NUCLEAR STRUCTURE AND RARE ISOTOPES

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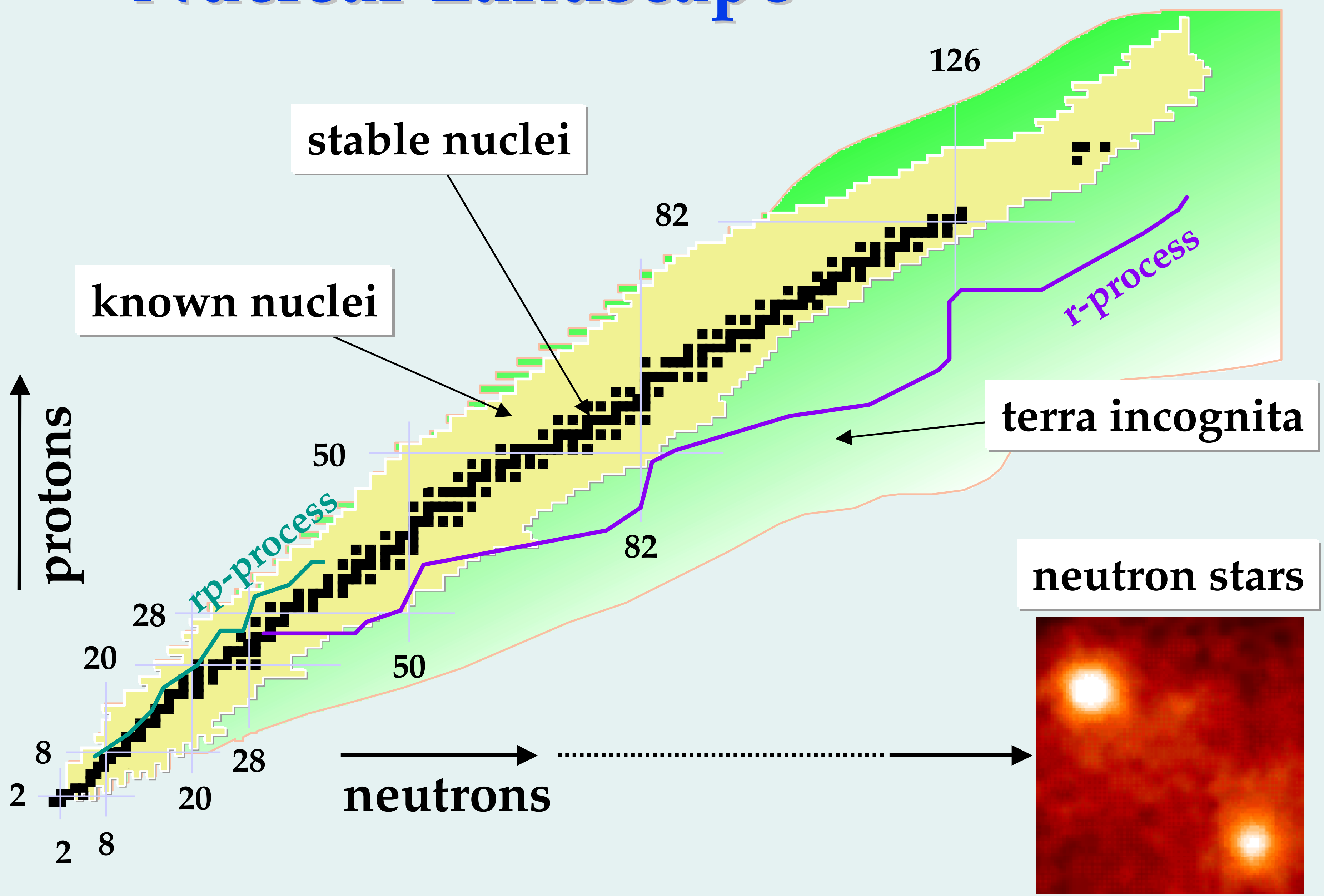
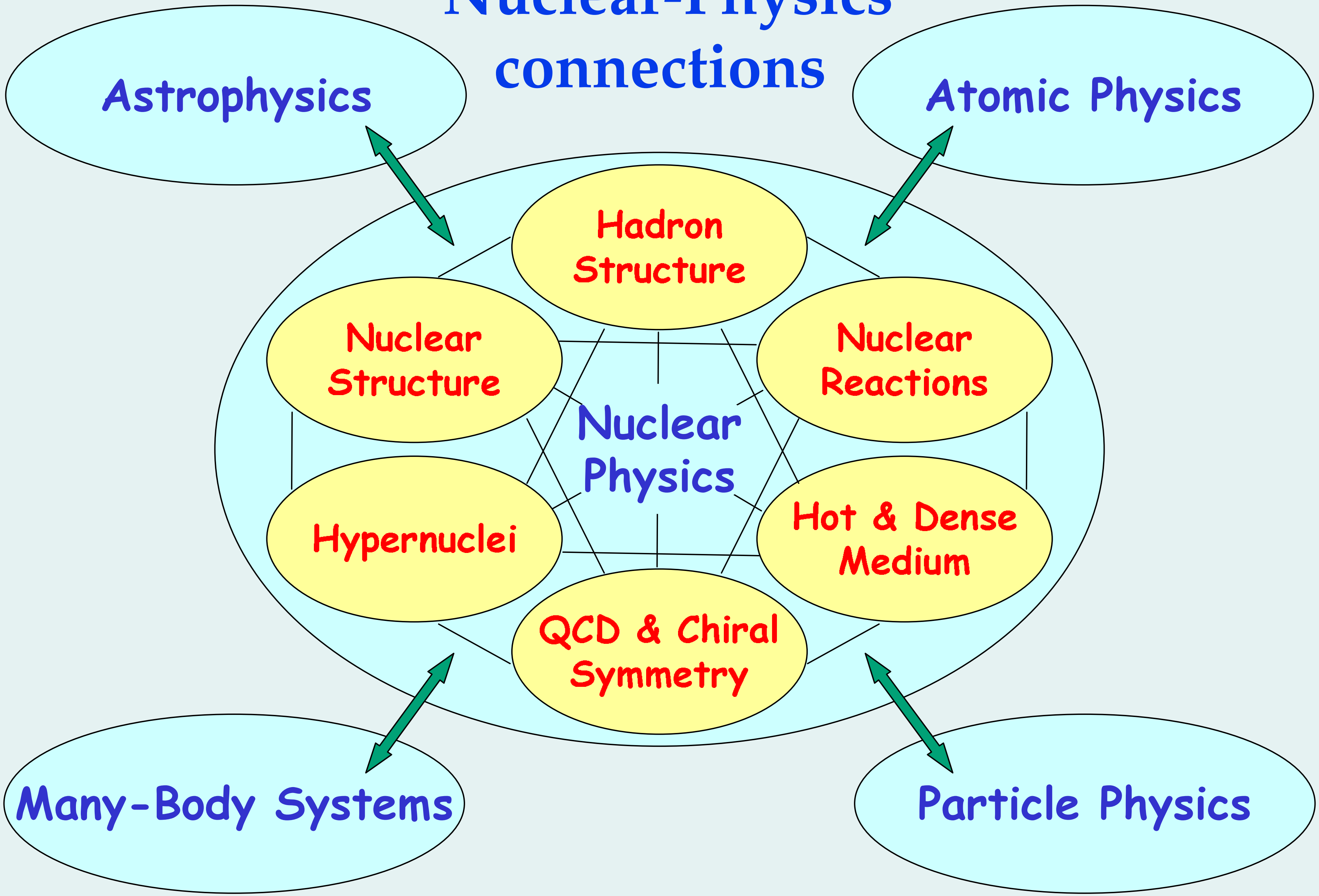
Nuclear Landscape



Nuclear Landscape

There are only less than 300 stable nuclides in nature (black squares), while already about 3000 other ones have been synthesized and studied in nuclear structure laboratories (yellow zone). However, the nuclear landscape extends further away into uncharted territories (green zone), where probably double of that await discovery. Properties of these exotic systems cannot be at present reliably derived from theoretical models, because our knowledge of basic ingredients thereof is still quite rudimentary. Derivations form first principles allow us already now to recognize general features of nuclear forces, energy-density functionals, or shell-model interactions, however, plenty of these features require careful adjustment to precise nuclear data (RIA!). Such adjustments, especially when performed for exotic, extreme systems, provide invaluable information, and then in turn allow for more reliable extrapolations.

Nuclear-Physics connections



Nuclear-Physics connections

The present-day nuclear physics contains not only the traditional subdomains, such as the **nuclear structure**, **nuclear reactions**, and physics of **hypernuclei**, but also those that not-so-long-ago where part and parcel of the particle physics, like the **hadron structure**, **hot and dense nuclear matter**, and **low-energy aspects of the QCD**.

Nuclear physics not only has very strong ties to **astrophysics** (stars are born and die via nuclear processes) and **particle physics** (weak interactions and neutrinos), but also to **atomic physics** (tests of fundamental interactions through the electron-nucleus interaction), and to the general physics of **many-body systems** (condensed matter and solid state physics).

Outline

Lecture I: Quarks, gluons, mesons, nucleons, and nuclei
- QCD fundamentals
- Composite particles
- NN interactions

Lecture II: Spontaneous symmetry breaking
- Parity – NH_3 molecule
- Chiral – pion condensate
- Rotational – deuteron & nuclei

Lecture III: Physics of exotic nuclei
- Matter distributions
- Shell structure
- Pairing

1) Erich Ormand: 2002 RIA Summer School
http://www.orau.org/ria/ria02/notes/ormand.ppt
2) Stuart Pittel: 2003 RIA Summer School
http://www.orau.org/ria/ria03/lecturenotes/pittel.pdf
3) Jacek Dobaczewski:2002 Ecole Internationale Joliot-Curie:
http://arxiv.org/abs/nucl-th/0301069

Fundamental matter and interaction fields

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W$^-$	80.4	–1
W$^+$	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Nuclear Physics studies composite objects that are built of light quarks **uds** and interact by exchanging gluons **g**. The complete theory is defined by the QCD Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu}_{\alpha} - \sum_n \bar{\psi}_n\gamma^\mu[\partial_\mu - igA^\alpha_\mu t_\alpha]\psi_n - \sum_n m_n\bar{\psi}_n\psi_n$$

where

$$F^{\alpha}_{\mu\nu} = \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + C^\alpha_{\beta\gamma}A^\beta_\mu A^\gamma_\nu$$

Quarks and gluons also interact electro-weakly with electrons **e** and neutrinos, ν_e and ν_μ, by exchanging photons γ and bosons W^-, W^+, Z^0.

