Sense and sensitivity: „robust'' quantum phase estimation

R. Demkowicz-Dobrzański[1], K. Banaszek[1], U. Dorner[2], I. A. Walmsley[2], W. Wasilewski[1], B. Smith[2], J. Lundeen[2], M. Kacprowicz[3], J. Kołodyński[1]

[1] *Faculty of Physics, Warsaw University, Poland*

[2] *Clarendon Laboratory, University of Oxford, United Kingdom*

[3] *Institute of Physics, Nicolaus Copernicus University, Toruń, Poland*

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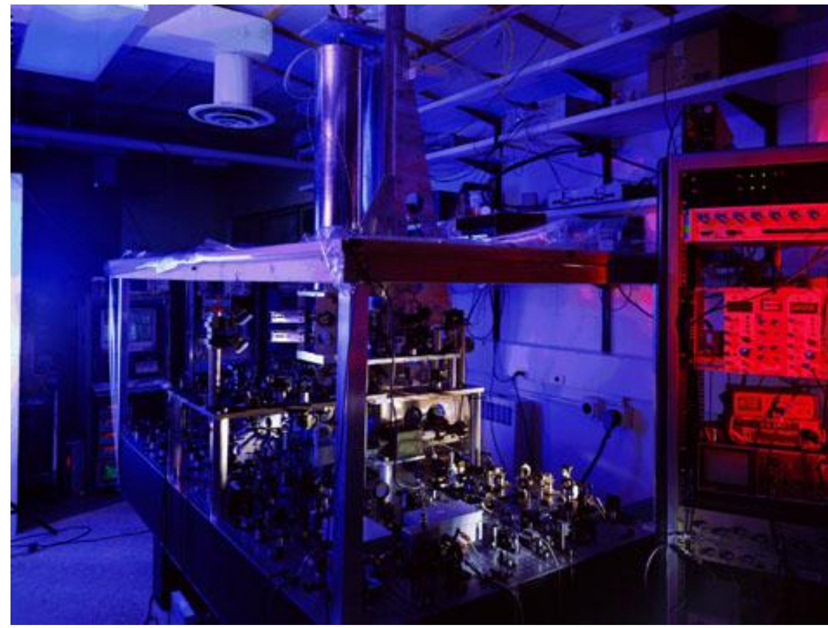
Interferometry at its (classical) limits

LIGO - gravitational wave detector

Michelson interferometer

$\Delta L/L \approx 10^{-22}$

NIST - Cs fountain atomic clock



Ramsey interferometry

$\Delta t/t \approx 10^{-16}$

Precision limited by:

