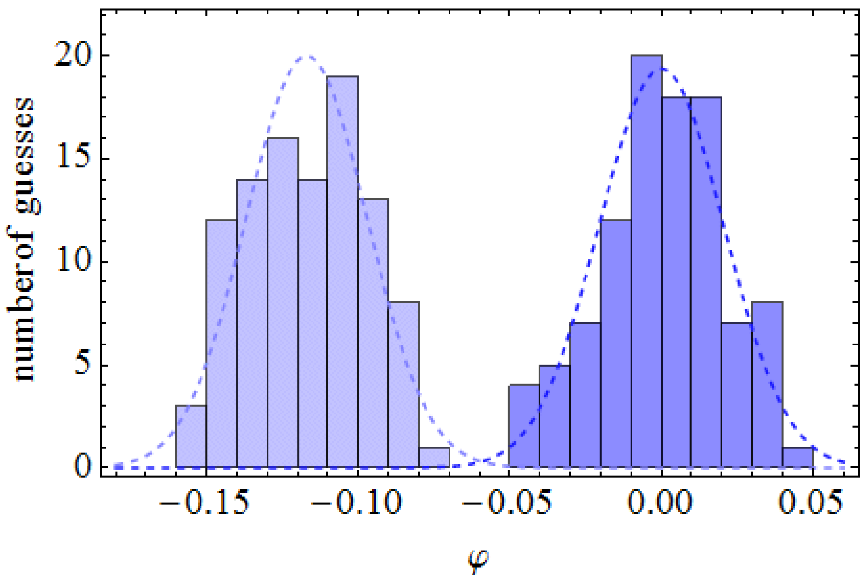
- η =0.361



Estimation for two phases separated by 0.12 rad

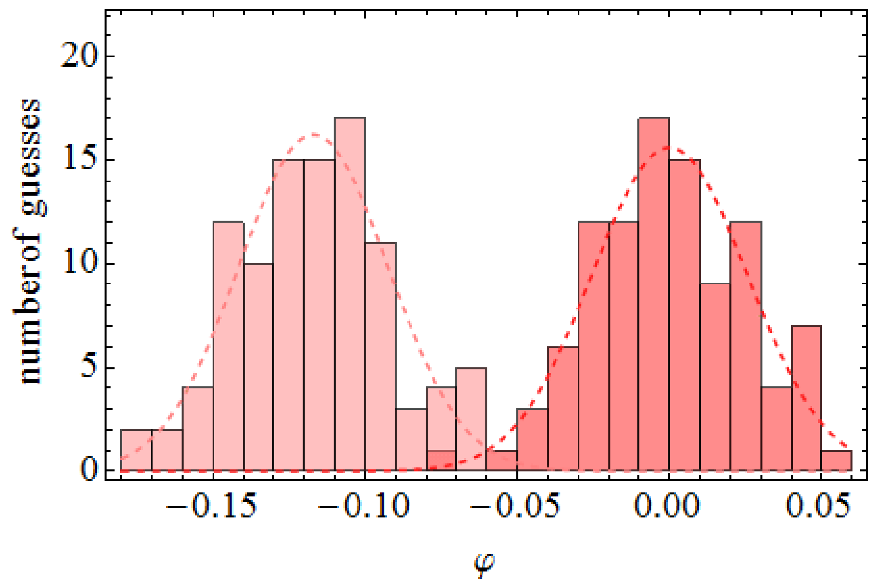
• **After estimating phase 100 times using the maximum likelihood estimator:**