http://www.cpepweb.org/

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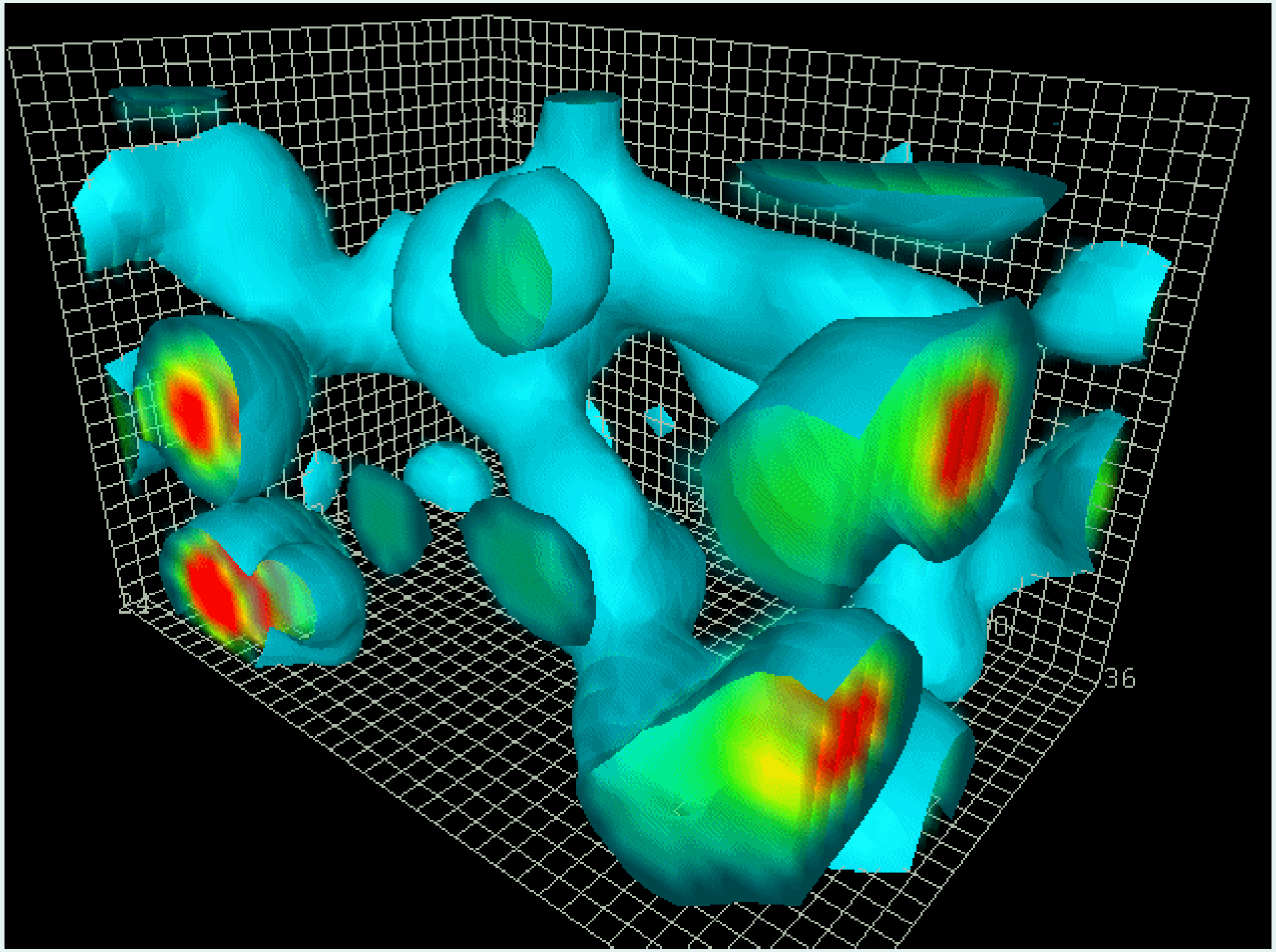
IFT UW · Oak Ridge National Laboratory · The University of Tennessee · Rare Isotope Accelerator (RIA) RIA: CONNECTING NUCLEI with the UNIVERSE

The QCD vacuum



Derek B. Leinweber

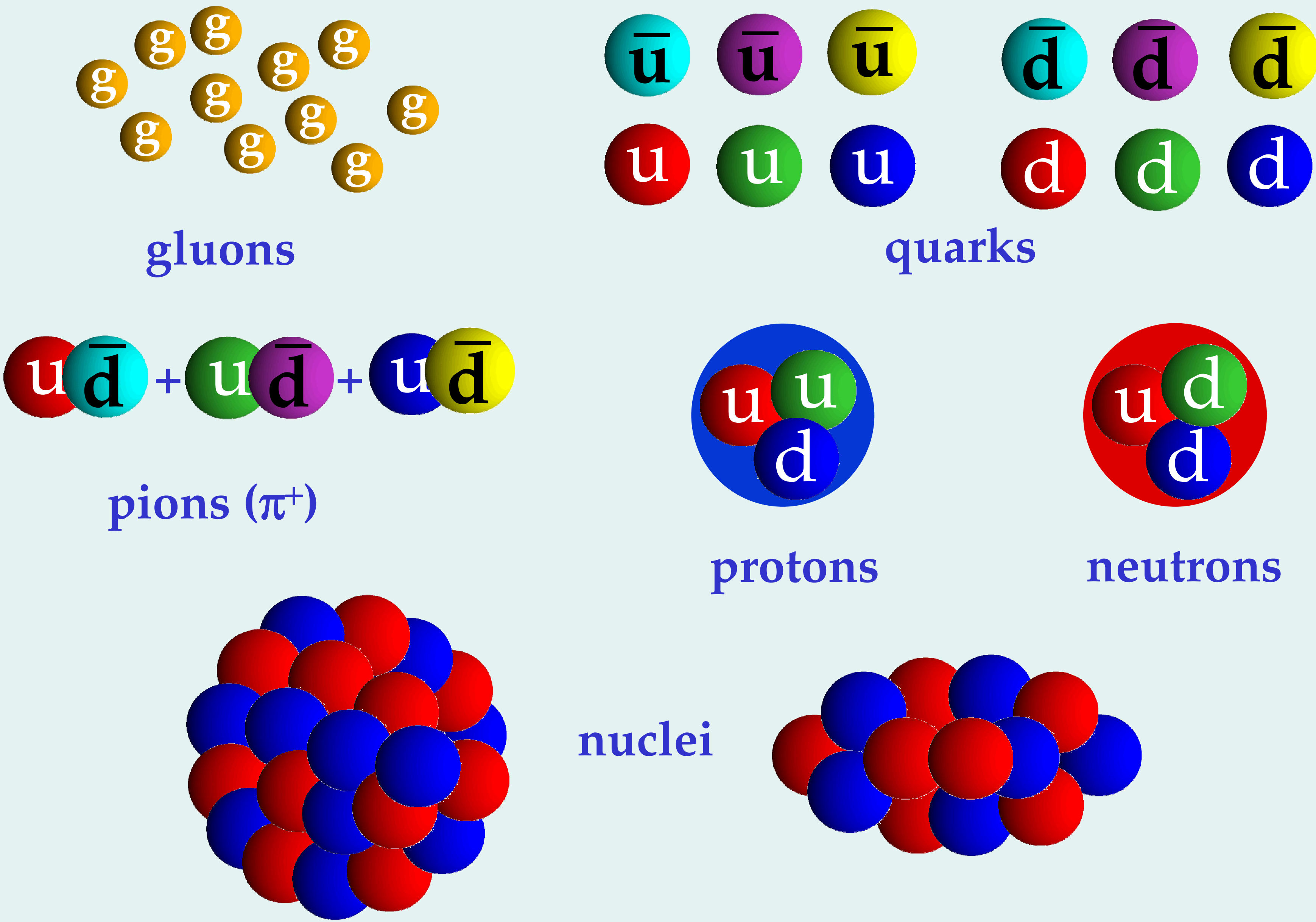
http://hermes.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/welcome.html

The QCD vacuum

The empty space is not empty at all! The vacuum is one of the solutions of the field equations which minimizes the energy for a state with baryon and lepton numbers equal to zero. Such a state may contain arbitrary numbers of particle-antiparticle pairs that can spontaneously appear in the empty space. On the average, there can be non-zero numbers of these pairs at any time and point in space. Therefore, the vacuum can be an enormously complicated state with a non-zero energy density.

This is a well-known and experimentally verified effect in QED (quantum electrodynamics) called the Casimir effect, whereupon two conducting parallel plates attract each other when put in otherwise completely empty space. The attraction is caused by electron-positron pairs that spontaneously appear in the empty space between the plates (the so-called vacuum polarization effect).

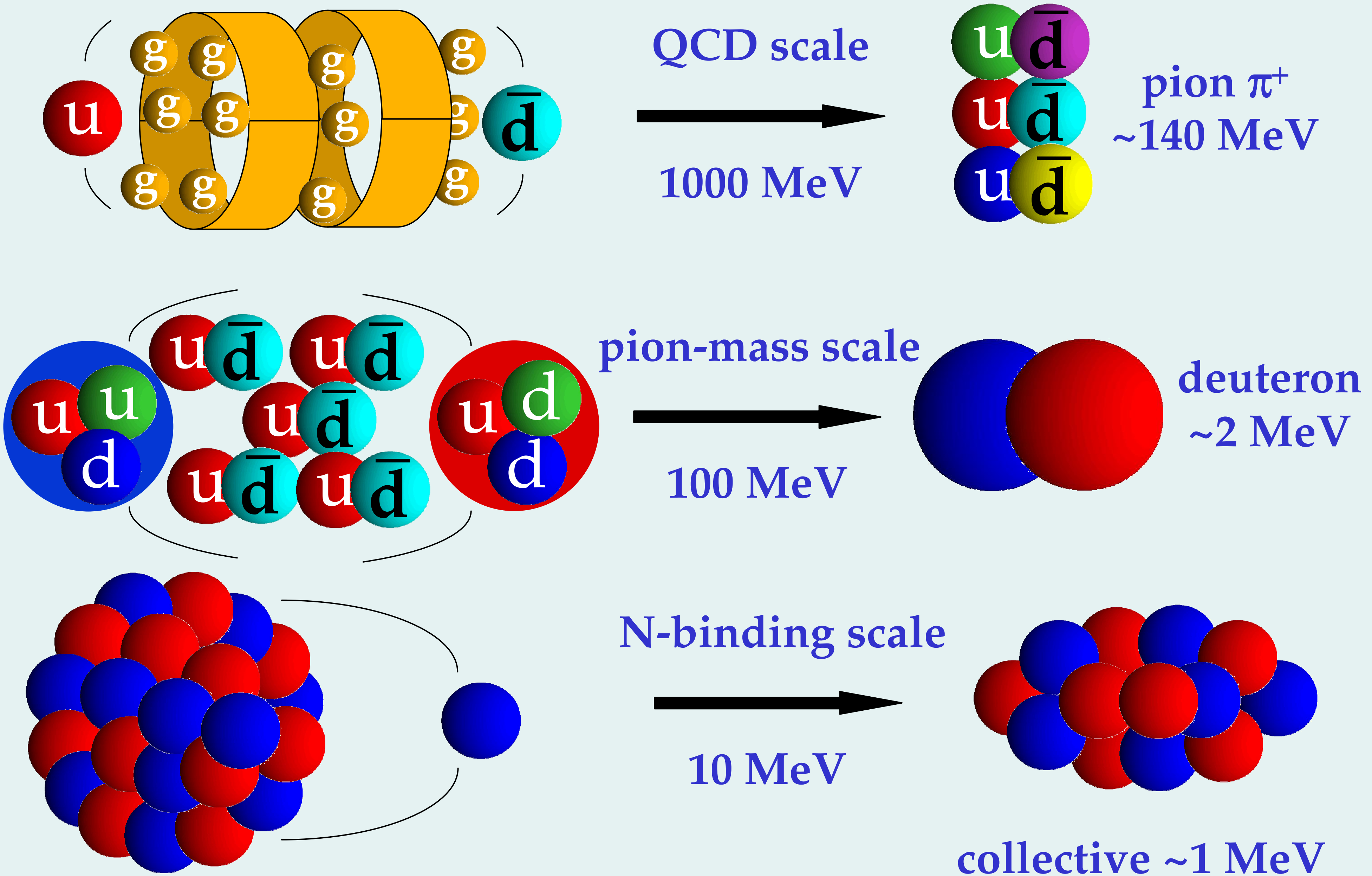
Main players in Nuclear Physics



Main players in Nuclear Physics

Quarks and gluons are fundamental fields that appear in the QCD Lagrangian. All other elementary hadrons are composite objects that are solutions of the QCD for specific baryon numbers. Quarks and gluons belong to three and eight dimensional representations of the color SU(3) group; whereupon quarks are traditionally referred by the red, green, and blue (RGB) basic colors, while antiquarks by cyan, magenta, and yellow (CMY) complementary colors. Only color-SU(3) scalars (white composite particles) can propagate as free particles; all colored fields are confined within white objects and cannot be separated. White combinations of the quark-antiquark pairs are called mesons. Nucleons are white three-quark composite particles. Composite particles are known from experiment and cannot yet be fully calculated within the QCD. Nuclei are composite particles built of nucleons, of which the quark constituents are unresolved.