shot noise $\propto 1/\sqrt{N}$

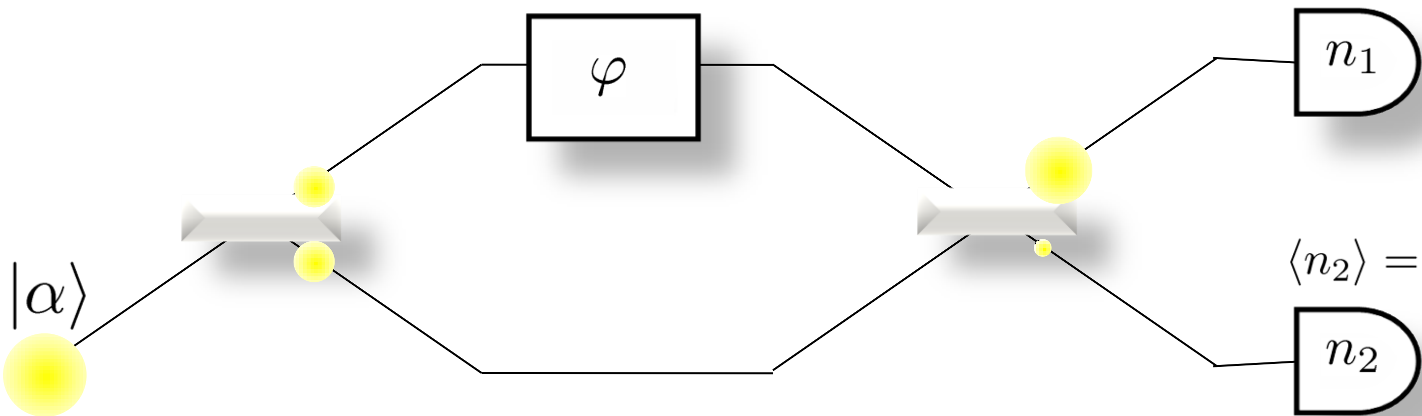
N - number of photons

projection noise $\propto 1/\sqrt{N}$

N - number of atoms

Classical phase estimation

$$\langle n_1 \rangle = |\alpha|^2 (1 + \cos\varphi)/2$$



$$\langle n_2 \rangle = |\alpha|^2 (1 - \cos\varphi)/2$$

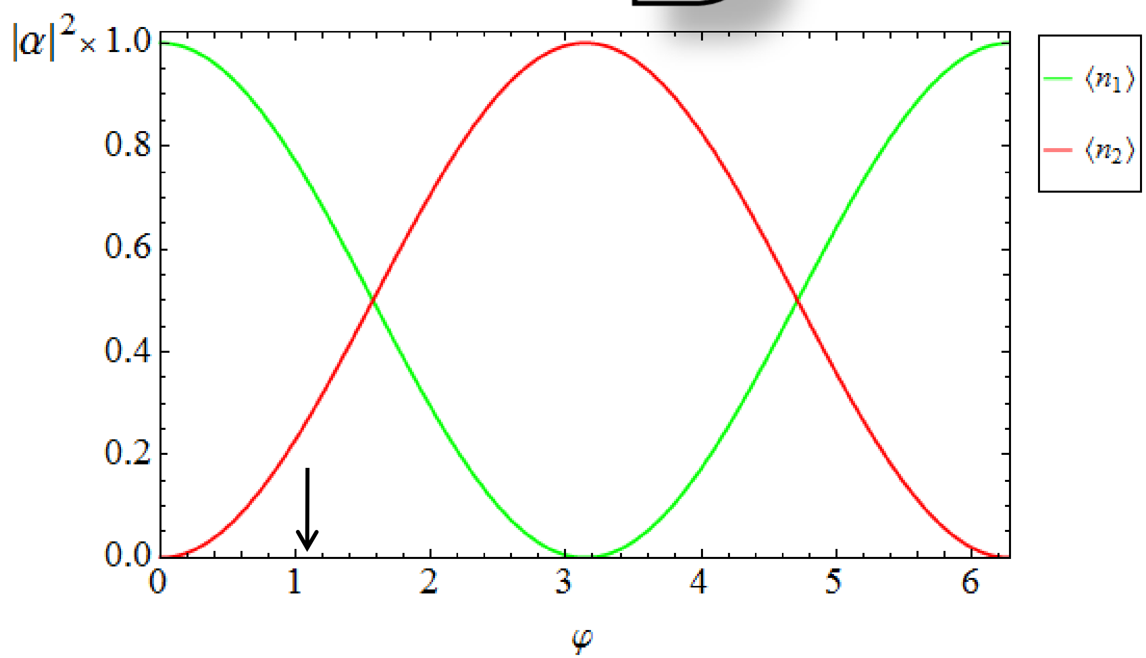
detecting n_1 and n_2

+

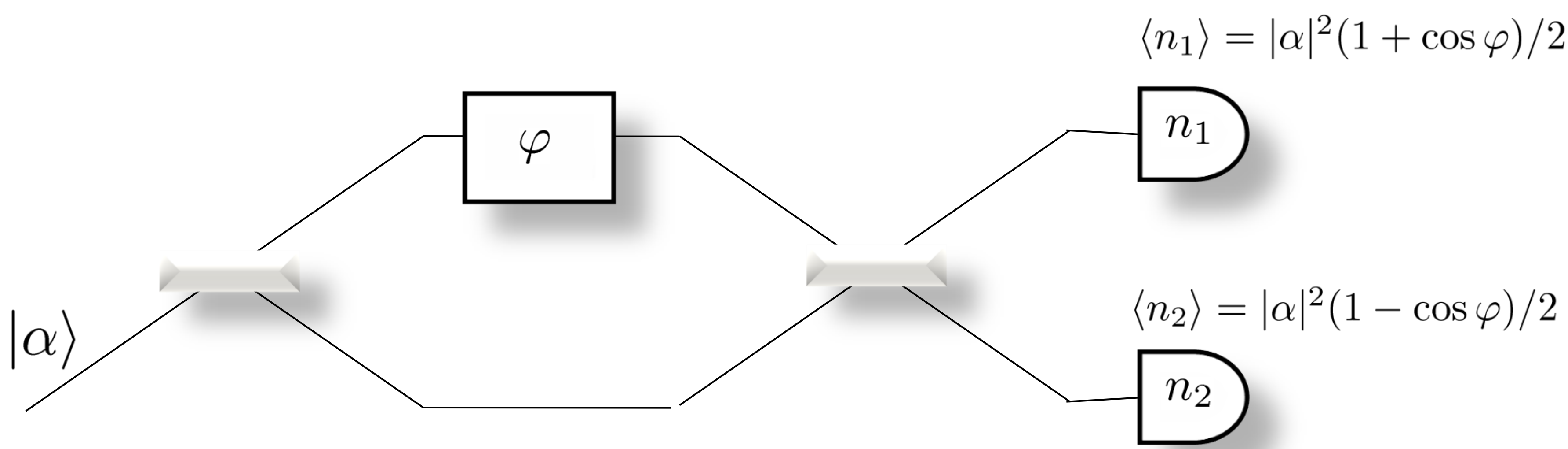
knowing theoretical dependence of n_1, n_2 on φ

