Quantum Estimation and Measurement Theory

Problem set 6

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Problem 1   Consider the following family of qubit states:

$$|\psi_\varphi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)\exp(i\varphi)|1\rangle, \tag{1}$$

where parameter θ we regard as known while our goal is to estimate φ.

a) What does quantum Cramer-Rao bound tell us about achievable estimation precision if we were given N copies of the above state: $|\psi_\varphi\rangle^{\otimes N}$

b) What would be the optimal measurement that guarantees saturation of the Cramer-Rao bound for large N—does the measurement depend on the value of estimated parameter φ?

c) Repeat the above points, in case where instead of N copies of the pure state $|\psi_\varphi\rangle$ we get N noisy copies each described by the following mixed state: $\rho_\varphi = p|\psi_\varphi\rangle\langle\psi_\varphi| + (1-p)\mathbb{1}/2$, where p is a known parameter.