Scales of energy in Nuclear Physics



Scales of energy in Nuclear Physics

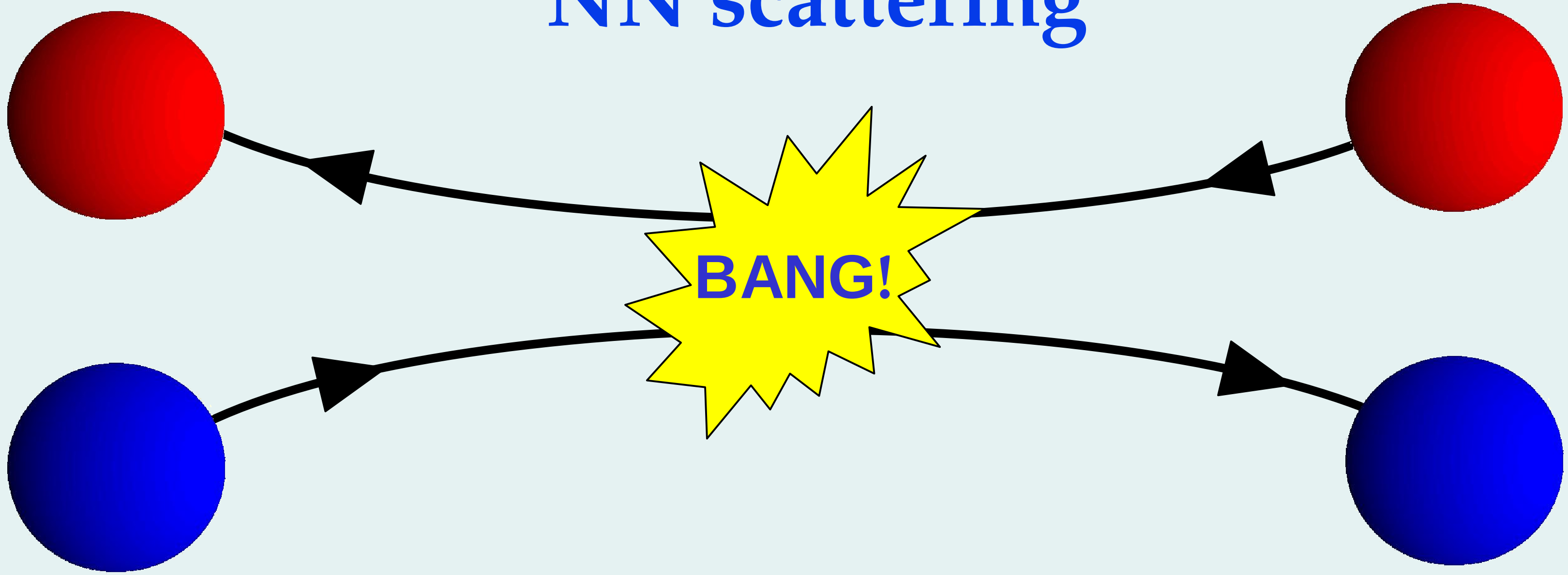
1000 MeV QCD scale: When two valence quarks are separated, they are connected by a tube of gluons and quark-antiquark pairs (the flux tube) that provides an interaction "potential" linearly growing with the distance. The same gluon-quark-antiquark soup binds the valence quarks into white composite particles and provides most of their mass.

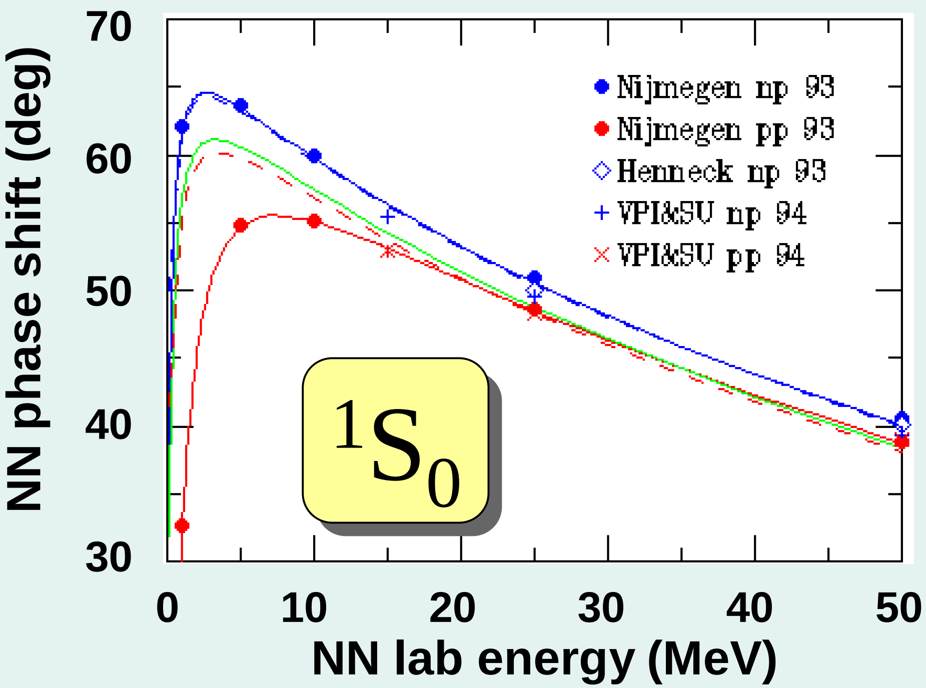
100 MeV pion-mass scale: When two nucleons are separated, they interact by exchanging one-, two-, ore several pions. Nuclei are bound as a result of pion exchanges within the background of the so-called chiral condensate.

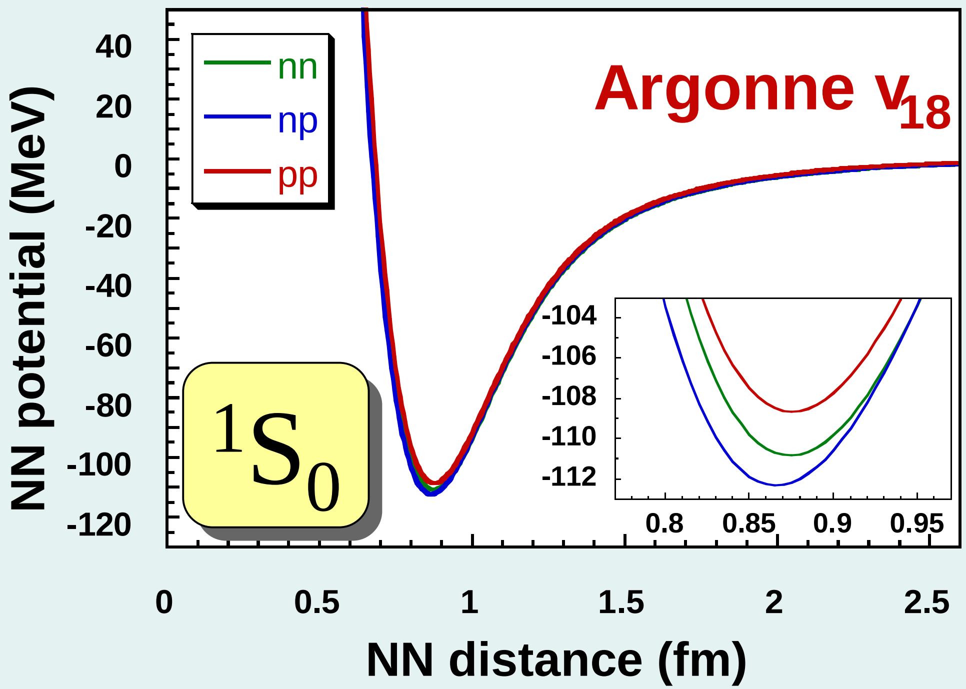
10 MeV N-binding scale: When a nucleon is separated from the nucleus, it interacts with the average field of all the remaining nucleons; its binding energy is a result of strong cancellation between the kinetic and interaction energies.

1 MeV collective scale: When the collective excitation of a nucleus is created, its energy is a coherent sum of small excitation energies of all or many constituent nucleons.

NN scattering







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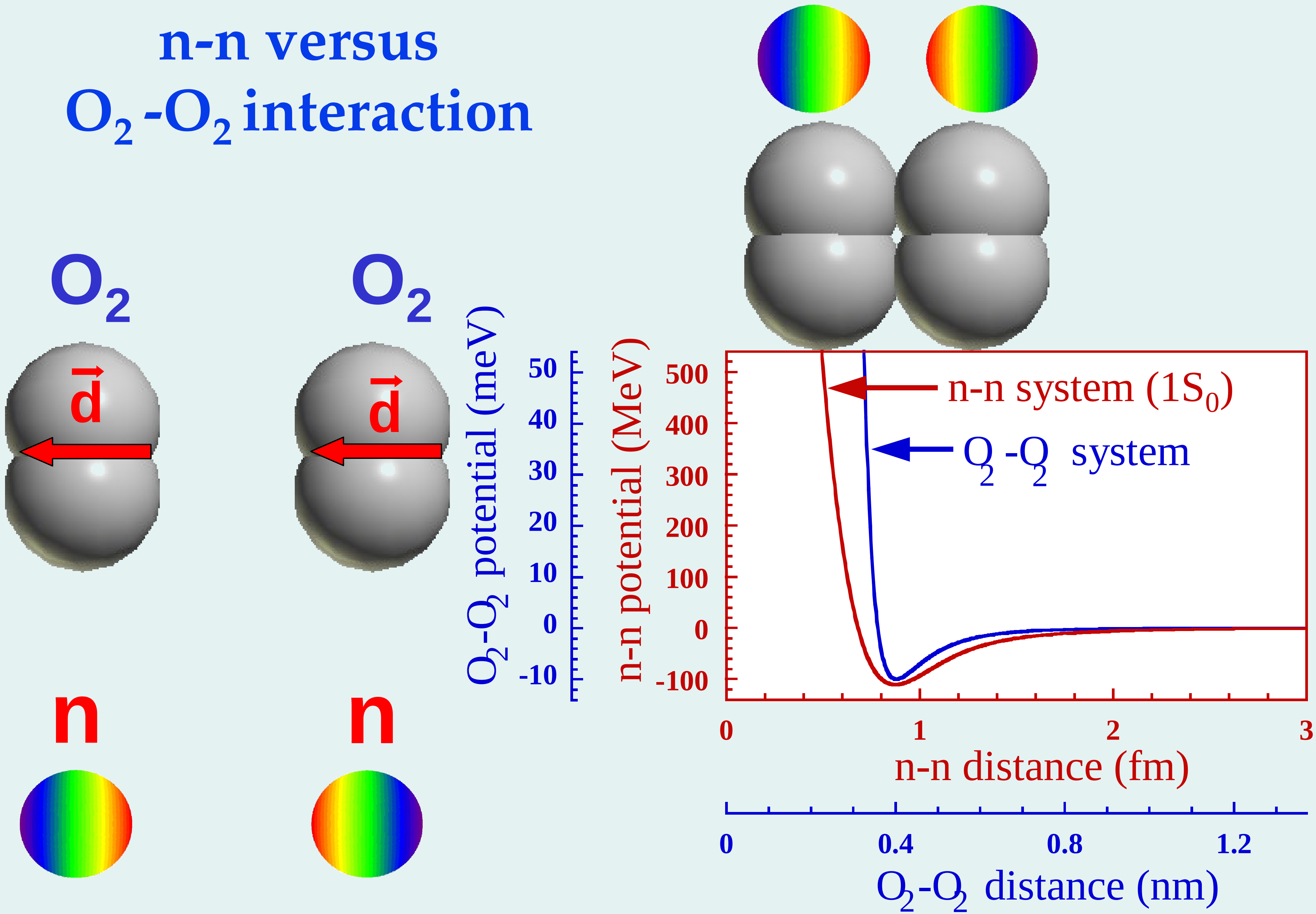
NN scattering

At low energies below 300 MeV, the quark structures of nucleons are not resolved and interactions between the nucleons can be approximated by potentials that have long attractive tails and strong repulsive cores. These potentials are best adjusted to the NN scattering data (phase shifts) in specific channels defined by L, S, and J, described by the spectroscopic notation of $^{2S+1}L_J$. The Pauli principle requires that the isospin of each channel be confined to T=0 for odd J and T=1 for even J. Interactions are very weakly depending on the isospin – mostly, but not uniquely, through the Coulomb force. Channels of J>0 mix two values of L=J±1, and therefore potentials in these channels are 2×2 matrices. Therefore, e.g., the deuteron wave function is a mixture of the L=0 (S) and L=2 (D) components.

n-n versus O_2-O_2 interaction



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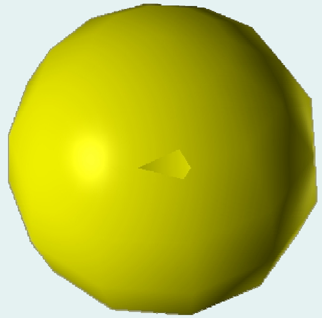
n-n versus O_2-O_2 interaction

Neutrons are white (color singlets). When two are brought together they **attract** each other through the Yukawa force ($\sim e^{-r}/r$), (one-pion exchange) and higher forces (two-pion exchanges). At smaller distances, heavier mesons are exchanged and the Pauli blocking sets in, which can be modeled by adding a phenomenological **repulsive** hard core – the Argonne v_{18} **potential**. At low energies, the interaction can be approximated by a potential without any reference to the meson exchanges. Can this potential be expressed as resulting from polarized **color/flavor distributions** (nuclear Van der Waals force)?