↓

we can estimate φ



Classical phase estimation

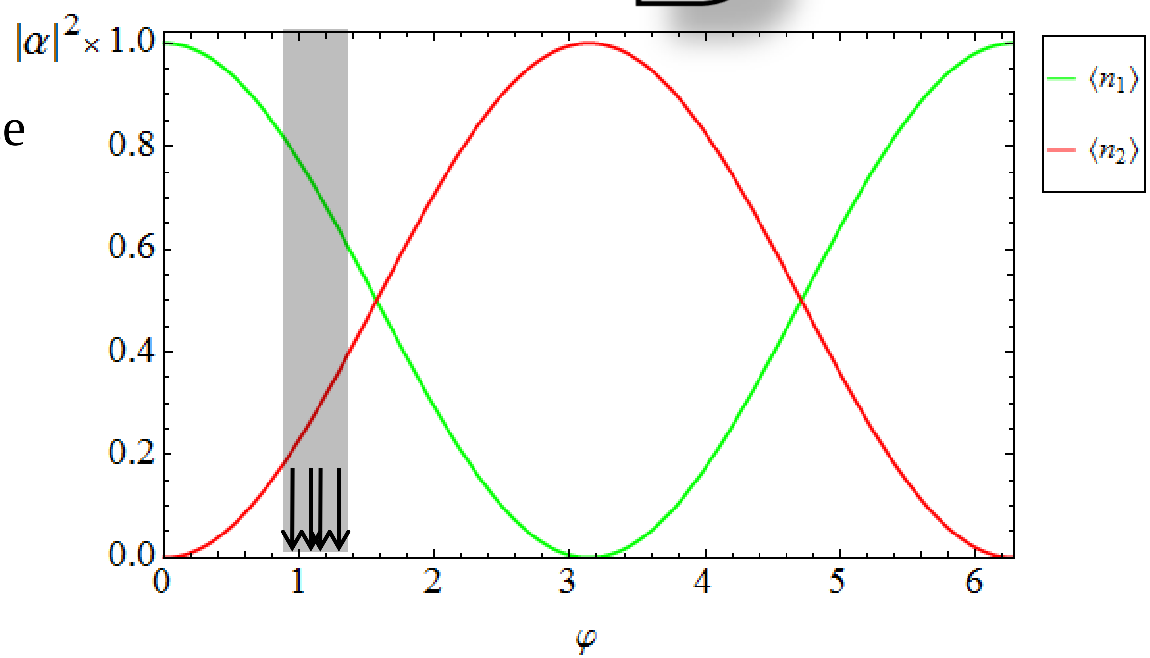


n_1 and n_2 are subject to shot noise

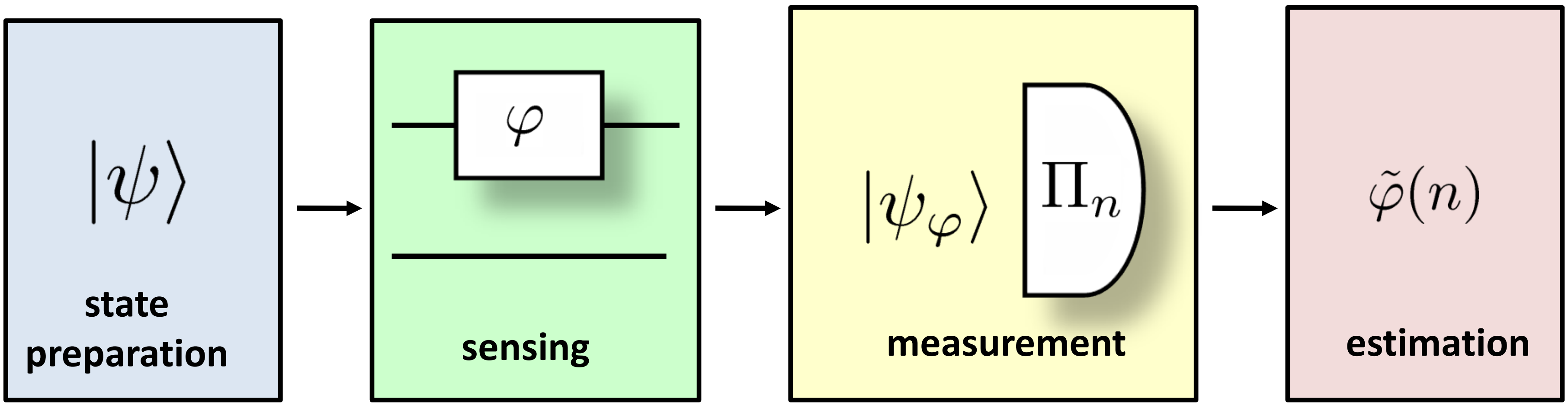
each measurement yields a bit different φ

$$\Delta\varphi \propto \frac{1}{|\alpha|} = \frac{1}{\sqrt{\bar{n}}}$$

Shot noise scaling



Quantum phase estimation



Minimize $\langle(\tilde{\varphi}-\varphi)^2\rangle$ over the choice of $|\psi\rangle$, Π_n and $\tilde{\varphi}$

$$\Delta^2\varphi = \langle(\tilde{\varphi}-\varphi)^2\rangle = \int d\varphi\, p(\varphi) \sum_n p(n|\varphi)[\tilde{\varphi}(n)-\varphi]^2$$

a priori knowledge $\qquad \langle\psi_\varphi|\Pi_n|\psi_\varphi\rangle \qquad 4\sin^2\left[\frac{\tilde{\varphi}(n)-\varphi}{2}\right]$

In general a very hard problem!

$$\Delta^2\varphi = \int d\varphi\; p(\varphi) \sum_n \langle \psi_\varphi | \Pi_n | \psi_\varphi \rangle [\tilde{\varphi}(n) - \varphi]^2$$

Local approach

we want to sense small fluctuations around a known phase

$$p(\varphi) \approx \delta(\varphi - \varphi_0)$$

Tool: Fisher Information, Cramer-Rao bound

$$\Delta\tilde{\varphi} \geq \frac{1}{\sqrt{F}}$$

$$F = 4[\langle \psi_\varphi | \hat{n}_1^2 | \psi_\varphi \rangle - \langle \psi_\varphi | \hat{n}_1 | \psi_\varphi \rangle^2]$$

The optimal N photon state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle)$$

