4 Wiele qubitów

25 października 2015
23:04

Dwa qubity: $|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle = \begin{bmatrix}\psi_0\\ \psi_1\end{bmatrix}$.
$|\varphi\rangle = \varphi_0|0\rangle + \varphi_1|1\rangle = \begin{bmatrix}\varphi_0\\ \varphi_1\end{bmatrix}$

Il. tensorowy:

$$|\psi\rangle\otimes|\varphi\rangle = \psi_0\varphi_0|0\rangle\otimes|0\rangle + \psi_0\varphi_1|0\rangle\otimes|1\rangle + \psi_1\varphi_0|1\rangle\otimes|0\rangle + \psi_1\varphi_1|1\rangle\otimes|1\rangle = \begin{bmatrix}\psi_0\varphi_0\\ \psi_0\varphi_1\\ \psi_1\varphi_0\\ \psi_1\varphi_1\end{bmatrix}$$

Ogólny stan:

$$|\Psi\rangle_{12} = \Psi_{00}|0\rangle\otimes|0\rangle + \Psi_{01}|0\rangle\otimes|1\rangle + \Psi_{10}|1\rangle\otimes|0\rangle + \Psi_{11}|1\rangle\otimes|1\rangle$$

Np. $|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle\otimes|1\rangle - |1\rangle\otimes|0\rangle\right) \neq |\psi\rangle\otimes|\varphi\rangle$

nie da się myśleć że oba fotony mają określone stany.

Jak opisać pomiar w którym mierzymy polaryzację fotonu 1 w bazie $|\pm a\rangle$ a foton 2 w bazie $|\pm b\rangle$

$$p(a,b) = \left|\langle a|\otimes\langle b|\,|\Psi\rangle_{12}\right|^2$$

<u>Przykład:</u>

$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle - |1\rangle|0\rangle\right)$ $\quad |a\rangle = a_0|0\rangle + a_1|1\rangle$

$$\left|\langle a|\otimes\langle b|\frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle - |1\rangle|0\rangle\right)\right|^2 =$$

$$\begin{cases}|a\rangle = \cos\frac{\theta_a}{2}|0\rangle + \sin\frac{\theta_a}{2}e^{i\varphi_a}|1\rangle\\ |b\rangle = \cos\frac{\theta_b}{2}|0\rangle + \sin\frac{\theta_b}{2}e^{i\varphi_b}|1\rangle\end{cases}$$

$$= \frac{1}{2}\left|\cos\frac{\theta_a}{2}\cdot\sin\frac{\theta_b}{2}e^{-i\varphi_b} - \sin\frac{\theta_a}{2}e^{-i\varphi_a}\cos\frac{\theta_b}{2}\right|^2 =$$

$$= \frac{1}{2}\Big(\cos^2\frac{\theta_a}{2}\sin^2\frac{\theta_b}{2} + \sin^2\frac{\theta_a}{2}\cos^2\frac{\theta_b}{2} - \cos\frac{\theta_a}{2}\sin\frac{\theta_b}{2}\sin\frac{\theta_a}{2}\cos\frac{\theta_b}{2}\left(e^{i(\varphi_a-\varphi_b)} + e^{i(\varphi_b-\varphi_a)}\right)\Big) =$$

$$= \frac{1}{2}\Big(\frac{1}{2}(1+\cos\theta_a)\frac{1}{2}(1-\cos\theta_b) + \frac{1}{2}(1-\cos\theta_a)\frac{1}{2}(1+\cos\theta_b) - \frac{1}{4}\sin\theta_a\sin\theta_b\, 2\cdot\cos(\varphi_a-\varphi_b)\Big) =$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}\cos\theta_a\cos\theta_b - \frac{1}{2}\sin\theta_a\sin\theta_b\cos(\varphi_a-\varphi_b)\right) =$$

$$= \frac{1}{4}\left(1 - \vec{a}\circ\vec{b}\right)$$

Polaryzacje przeciwnie skierowane jeśli mierzymy w dowolnych identycznych bazach.

Wprowadźmy obserwable

$$\sigma_{\vec{a}} = |\vec{a}\rangle\langle\vec{a}| - |-\vec{a}\rangle\langle-\vec{a}|$$

$$\langle\Psi_-|\sigma_{\vec{a}}\otimes\sigma_{\vec{b}}|\Psi_-\rangle = -\vec{a}\circ\vec{b}$$

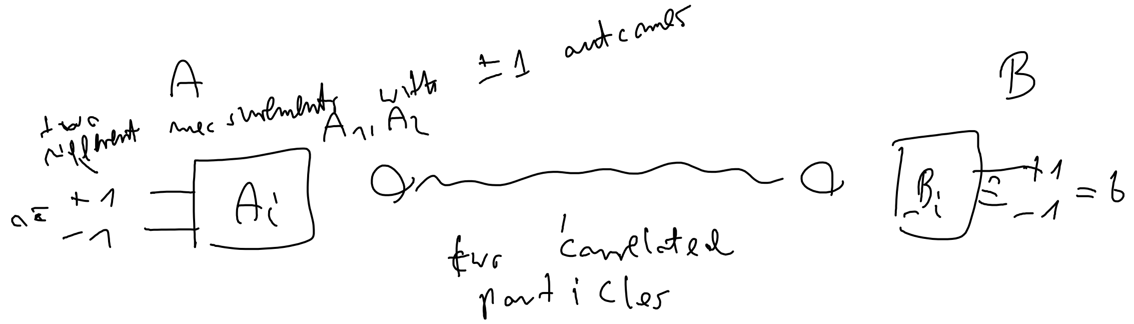
$$\langle \psi_- | \sigma_{\vec a} \otimes \sigma_{\vec b} | \psi_- \rangle = -\vec a \cdot \vec b$$

Nierówności Bella (1964)

Hot debate:

How do we know that entangled states are not just some hidden classical correlations? and that the measurement just reveals this state.