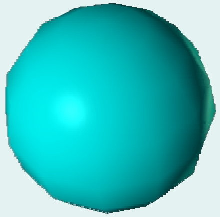
O_2 molecules are neutral (have zero net charge) and non-polar (have zero dipole moment). When two are brought together they polarize each other and **attract** through a dipole-dipole interaction E· d where E ($\sim 1/r^3$) induces d ($\sim 1/r^3$), and hence the resulting Van der Waals force decreases as ($\sim 1/r^6$). At smaller distances, higher multipoles and Pauli blocking set in, which can be modeled by an ad hoc **repulsive** term ($\sim 1/r^{12}$) - the Lennard-Jones **potential**. EM interactions result from exchanging photons, but at low energies they can be approximated by the Coulomb force acting between the **charge distributions**.

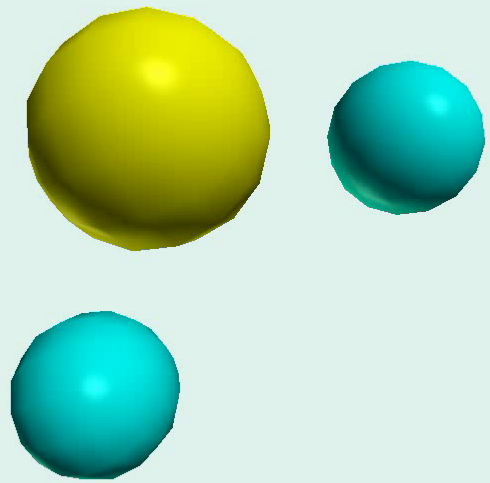
Ammonia molecule NH_3

Nitrogen atom

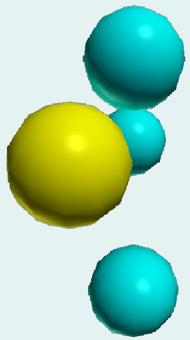


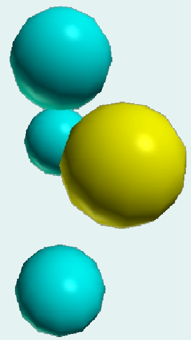
Hydrogen atom





$|L\rangle =$



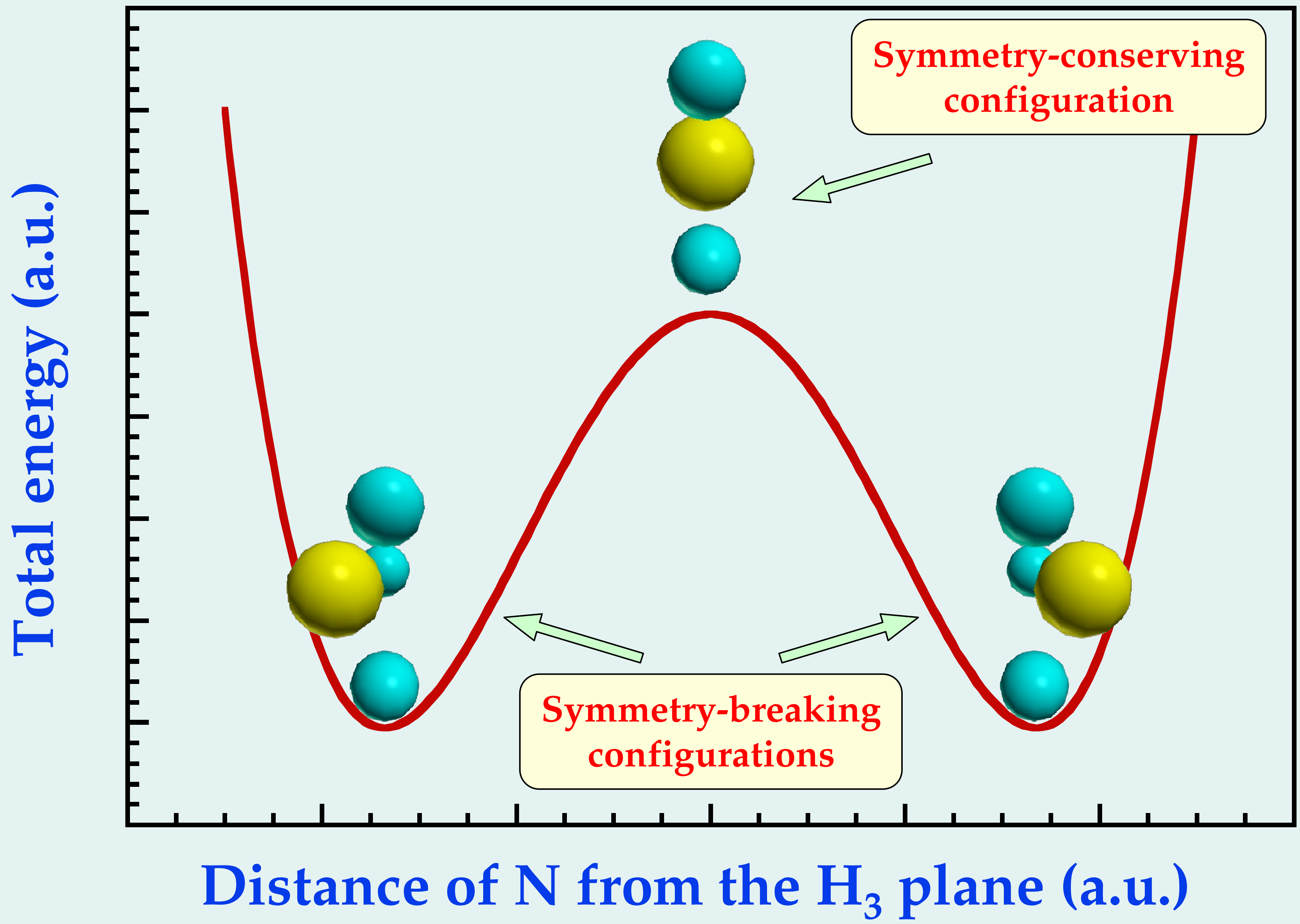


$= |R\rangle$

left state

right state

Ammonia molecule NH_3



Let P be the plane-reflection operator with respect to the H_3 plane, then

$$\begin{aligned} P|R\rangle &= |L\rangle \\ P|L\rangle &= |R\rangle \end{aligned}$$

Let us denote overlaps and matrix elements by

$$\begin{aligned} 1 &= \langle L|L\rangle = \langle R|R\rangle \\ \epsilon &= \langle L|R\rangle \\ E_0 &= \langle L|H|L\rangle = \langle R|H|R\rangle \\ \Delta &= \langle L|H|R\rangle \end{aligned}$$

In the non-orthogonal basis of $|L\rangle$, $|R\rangle$ the Hamiltonian matrix reads

$$H = \begin{pmatrix} E_0 & \Delta \\ \Delta & E_0 \end{pmatrix}$$

The eigenstates must correspond to the restored-symmetry states

$$|\pm\rangle = \frac{1}{\sqrt{2\pm 2\epsilon}}(|L\rangle \pm R\rangle)$$

i.e.,

$$P|\pm\rangle = \pm|\pm\rangle$$

The eigenenergies read

$$E_\pm = \langle\pm|H|\pm\rangle = \frac{E_0 \pm \Delta}{1 \pm \epsilon}$$

States $|L\rangle$ and $R\rangle$ are wave packets, e.g.,

$$|L\rangle = \frac{1}{2}\left(\sqrt{2+2\epsilon}|+\rangle + \sqrt{2-2\epsilon}|-\rangle\right)$$

which evolve in time ($\epsilon << \Delta/E_0$ assumed) as:,

$$|L,t\rangle = e^{iE_0t/\hbar}\left(\cos(\Delta t/\hbar)|L,0\rangle + i\sin(\Delta t/\hbar)|R,0\rangle\right)$$

Spontaneous symmetry breaking

Spontaneous symmetry breaking gives a description of the system in terms of wave packets instead of eigenstates. The wave packets corresponds to given configurations of constituents. If the configuration interaction energy is very small, the wave packets live a very long time and behave like classical objects. If configurations are orthogonal and degenerate, the wave packets are also eigenstates **(infinite systems)**.

Symmetry restoration amounts to projecting states with good quantum numbers from the symmetry-breaking states. If the configuration interaction energy is very small, the energies of projected states are very close to those of the symmetry-breaking states. Symmetry restoration can be essential for calculating average values of symmetry-conserving observables other than the Hamiltonian. **After the symmetry restoration, the symmetry-breaking solutions do not break symmetry!**

Don't let yourself confuse by the confusing traditional terminology

When you hear about:		Think about:
State in the intrinsic reference frame	→	State before the symmetry restoration
State in the laboratory reference frame	→	State after the symmetry restoration

Chiral symmetry and isospin

$$\mathcal{L}_\chi = -\bar{q}\gamma^\mu D_\mu q = -\bar{u}\gamma^\mu D_\mu u - \bar{d}\gamma^\mu D_\mu d$$

- Covariant derivative: $D_\mu = \partial_\mu - igA^\alpha_\mu t_\alpha$
- Quark iso-spinor: $q = \begin{pmatrix} u \\ d \end{pmatrix}$
- Quark masses neglected for a moment.
- $\mathcal{L}_\chi$ is invariant with respect to the SU(2)×SU(2) group generated by the mixing of the u and d quarks.
- SU(2)×SU(2) generators:
  1° the isospin matrices: $\vec{t} = \frac{1}{2}\vec{\tau}$

  $$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  2° the γ_5×isospin matrices: $\vec{x} = \gamma_5 \vec{t}$ (remember that $(\gamma_5)^2 = 1$)
- SU(2)×SU(2) commutation relations:
  $\vec{t}_L = \frac{1}{2}(1+\gamma_5)\vec{t} = \frac{1}{2}(\vec{t} + \vec{x})$ ⟵ left-handed
  $\vec{t}_R = \frac{1}{2}(1-\gamma_5)\vec{t} = \frac{1}{2}(\vec{t} - \vec{x})$ ⟵ right-handed
  $[t_{Li}, t_{Lj}] = i\epsilon_{ijk} t_{Lk}$, $[t_{Ri}, t_{Rj}] = i\epsilon_{ijk} t_{Rk}$, $[t_{Li}, t_{Rj}] = 0$
- Group isomorphism: SU(2)×SU(2) $\equiv$ O(4)

Chiral symmetry breaking

$$\mathcal{L}_\chi = -g\bar{u}\gamma^\mu D_\mu u - \bar{d}\gamma^\mu D_\mu d - m_u\bar{u}u - m_d\bar{d}d$$