$$\boxed{\Delta\tilde{\varphi} \approx \frac{1}{N}}$$

J. J. . Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, *Phys. Rev. A* **54**, R4649 (1996).

Global approach

no a priori knowledge about the phase

$$p(\varphi) \approx \frac{1}{2\pi}$$

Tool: Symmetry implies a simple structure of the optimal measurement

Optimal state: $|\psi\rangle = \sum_{n=0}^{N} \alpha_n |n, N-n\rangle$

$$\alpha_n = \sqrt{\tfrac{2}{N+2}} \sin\left[\tfrac{(n+1)\pi}{N+2}\right]$$

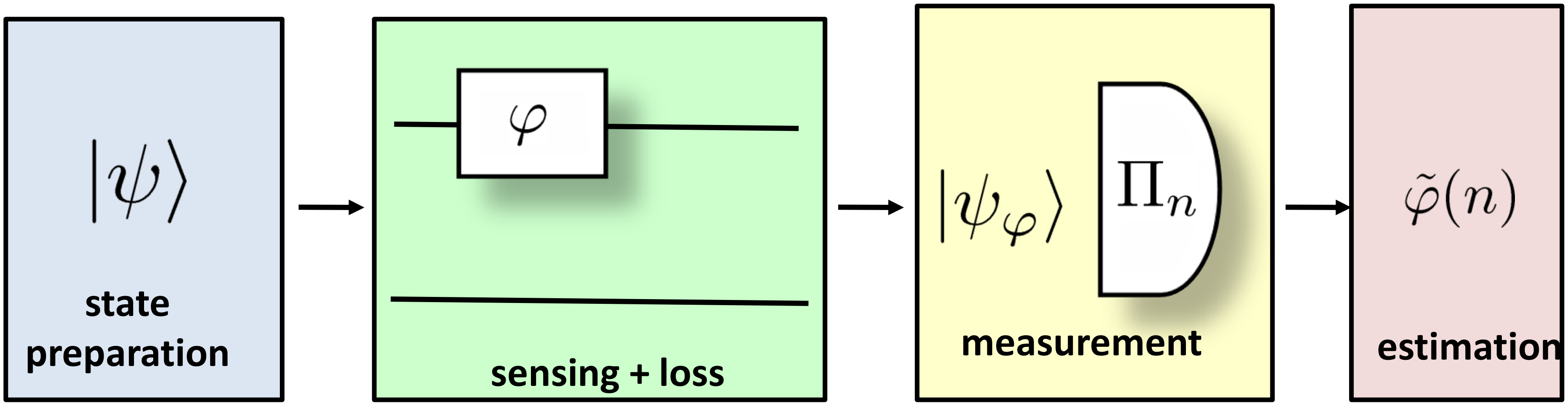
$$\boxed{\Delta\tilde{\varphi} \approx \frac{\pi}{N+2}}$$