Is there a simple experimental proof of this



A, and B each randomly chooses to perform one of two measurements.

We assume that there is some parameter λ that determines how particles will behave in the particular measurement

$$a_i = a(A_i, \lambda) = \pm 1 \qquad b_i = b(B_i, \lambda) = \pm 1$$

correlations in measurement of A and B are due classical correlation imprinted during preparation

Consider a quantity:

$$C = a_1 b_1 + a_1 b_2 + a_2 b_1 - a_2 b_2 =$$
$$= a_1(b_1 + b_2) + a_2(b_1 - b_2)$$

for every combination of ± 1 $\quad |C| \leq 2$

$$\langle C \rangle = \langle a_1 b_1 \rangle + \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle - \langle a_2 b_2 \rangle$$

$\langle C \rangle = \int d\lambda\, \rho(\lambda) \ldots$

$$\langle C\rangle = \int d\lambda\, \rho(\lambda) \Big(a(A_1,\lambda)\, b(B_1,\lambda) + a(A_1,\lambda)\, b(B_2,\lambda) + a(A_2,\lambda)\, b(B_1,\lambda) - a(A_2,\lambda)\, b(B_2,\lambda) \Big)$$

$$|\langle C\rangle| \leqslant 2$$

what are the assumptions:

- reality (measurements just reveal preexisting quantities $a(A_i,\lambda)$, $b(B_i,\lambda)$)
- locality (measurements of A does not depend on measurement of B) without locality we could have $a_i = a(A_i, B_j, \lambda)$

Local realism.

Is Quantum Mechanics a local realistic theory?

$$|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle \right) \qquad \begin{cases} |0\rangle = |-\tfrac{1}{2}\rangle \\ |1\rangle = |+\tfrac{1}{2}\rangle \end{cases} \text{ spin } \tfrac{1}{2}$$

$$\sigma_{\vec{n}} = \vec{\sigma} \cdot \vec{n} = \sigma_x n_x + \sigma_y n_y + \sigma_z n_z$$

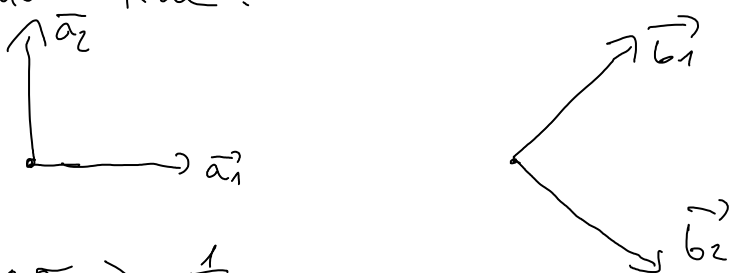
$$\sigma_{\vec{n}} = |\vec{n}\rangle\langle\vec{n}| - |\vec{n}^\perp\rangle\langle\vec{n}^\perp|$$ – observable with measurement results ± 1

$$\langle \sigma_{\vec{n}} \otimes \sigma_{\vec{n}'} \rangle = \langle\psi_-| \sigma_{\vec{n}} \otimes \sigma_{\vec{n}'} |\psi_-\rangle =$$

$$\left\{ \langle\psi_-| \sigma_i \otimes \sigma_j |\psi_-\rangle = -\delta_{ij} \right.$$

$$= -\vec{n} \cdot \vec{n}'$$

Let us take:



$$\langle \sigma_{\vec{a}_1} \otimes \sigma_{\vec{b}_1} \rangle = \frac{1}{\sqrt{2}}$$

$$\langle \sigma_{\vec{a}_1} \otimes \sigma_{\vec{b}_2} \rangle = \frac{1}{\sqrt{2}}$$

$$\langle \sigma_{\vec{a}_2} \otimes \sigma_{\vec{b}_1} \rangle = \frac{1}{\sqrt{2}}$$

$$\langle \sigma_{\vec{a}_2} \otimes \sigma_{\vec{b}_2} \rangle = -\frac{1}{\sqrt{2}}$$

$$|C| = |\langle \sigma_{a_1} \otimes \sigma_{b_1} \rangle + \langle \sigma_{a_1} \otimes \sigma_{b_2} \rangle + \langle \sigma_{a_2} \otimes \sigma_{b_1} \rangle - \langle \sigma_{a_2} \otimes \sigma_{b_2} \rangle|$$

$$= 2\sqrt{2} > 2$$

Nierówności Bella złamane,
trzeba odrzucić lokalność albo realizm ?