- Quark masses are small as compared to the QCD scale: $m_u \approx 3\,\mathrm{MeV}$, $m_d \approx 6\,\mathrm{MeV}$, $\Lambda_{QCD} \approx 1000\,\mathrm{MeV}$
- Quark masses weakly break the chiral symmetry
- Chiral symmetry is strongly broken in the real world: pairs of particles having similar masses and opposite parities are not observed.
- Strong chiral symmetry breaking: SU(2)×SU(2) broken while the isospin SU(2) conserved.
- Effective theories are needed to describe complicated composite objects like mesons and nucleons.
- Fields of composite objects can be treated as elementary fields.
- Lagrangians of effective fields can be built based on the symmetry requirements.
- Pion mass is given by the quark masses: $m_\pi = -4(m_u + m_d)\langle\Phi_4^+\rangle_{\mathrm{vac}}/F_\pi^2$

Linear σ model

Let ϕ_n, n=1,2,3,4, denote the pseudoscalar (real) fields described by the Lagrangian

$$\mathcal{L}_\sigma = -\tfrac{1}{2}\partial_\mu\phi_n\partial^\mu\phi_n - \tfrac{1}{2}\mathcal{M}^2\phi_n\phi_n - \tfrac{1}{4}g(\phi_n\phi_n)^2$$

$\mathcal{L}_\sigma$ is explicitly invariant with respect to rotations in 4 dimensions: O(4) ≡ SU(2)×SU(2) symmetry. However, the potential energy depends only on the radial variable $\sigma^2=\phi_n\phi_n$

$$V(\phi) = V(\sigma) = \tfrac{1}{2}\mathcal{M}^2\sigma^2 + \tfrac{1}{4}g\sigma^4$$

For $g>0$ and $\mathcal{M}^2<0$ it has the minimum at

$$\sigma = |\mathcal{M}|/\sqrt{g}$$

and does not depend on the orientation of ϕ in the 4-dim space.

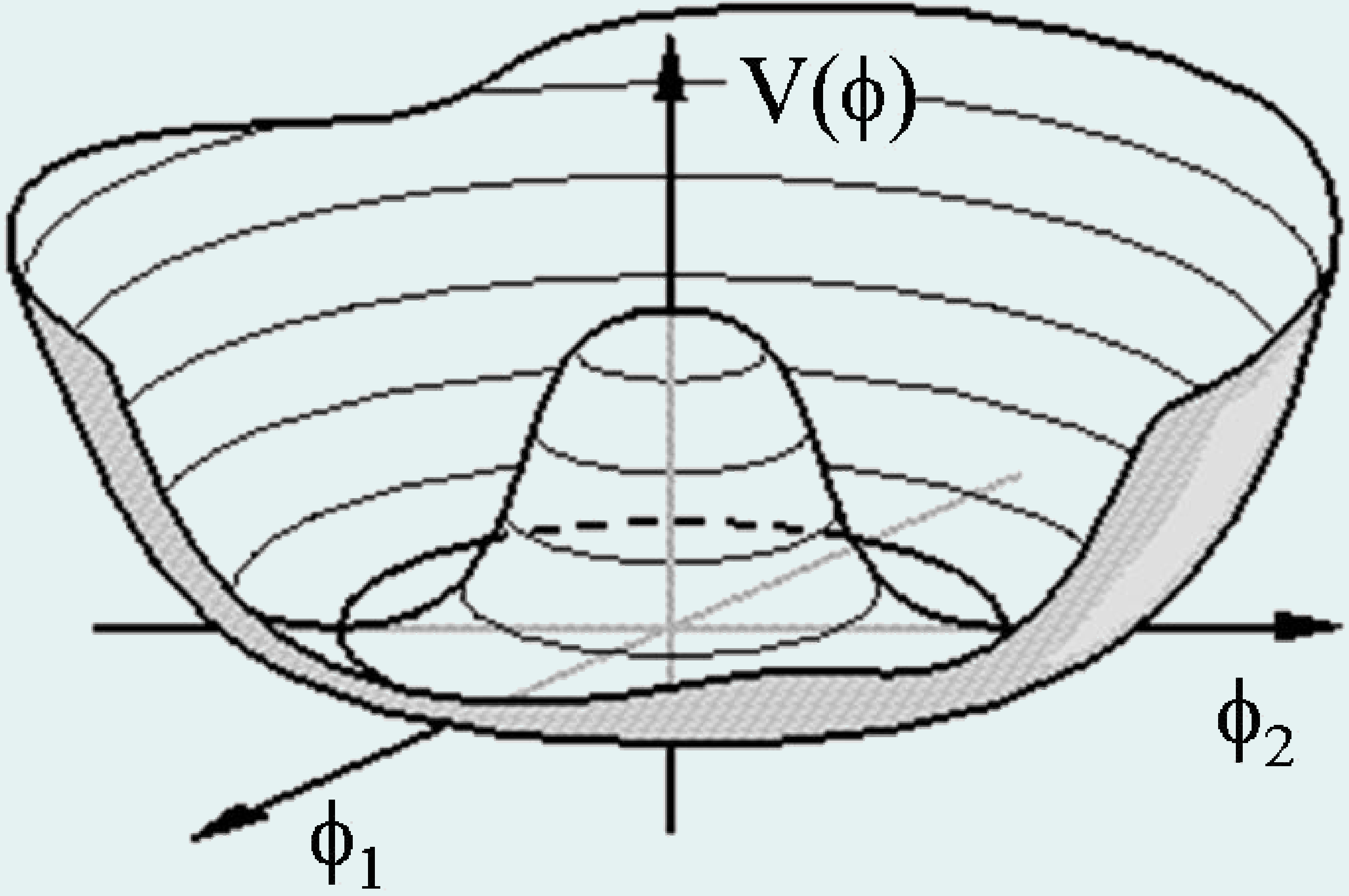
By picking one solution $\bar{\phi}$ out of the infinitely-many existing ones, we break the $O(4)$ symmetry.

The mass-matrix (the stiffness of the potential) calculated at $\bar{\phi}$ reads:

$$\mathcal{M}_{nm} = \frac{\partial^2 V}{\partial\phi_n\partial\phi_m} = 2g\bar{\phi}_n\bar{\phi}_m$$

It has one eigenvalue equal to $m^2 = 2g\sigma^2 = -2\mathcal{M}^2$ (eigenvector $\bar{\phi}$) and three eigenvalues m^2=0 (three vectors orthogonal to $\bar{\phi}$) — the Goldstone bosons — the three pions: $\vec{\pi} = (\pi_+, \pi_0, \pi_-)$.

Mexican-hat potential



Non-linear σ model

In order to separate out the radial variable σ in the 4-dimensional space we set:

$$\vec{\phi} = \frac{2\vec{z}}{1+\vec{z}^2}\sigma \qquad (n=1,2,3)$$

$$\phi_4 = \frac{1-\vec{z}^2}{1+\vec{z}^2}\sigma$$

The Lagrangian expressed in the new fields $\vec{z}$ and σ reads

$$\mathcal{L}_\sigma = -2\sigma^2\frac{\partial_\mu\vec{z}\partial^\mu\vec{z}}{(1+\vec{z}^2)^2} - \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{2}\mathcal{M}^2\sigma^2 - \tfrac{1}{4}g\sigma^4$$

- Pion field $\vec{\pi}$ is equal to $\vec{z}$ up to a normalization constant:

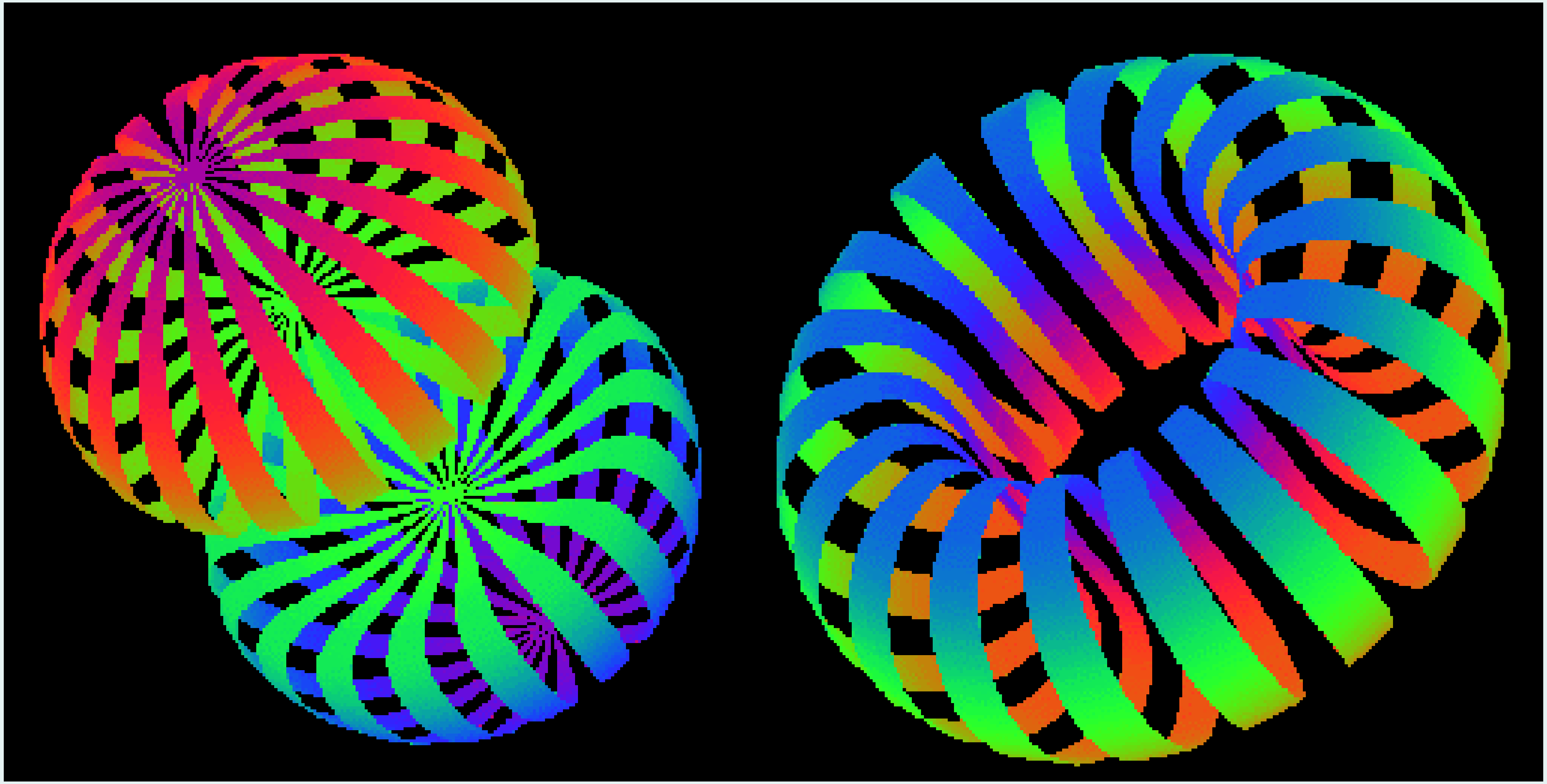
$$\vec{\pi} = F\vec{z}$$

- Details of the part depending on σ are irrelevant (high-energy part).
- Pions fields must couple to other fields (e.g. to nucleons) through the covariant derivatives $\vec{D}_\mu = \frac{\partial_\mu\vec{z}}{1+\vec{z}^2}$

Chiral symmetry breaking

The chiral symmetry — basic property of the QCD Lagrangian for massless quarks— must be obeyed by all composite fields made of quarks and gluons. Had it been unbroken in Nature, all observed fields would have appeared in pairs of opposite parities, which is obviously not the case; hence it must be spontaneously broken. The so-called σ model provides a simple illustration of the breaking mechanism, whereupon the four-dimensional chiral-invariant real fields become separated into the radial field, which spontaneously acquires non-zero value in the vacuum — the chiral condensate — and three orthogonal fields that represent the massless Goldstone bosons related to the broken chiral symmetry — the pions. The mass of physical pions results from the explicit breaking of the chiral symmetry caused by non-zero masses of quarks. Differences in masses of the three pions result from the coupling to the electromagnetic field.