D. W. Berry and H. M. Wiseman, *Phys. Rev. Lett*. **85**, 5098 (2000).

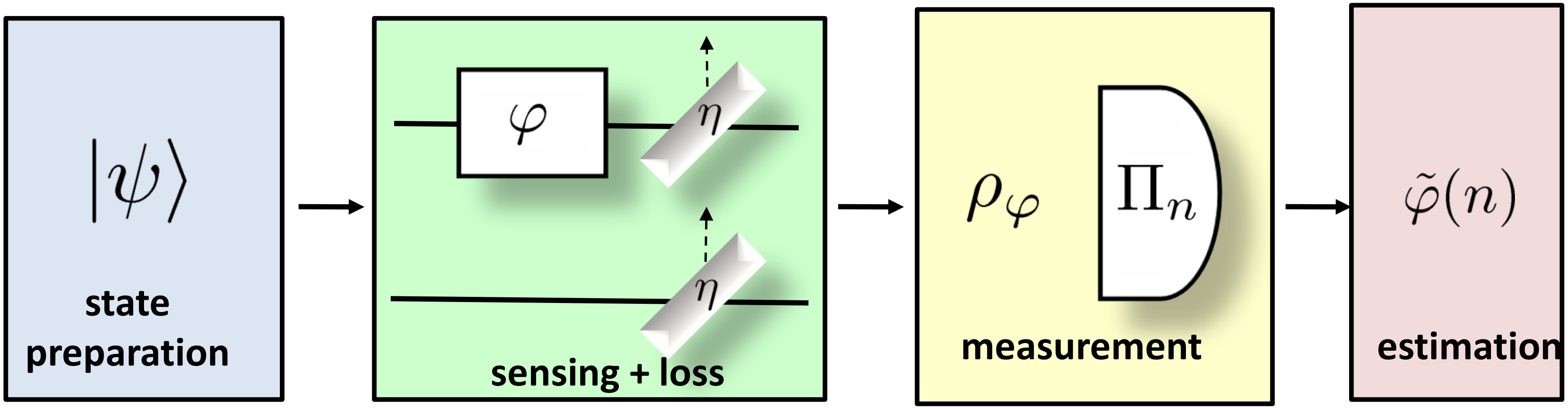
Heisenberg scaling

In reality there is loss…

Phase estimation in the presence of loss



Phase estimation in the presence of loss

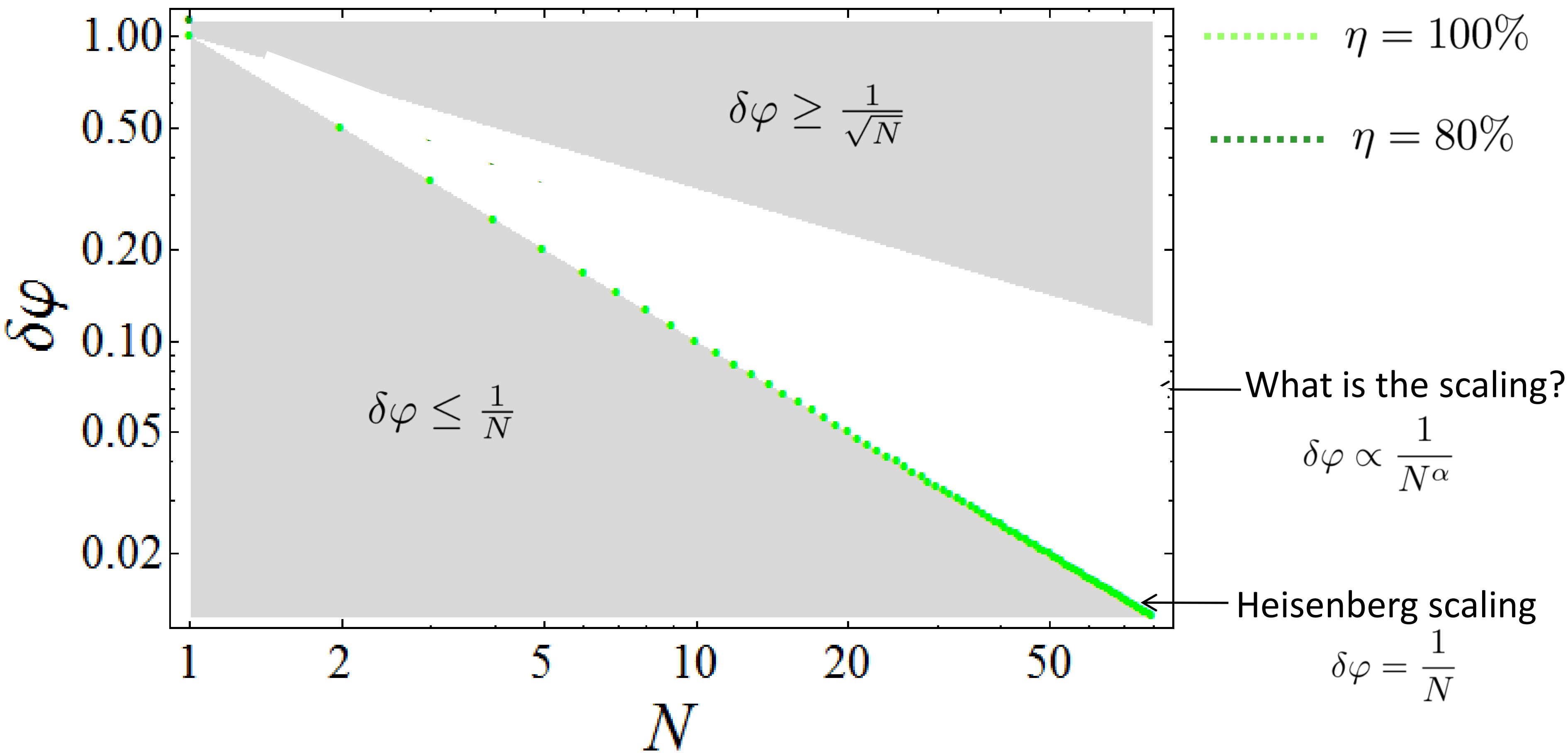


- no analytical solutions for the optimal states and precission
- calculating Fisher information not trivial (symmetric logarithmic derrivative)

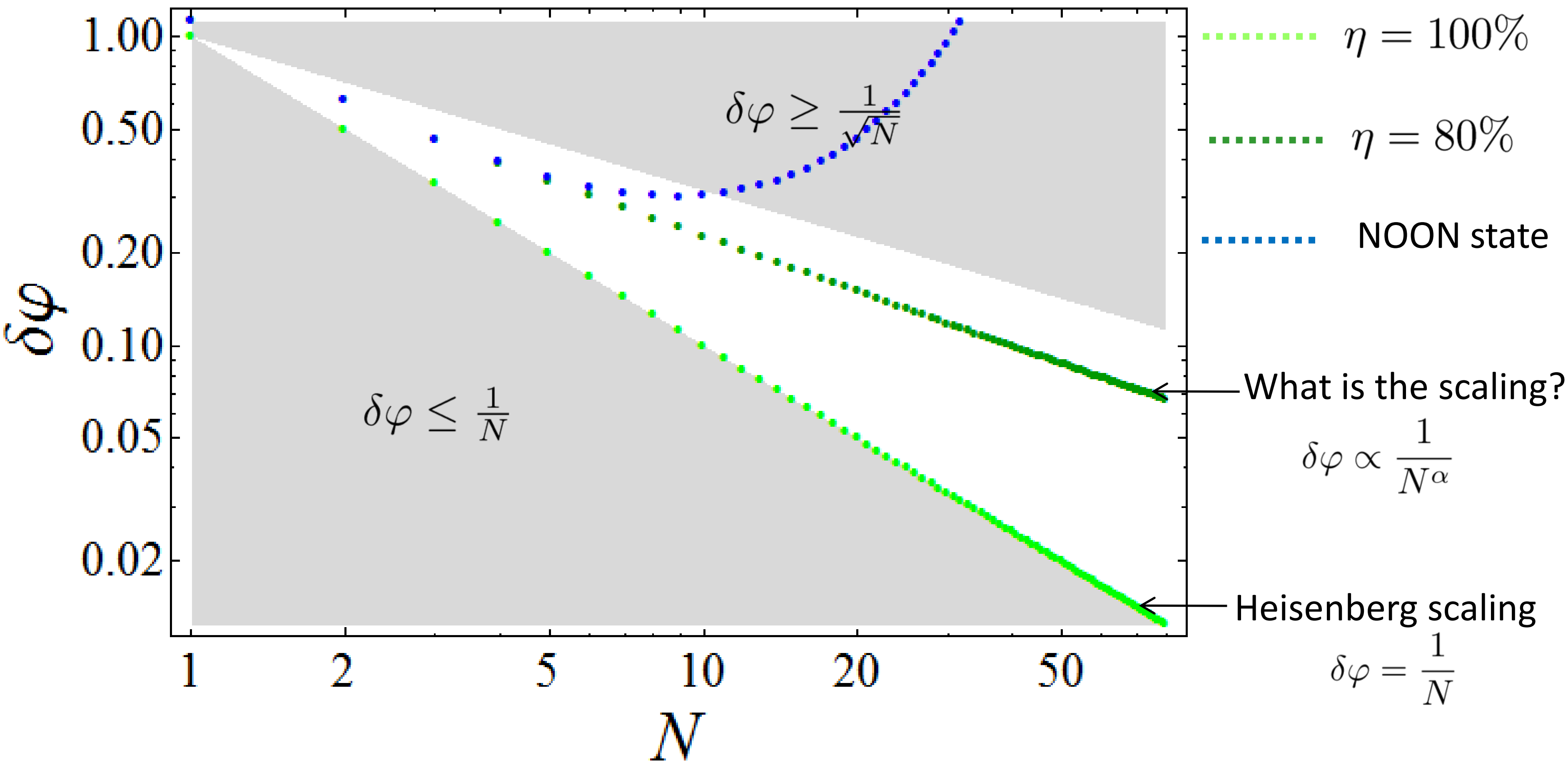
- phase sensing and loss commute (no ambiguity in ordering)
- in the global approach the optimal measurements is not altered – the solution is obtained by solving an eigenvalue problem (fast)
- effective numerical optimization procedures yielding global minima