the optimal state



$\delta\varphi = 0.02060$

the NOON state: $\frac{1}{\sqrt{2}}(|2,0\rangle + |0,2\rangle)$



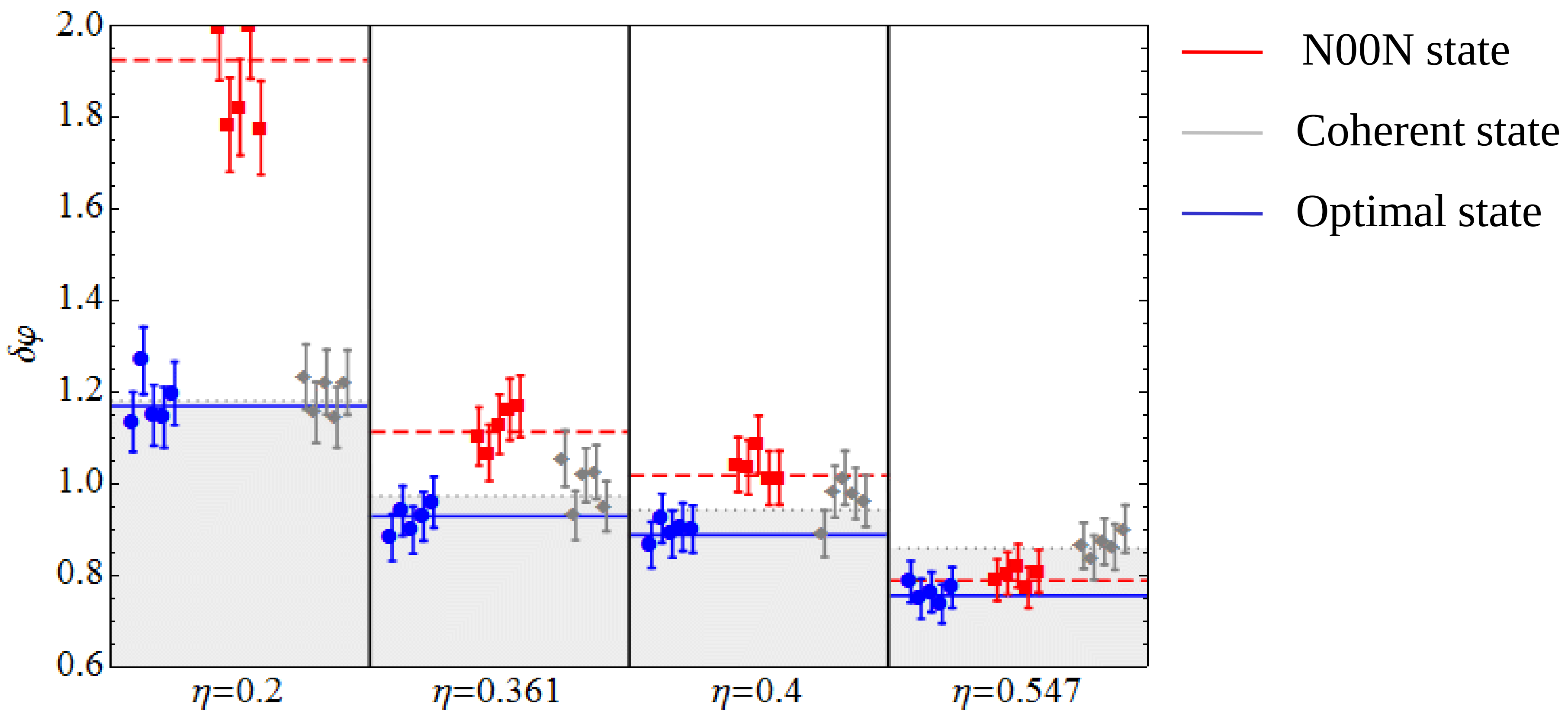
$\delta\varphi = 0.02556$

the Cramer-Rao bound: $\delta\varphi \geq 0.01906$

the Cramer-Rao bound (with 98% interferometer visibility): $\delta\varphi \geq 0.02001$

Achieving the Cramer-Rao bound

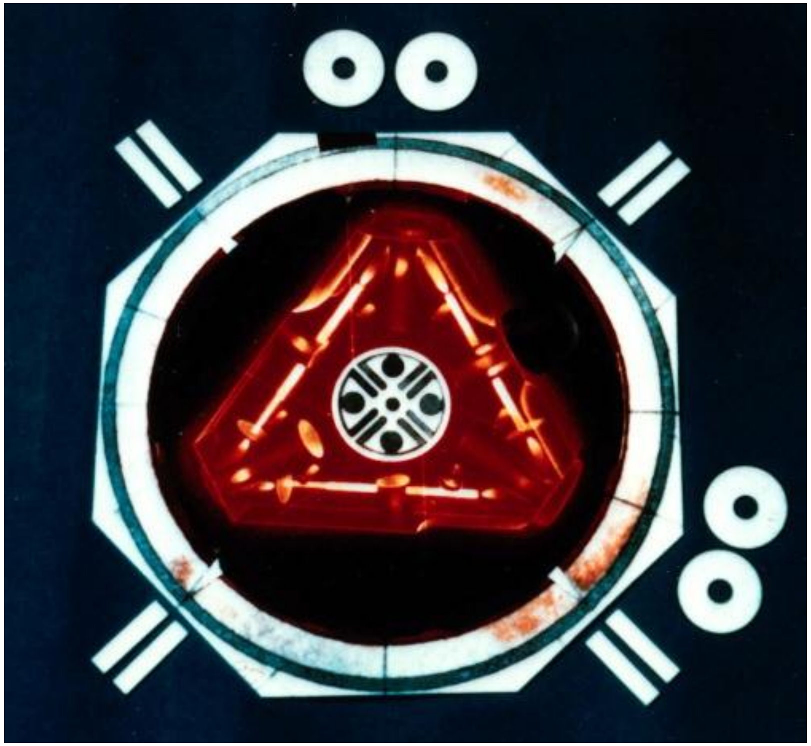
- **Precision rescaled by the square root of the number of coincidences for various transmissions *η*, for five phases: 0, ±0.02, ±0.04 rad**



Maybe one day...

Gravitational wave detectors

Laser gyroscopes



Summary

Theory [Phys. Rev. Lett. 102, 040403 (2009), arXiv:0904.0456 (2009)]

- Optimal *N*-photon states for lossy phase estimation found
- Numerical evidence for lack of asymptotic better-than-standard scaling of precision when losses are present
- ...

Experiment [arXiv: 0906.NEXT_WEEK (2009)]

- Design of preparation and measurement scheme able to reach the Cramer-Rao bound
- Experimental two photon phase estimation performed using coherent, NOON and the optimal states