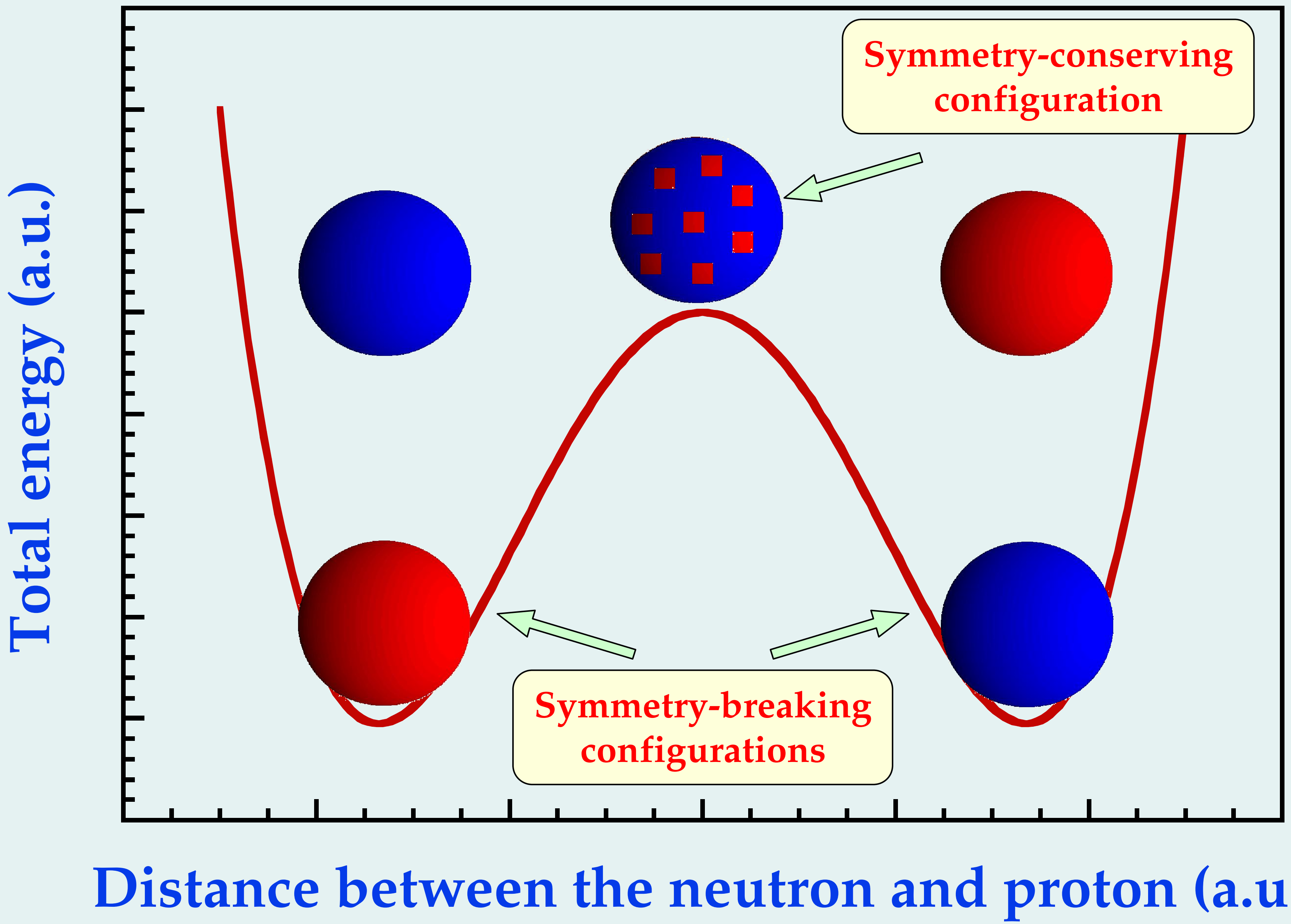
Structure of the deuteron



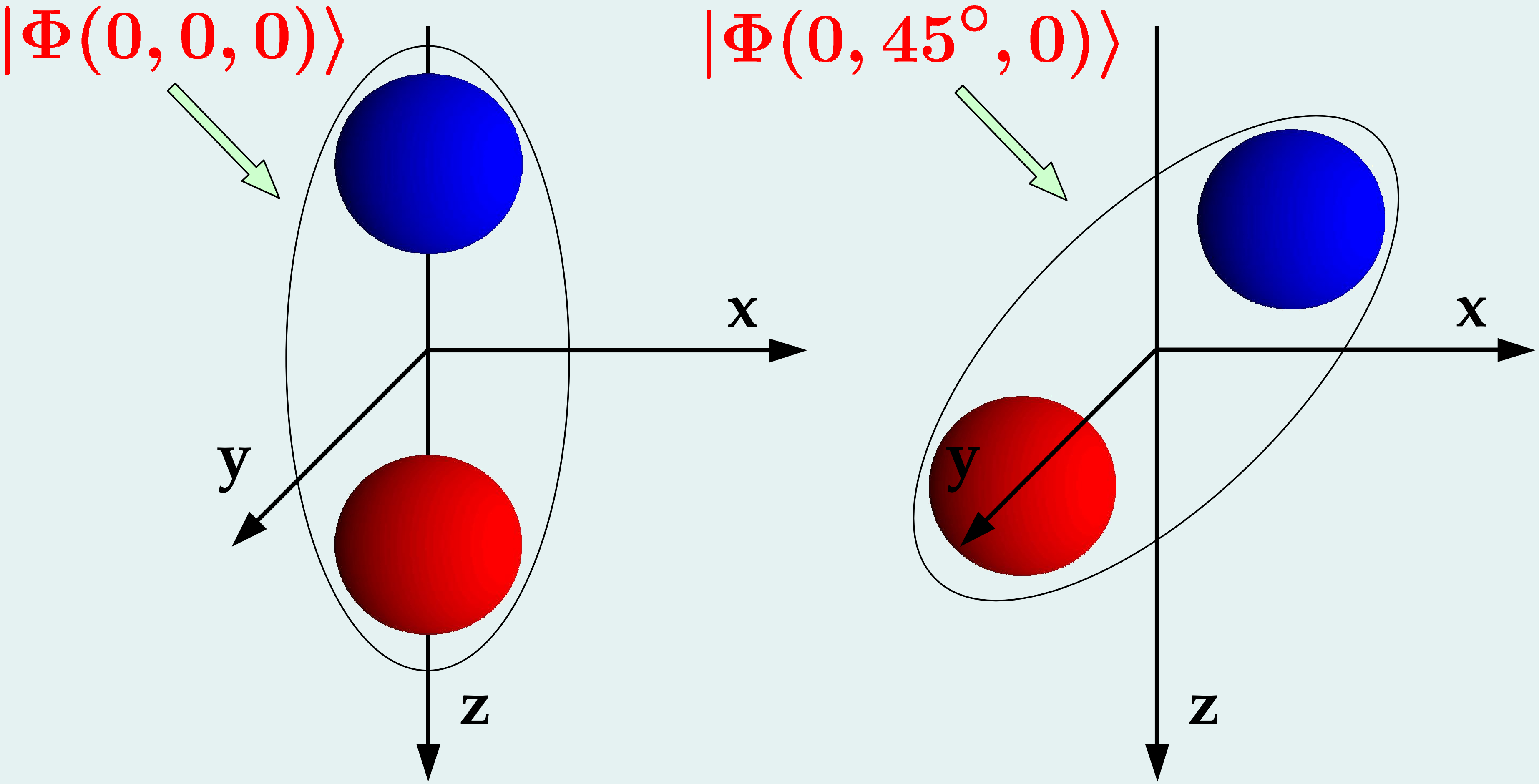
M=1 J=1 M=0

http://www.phy.anl.gov/theory/movie-run.html

Deuteron breaks the spherical symmetry



Restoration of the spherical symmetry



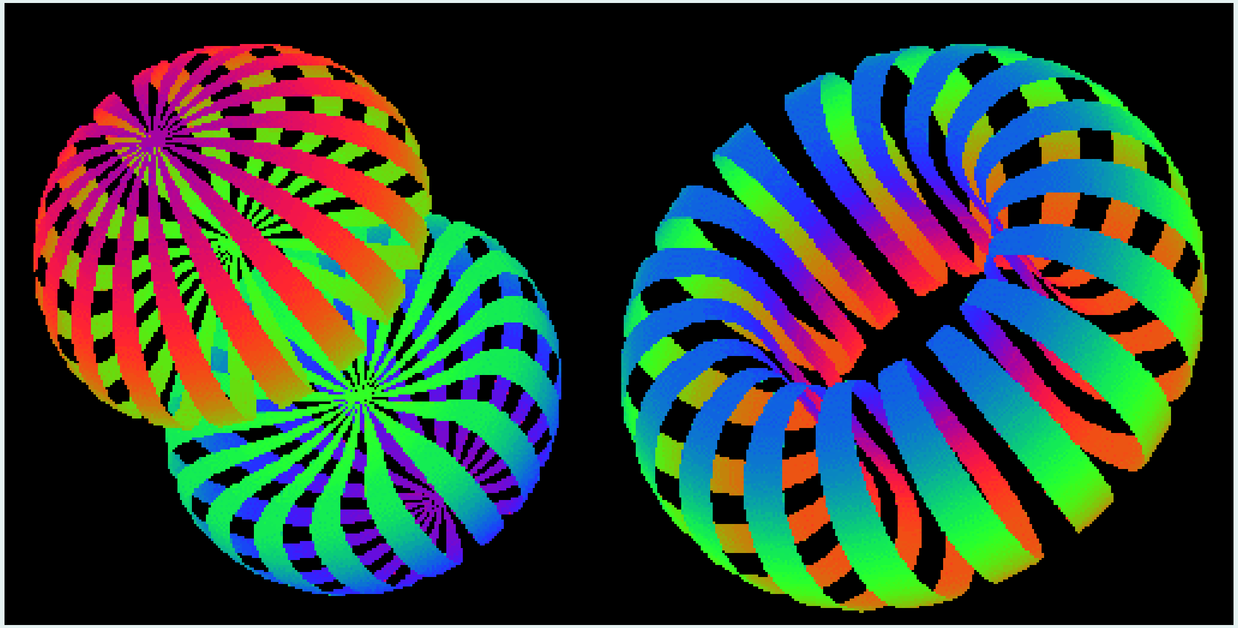
$$|\Psi^J_M\rangle = \int d\psi d\theta d\phi\, D^{J*}_{MK}(\psi,\theta,\phi)\,|\Phi(\psi,\theta,\phi)\rangle$$

for the deuteron: $J=1, K=1$

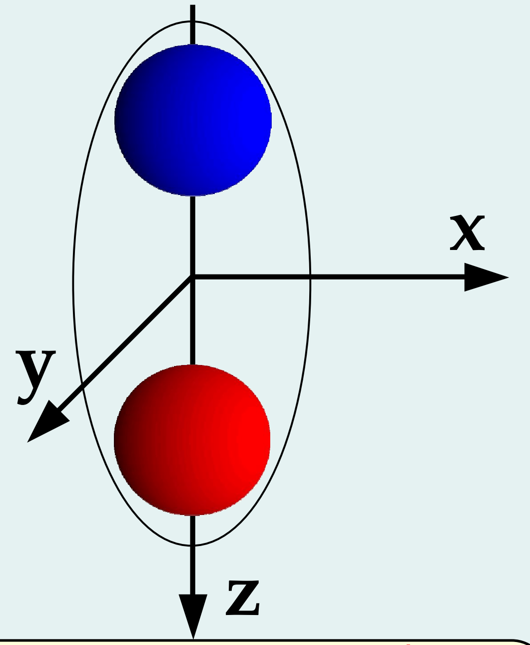
Shapes of the deuteron

"Laboratory" frame

"Intrinsic" frame



M=1 J=1 M=0



$3^\circ:\psi$ (z) $2^\circ:\theta$ (y) $1^\circ:\phi$ (z)

$$|\Psi^1_1\rangle = \int d\psi d\theta d\phi\, D^{1*}_{11}(\psi,\theta,\phi)|\Phi(\psi,\theta,\phi)\rangle \quad \text{for} \quad D^{1*}_{11} = e^{i\psi}\left(\frac{1+\cos\theta}{2}\right)e^{i\phi}$$