R. Demkowicz-Dobrzanski, et al. ***Phys. Rev.*** **A 80, 013825 (2009)**
U. Dorner, et al.., ***Phys. Rev. Lett.*** **102, 040403 (2009)**

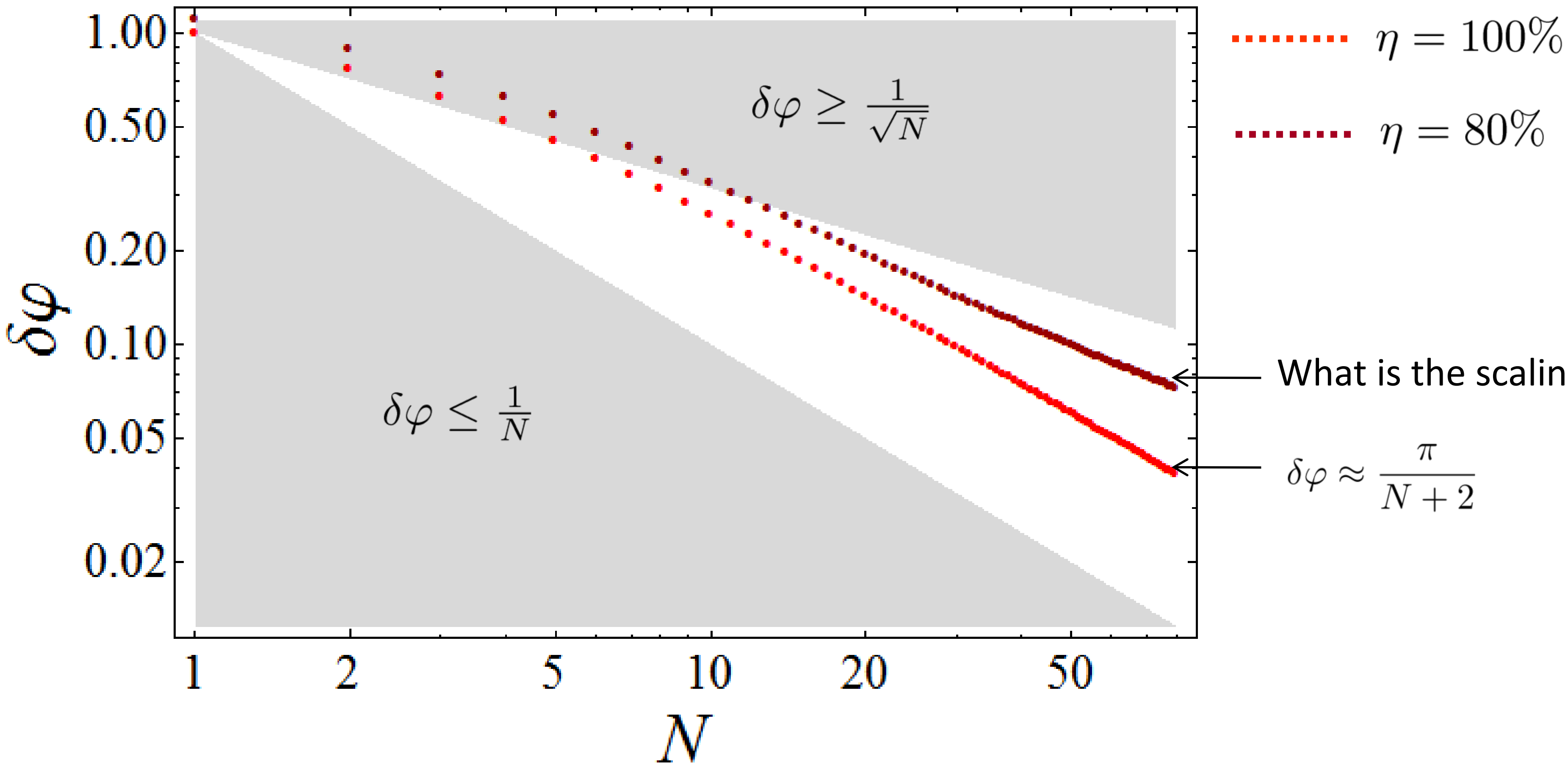
Estimation uncertainty with the number of photons used (local approach)



Estimation uncertainty with the number of photons used (local approach)



Estimation uncertainty with the number of photons used (global approach)

