Quantum Estimation and Measurement Theory

Problem set 7

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Problem 1 Consider multiparameter estimation case of the qubit estimation problem from the previous Problem set, where we assume that apart from φ also θ and p are unknown parameters to be estimated. Write down the QFI matrix and try to conjecture whether there is measurement that allows to saturation of the CR bound for all parameters simultaneously.

Problem 2 We have introduced the Bures metric, where the distance element is defined through:

$$d_B^2\rho = \frac{1}{4}\mathrm{Tr}(\rho\, \mathrm{d}\Lambda^2), \quad d\rho = \frac{1}{2}(\mathrm{d}\Lambda\rho + \rho\mathrm{d}\Lambda). \tag{1}$$

Prove that in case of a qubit, for which a general state is parameterized as:

$$\rho = \frac{1}{2}(\mathbb{1} + \vec{r}\cdot\vec{\sigma}), \quad \vec{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta), \tag{2}$$

Bures metric takes the form:

$$d_B^2\rho = \frac{1}{4}\left[\frac{d^2r}{1-r^2} + r^2(d^2\theta + \sin^2\theta d^2\varphi)\right]. \tag{3}$$

Hint: Start by parameterizing the state using Cartesian coordinates (x,y,z) and try to find Λ from condition (1)—remember that Λ is hermitian which means that it can be written as$\Lambda = \sum_{i=0}^{3}\lambda_i\sigma_i$, where $\lambda_i \in \mathbb{R}$, and $\sigma_0 = \mathbb{1}$.