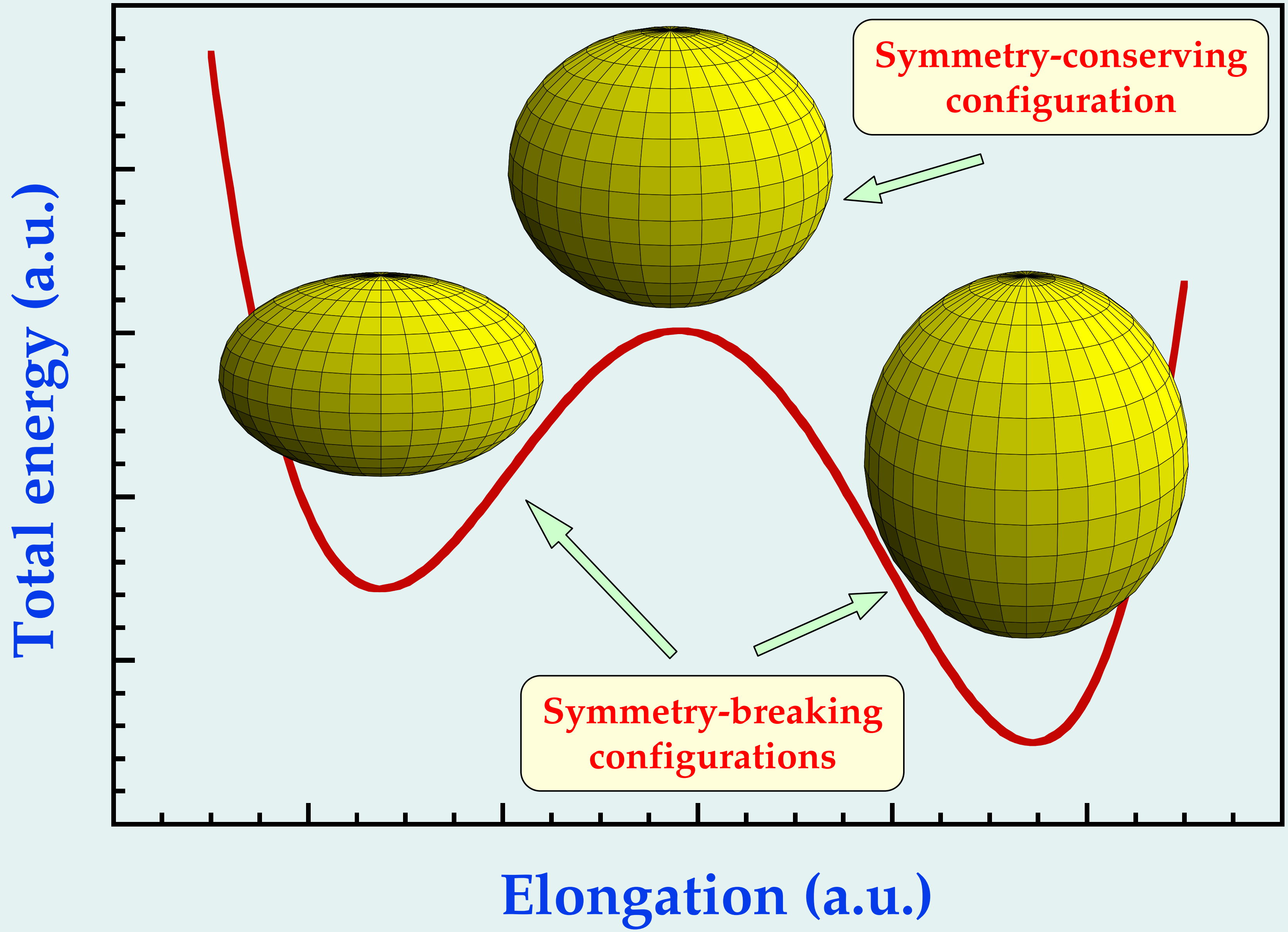
$$|\Psi^1_0\rangle = \int d\psi d\theta d\phi\, D^{1*}_{01}(\psi,\theta,\phi)|\Phi(\psi,\theta,\phi)\rangle \quad \text{for} \quad D^{1*}_{01} = \left(\frac{\sin\theta}{\sqrt{2}}\right)e^{i\phi}$$

Shapes of the deuteron

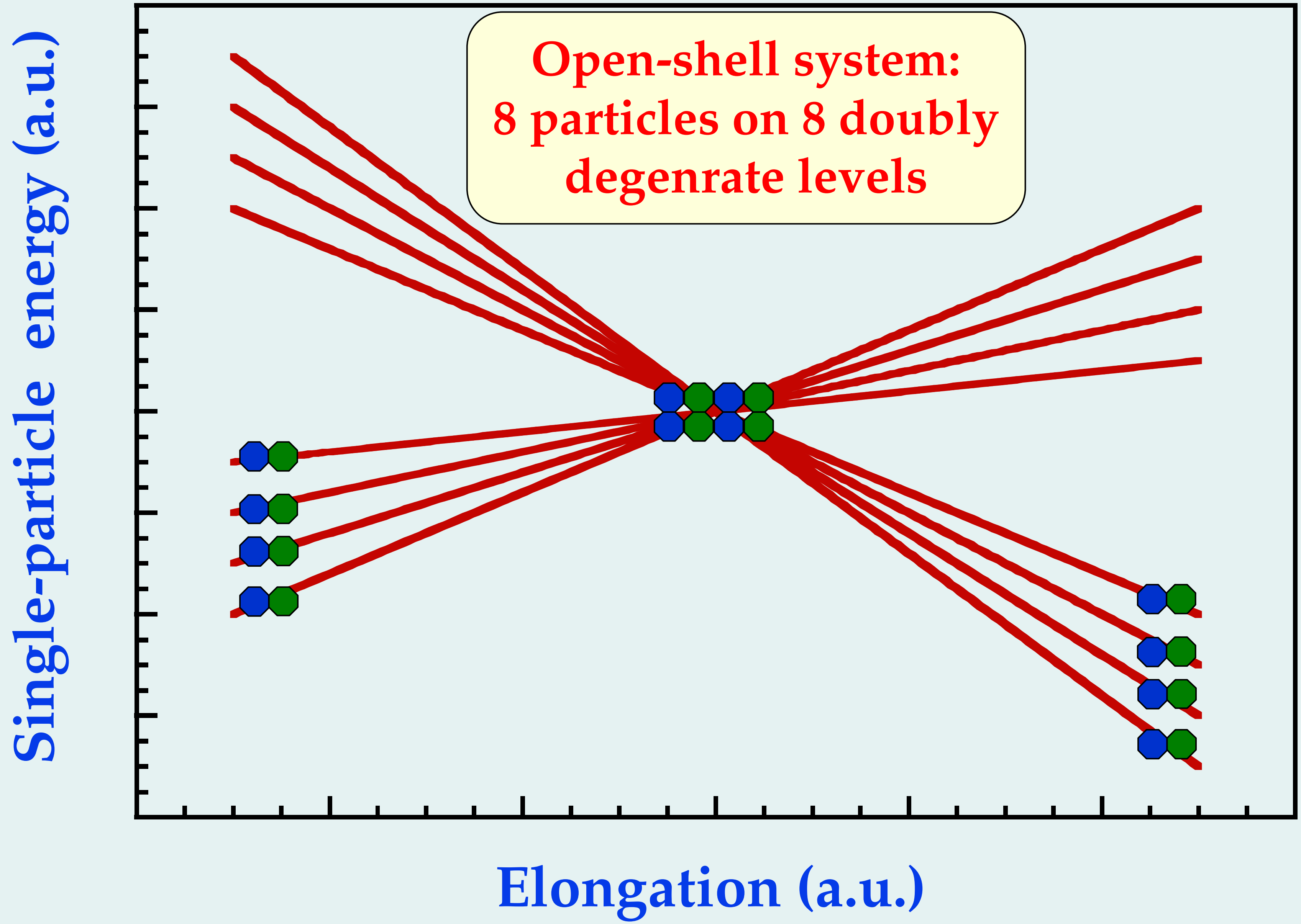
Before the symmetry restoration the deuteron wave function is built of the proton wave function localized at a given point in space and a neutron wave function localized 2.3 fm away north, minus the same piece with the proton and neutron wave functions exchanged. This wave function does not have good angular momentum but represents a **wave packet with good orientation** angle towards one spontaneously chosen spatial direction.

After the symmetry restoration the deuteron M=1 wave function looks like a dumbbell and the M=0 wave function like a torus. These wave functions have good angular momentum J=1 but **undetermined orientation** angle in space. The symmetry axis of the wave function is just a quantization axis, which can be arbitrarily chosen in space.

Nuclear deformation



Origins of nuclear deformation



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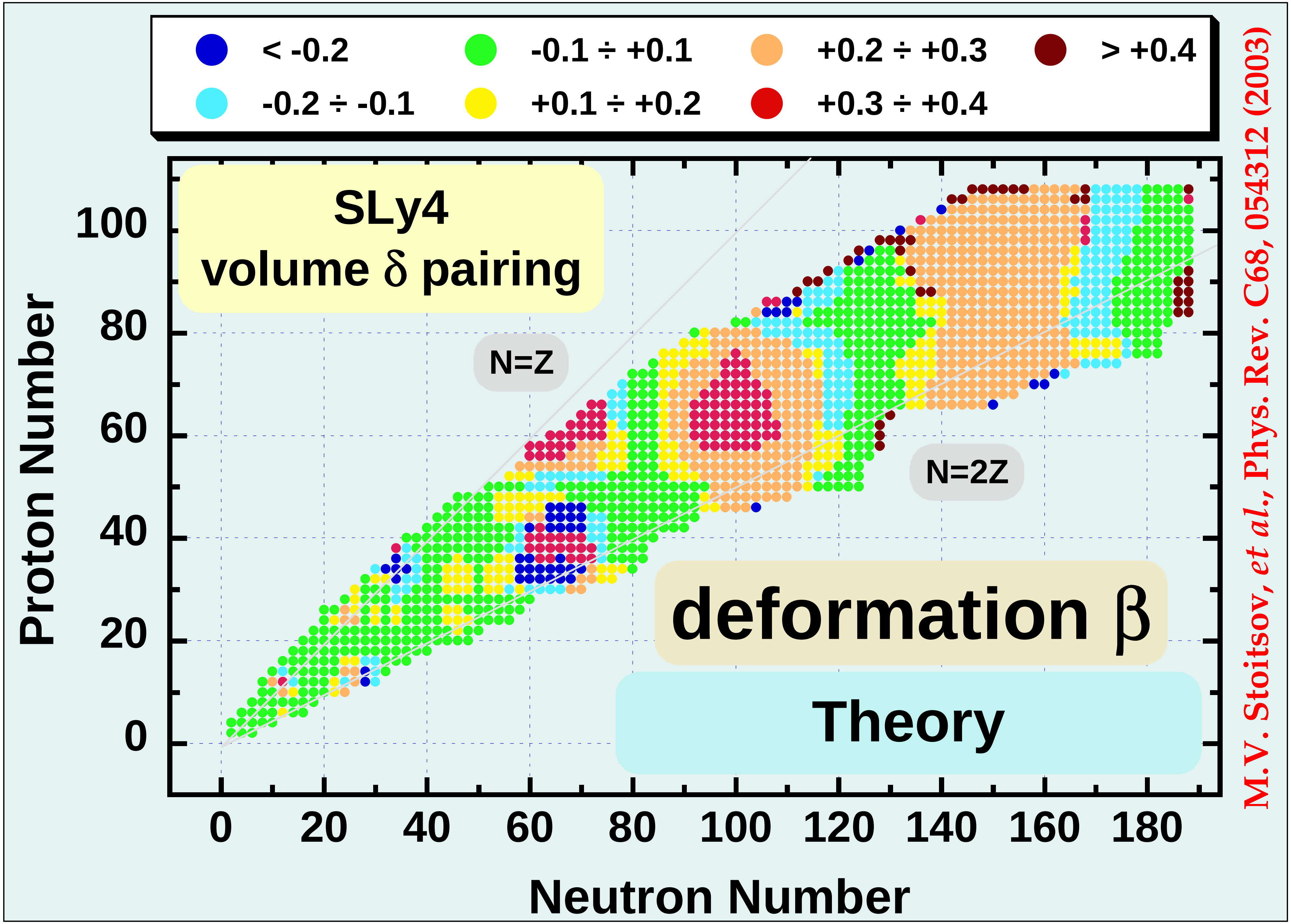
Nuclear deformation