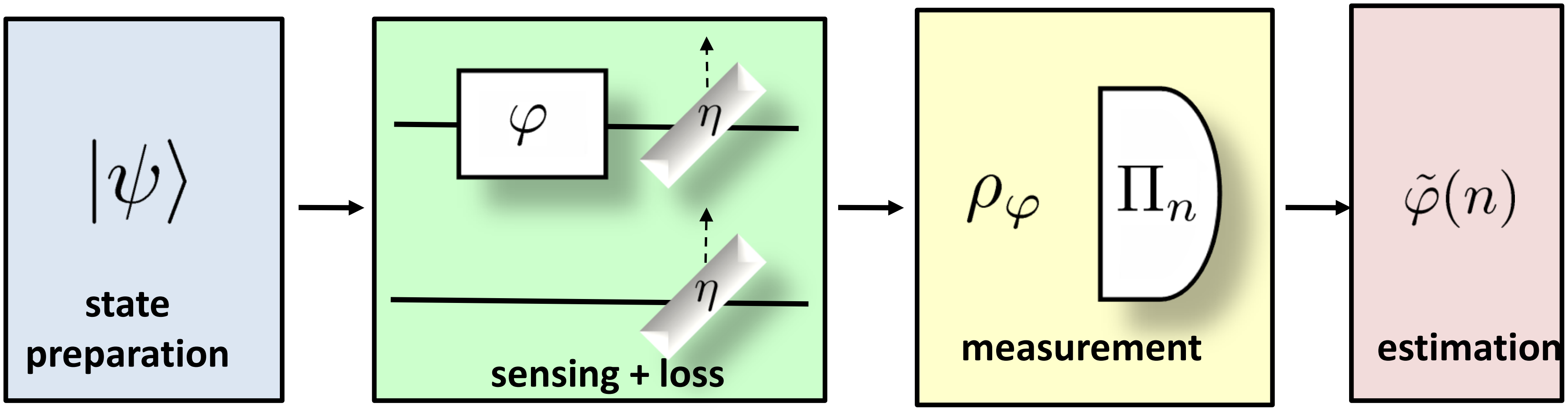


What is the scaling?

$\delta\varphi \approx \frac{\pi}{N+2}$

Do quantum states provide beter scaling exponent in the presence of loss?

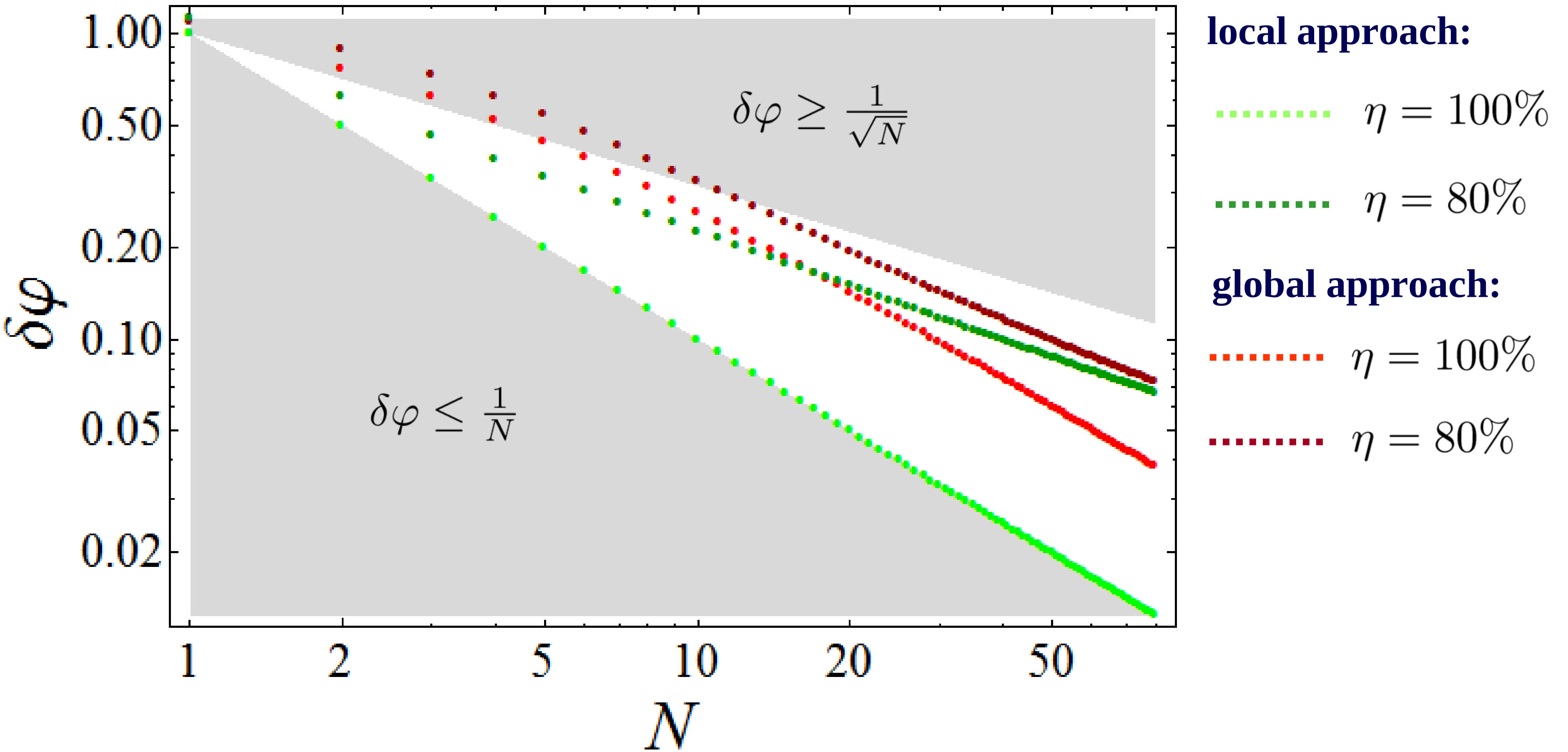
Fundamental bound on uncertainty in the presence of loss (global approach)



