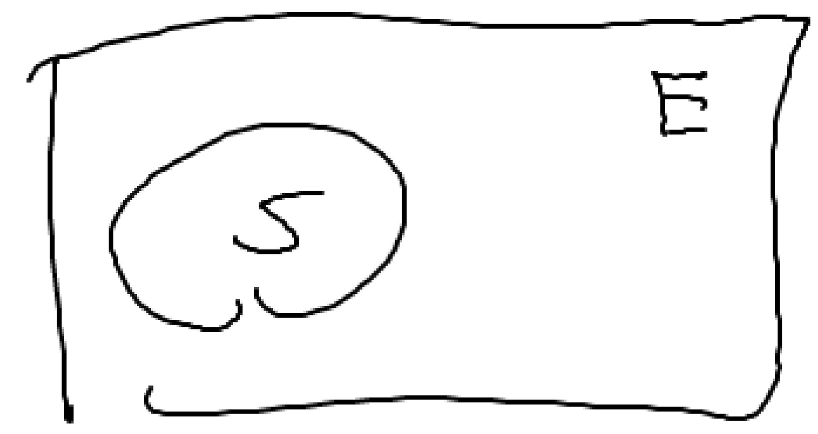
- Quantum master equation

We want to introduce some additional assumptions on the open system dynamics and arrive at a differential equation for time evolution of ρ



Assumptions:

- Hamiltonian H_{SE} is time invariant (uniformity)
- The state of the environment is practically unaffected by the evolution - environment is big and quickly relaxes to initial state

$$\tau_E \ll \delta t$$

δt ← evolution scale of the system

τ_E: E relaxation time

(Markovianity)

$$\rho_S(t) \otimes \rho_E \xrightarrow{\delta t} \approx \rho_S(t+\delta t) \otimes \rho_E$$

interaction of S&E + relaxation of E

Consider an evolution for time δt

$$\rho(t+\delta t) \approx \sum_i K_i(\delta t)\, \rho(t)\, K_i^\dagger(\delta t)$$

(We have used uniformity & markovianity here)

Expand in lowest order in δt

$$\rho(t) + \delta t \cdot X = \sum_i K_i(\delta t)\, \rho(t)\, K_i^\dagger(\delta t) + O(\delta t^2)$$

For $\delta t = 0$ we need at least one K_i to be $\mathbb{1}$

$$K_0(\delta t) = \mathbb{1} - Y\, \delta t$$

then $K_0(\delta t)\rho(t)K_0^\dagger(\delta t) = \rho(t) + (-Y\rho(t) - \rho(t)Y^\dagger)\,\delta t + O(\delta t^2)$

{ note that more operators of the form $K_i = \sqrt{p_i}\,\mathbb{1} + Y_i\,\delta t$ will amount to a single operator with $Y = \sum_i \sqrt{p_i}\,Y_i$

But, $K_0^\dagger K_0 = \mathbb{1} - (Y + Y^\dagger)\,\delta t + O(\delta t^2) \neq \mathbb{1}$

so we need to have other K_i to guarantee trace preservation:

We can take $K_i = R_i\sqrt{\delta t} + O(\delta t^{\frac{3}{2}})$ $\quad i \geq 1$

If $Y = A + iH$, A, B hermitian then

$$\sum_i K_i^\dagger K_i = \mathbb{1} - 2A\delta t + \sum_i R_i^\dagger R_i\,\delta t + O(\delta t^2)$$

so everything is fine provided $A = \frac{1}{2}\sum_i R_i^\dagger R_i$

So

$$\rho(t+\delta t) - \rho(t) = \delta t\left(\sum_i R_i\rho R_i^\dagger - \frac{1}{2}\sum_i R_i R_i^\dagger \rho - \frac{1}{2}\rho\sum_i R_i^\dagger R_i - i[H,\rho]\right)$$

and finally:

$$\boxed{\frac{d\rho}{dt} = -i[H,\rho] + \sum_i \Big(R_i\rho R_i^\dagger \underbrace{- \tfrac{1}{2}R_i^\dagger R_i\rho - \tfrac{1}{2}\rho R_i^\dagger R_i}_{-\frac{1}{2}\{\rho,\, R_i^\dagger R_i\}}\Big)}$$

Lindblad-Gorini-Kossakowski-Sudarshan equation
- q. master equation

$$\frac{d\rho}{dt} = \underset{\substack{\uparrow \\ \text{linear operator}}}{\mathcal{L}}(\rho) \implies \rho(t) = \underbrace{e^{\mathcal{L}t}}_{\Lambda_t}\rho(0)$$

family of CP maps forming a semi-group