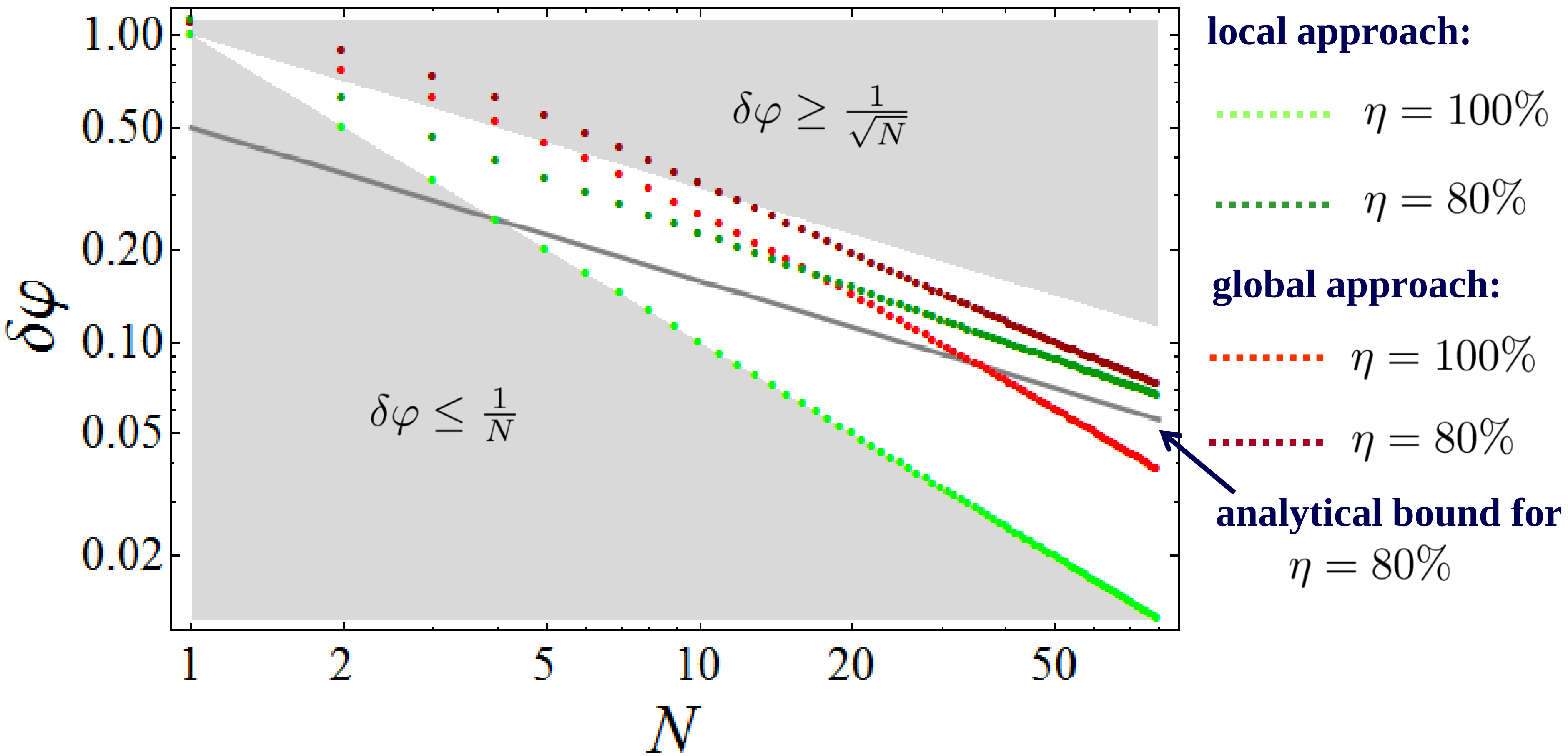
$$\delta\varphi_{\text{quantum}} \geq \sqrt{\frac{1-\eta}{\eta N}} + O\left(\frac{1}{N}\right)$$

J. Kolodynski and R.Demkowicz-Dobrzanski, **arXiv:1006.0734 (2010)**

Fundamental bound on uncertainty in the presence of loss (global approach)



Fundamental bound on uncertainty in the presence of loss (global approach)



Fundamental bound on asymptotic quantum gain in phase estimation

$$\delta\varphi_{\text{quantum}} \geq \sqrt{\frac{1-\eta}{\eta N}} + O\left(\frac{1}{N}\right) \qquad \delta\varphi_{\text{classical}} = \sqrt{\frac{1}{\eta N}}$$

$$\boxed{\lim_{N\to\infty} \frac{\delta\varphi_{\text{classical}}}{\delta\varphi_{\text{quantum}}} \leq \frac{1}{\sqrt{1-\eta}}}$$

Example: $\eta = 80\%$ $\quad 1/\sqrt{1-\eta} \approx 2.24$

even for moderate loss quantum gain degrades quickly

Summary

• Asymptotically, loss renders quantum phase estimation uncertainty scaling classical and destroys the Heisenberg scaling.

• Quantum state can be practically useful only for very small degree of loss (loss <1% implies gain> 10)

• Neither adaptive measurements, nor photon distinguishability can help

1. K. Banaszek, R. Demkowicz-Dobrzanski, and I. Walmsley, ***Nature Photonics* 3, 673 (2009)**
2. U. Dorner, R. Demkowicz-Dobrzanski, B. Smith, J. Lundeen, W. Wasilewski, K. Banaszek, and I..Walmsley, ***Phys. Rev. Lett.* 102, 040403 (2009)**
3. R. Demkowicz-Dobrzanski, U. Dorner, B. Smith, J. Lundeen, W. Wasilewski, K. Banaszek, and I. Walmsley, ***Phys. Rev.* A 80, 013825 (2009)**
4. M. Kacprowicz, R. Demkowicz-Dobrzanski, W. Wasilewski, and K. Banaszek, **Nature Photonics 4, 357(2010)**
5. J. Kolodynski and R.Demkowicz-Dobrzanski, **arXiv:1006.0734 (2010)**

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