Nuclear deformation results form residual two-body interactions between valence nucleons that favor configurations in which nucleons occupy single-particle orbitals in a deformed mean field. **Prolate** deformations are preferred at the beginning of large shells (almost empty shells) and **oblate** deformations at the and of large shells (almost full shells), although for detailed and realistic situations the prolate ones appear more often in Nature. **Deformed wave functions** do not have good angular momenta and represent **wave packets of very large widths** (very many angular-momentum components). The average angular momentum squared is always large, while the average angular momentum can be zero (non-rotating wave packets) or non-zero (rotating wave packets). In the cranking approximation a **rotating wave packet is stationary in the rotating reference frame.**



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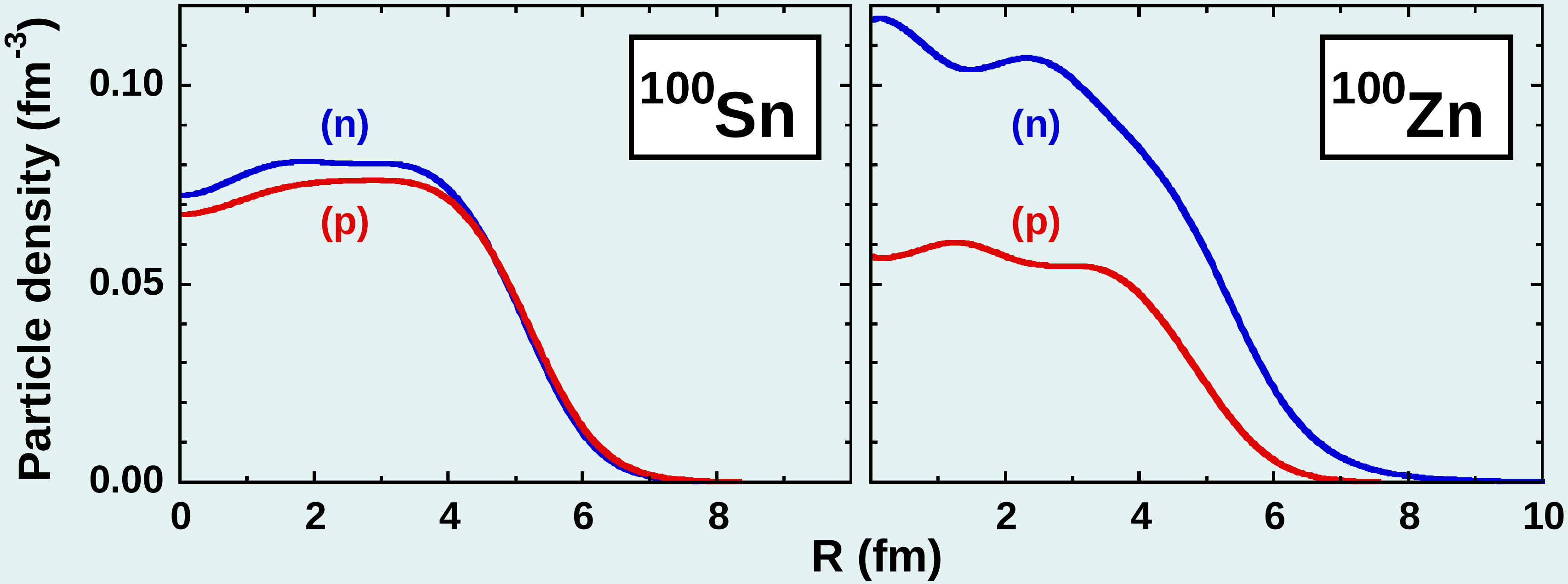
Nuclear densities as composite fields



Modern Mean-Field Theory ≡ Energy Density Functional

ρ, τ, $\overleftrightarrow{J}$, $\vec{j}$, $\vec{T}$, $\vec{s}$, $\vec{F}$,

- Hohenberg-Kohn
- Kohn-Sham
- Negele-Vautherin
- Landau-Migdal
- Nilsson-Strutinsky

mean field ⇒ one-body densities

zero range ⇒ local densities

finite range ⇒ non-local densities

Nuclear densities as composite fields

Density matrix:

$$\rho(\vec{r}\sigma,\vec{r}'\sigma') = \langle\Phi|a^+(\vec{r}'\sigma')a(\vec{r}\sigma)|\Phi\rangle$$

Scalar and vector part:

$$\rho(\vec{r},\vec{r}') = \sum_\sigma \rho(\vec{r}\sigma,\vec{r}'\sigma)$$

$$\vec{s}(\vec{r},\vec{r}') = \sum_{\sigma\sigma'} \rho(\vec{r}\sigma,\vec{r}'\sigma')\langle\sigma'|\vec{\sigma}|\sigma\rangle$$

Symmetries:

$$\rho^T(\vec{r},\vec{r}') = \rho^*(\vec{r},\vec{r}') = \rho(\vec{r}',\vec{r})$$

$$\vec{s}^T(\vec{r},\vec{r}') = -\vec{s}^*(\vec{r},\vec{r}') = -\vec{s}(\vec{r}',\vec{r})$$

Local densities:

Matter:	$\rho(\vec{r}) = \rho(\vec{r},\vec{r})$
Momentum:	$\vec{j}(\vec{r}) = (1/2i)[(\vec{\nabla}-\vec{\nabla}')\rho(\vec{r},\vec{r}')]_{r=r'}$
Kinetic:	$\tau(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\rho(\vec{r},\vec{r}')]_{r=r'}$
Spin:	$\vec{s}(\vec{r}) = \vec{s}(\vec{r},\vec{r})$
Spin momentum:	$J_{\mu\nu}(\vec{r}) = (1/2i)[(\nabla_\mu-\nabla'_\mu)s_\nu(\vec{r},\vec{r}')]_{r=r'}$
Spin kinetic:	$\vec{T}(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\vec{s}(\vec{r},\vec{r}')]_{r=r'}$
Tensor kinetic:	$\vec{F}(\vec{r}) = \frac{1}{2}[(\vec{\nabla}\otimes\vec{\nabla}'+\vec{\nabla}'\otimes\vec{\nabla})\cdot\vec{s}(\vec{r},\vec{r}')]_{r=r'}$

Local energy density: (no isospin, no pairing)

Density	Derivative	T	P	space	Energy density
$\rho(\vec{r})$		$+$	$+$	scalar	ρ^2
	$\vec{\nabla}\rho(\vec{r})$	$+$	$-$	vector	$\vec{\nabla}\rho\cdot\vec{J}$
	$\Delta\rho(\vec{r})$	$+$	$+$	scalar	$\rho\Delta\rho$
$\tau(\vec{r})$		$+$	$+$	scalar	$\rho\tau$
$J^{(0)}(\vec{r})$		$+$	$-$	scalar	$J^{(0)}J^{(0)}$
	$\vec{\nabla}J^{(0)}(\vec{r})$	$+$	$+$	vector	
$\vec{J}(\vec{r})$		$+$	$-$	vector	$\vec{J}^2$
	$\vec{\nabla}\cdot\vec{J}(\vec{r})$	$+$	$+$	scalar	$\rho\vec{\nabla}\cdot\vec{J}$
	$\vec{\nabla}\times\vec{J}(\vec{r})$	$+$	$+$	vector	
$J^{(2)}_{\mu\nu}(\vec{r})$		$+$	$-$	tensor	$\sum_{\mu\nu}J^{(2)}_{\mu\nu}J^{(2)}_{\mu\nu}$
$\vec{s}(\vec{r})$		$-$	$+$	vector	$\vec{s}^2$
	$\vec{\nabla}\cdot\vec{s}(\vec{r})$	$-$	$-$	scalar	$(\vec{\nabla}\cdot\vec{s})^2$
	$\vec{\nabla}\times\vec{s}(\vec{r})$	$-$	$-$	vector	$\vec{j}\cdot\vec{\nabla}\times\vec{s}$
	$\Delta\vec{s}(\vec{r})$	$-$	$+$	vector	$\vec{s}\cdot\Delta\vec{s}$
$\vec{j}(\vec{r})$		$-$	$-$	vector	$\vec{j}^2$
	$\vec{\nabla}\cdot\vec{j}(\vec{r})$	$-$	$+$	scalar	
	$\vec{\nabla}\times\vec{j}(\vec{r})$	$-$	$+$	vector	$\vec{s}\cdot\vec{\nabla}\times\vec{j}$
$\vec{T}(\vec{r})$		$-$	$+$	vector	$\vec{s}\cdot\vec{T}$
$\vec{F}(\vec{r})$		$-$	$+$	vector	$\vec{s}\cdot\vec{F}$

Complete local energy density

The energy density can be written in the following form:

$$\mathcal{H}(\vec{r}) = \frac{\hbar^2}{2m}\tau_0(\vec{r}) + \sum\nolimits_{t=0,1}\left(\chi_t(\vec{r}) + \breve{\chi}_t(\vec{r})\right),$$

The p-h and p-p interaction energy densities, $\chi_t(\vec{r})$ and $\breve{\chi}_t$, for t=0 depend quadratically on the isoscalar densities, and for t=1 – on the isovector ones. Based on general rules of constructing the energy density, one obtains

Mean field

$$\begin{aligned}
\chi_0(\vec{r}) &= C_0^\rho\rho_0^2 + C_0^{\Delta\rho}\rho_0\Delta\rho_0 + C_0^\tau\rho_0\tau_0 \\
&+ C_0^{J0}J_0^2 + C_0^{J1}\vec{J}_0^2 + C_0^{J2}\underline{\mathsf{J}}_0^2 + C_0^{\nabla J}\rho_0\vec{\nabla}\cdot\vec{J}_0 \\
&+ C_0^s\vec{s}_0^2 + C_0^{\Delta s}\vec{s}_0\cdot\Delta\vec{s}_0 + C_0^T\vec{s}_0\cdot\vec{T}_0 \\
&+ C_0^j\vec{j}_0^2 + C_0^{\nabla j}\vec{s}_0\cdot(\vec{\nabla}\times\vec{j}_0) \\
&+ C_0^{\nabla s}(\vec{\nabla}\cdot\vec{s}_0)^2 + C_0^F\vec{s}_0\cdot\vec{F}_0, \\
\chi_1(\vec{r}) &= C_1^\rho\vec{\rho}^2 + C_1^{\Delta\rho}\vec{\rho}\circ\Delta\vec{\rho} + C_1^\tau\vec{\rho}\circ\vec{\tau} \\
&+ C_1^{J0}\vec{J}^2 + C_1^{J1}\vec{\vec{J}}^2 + C_1^{J2}\vec{\underline{\mathsf{J}}}^2 + C_1^{\nabla J}\vec{\rho}\circ\vec{\nabla}\cdot\vec{\vec{J}} \\
&+ C_1^s\vec{\vec{s}}^2 + C_1^{\Delta s}\vec{\vec{s}}\cdot\circ\Delta\vec{\vec{s}} + C_1^T\vec{\vec{s}}\cdot\circ\vec{\vec{T}} \\
&+ C_1^j\vec{\vec{j}}^2 + C_1^{\nabla j}\vec{\vec{s}}\cdot\circ(\vec{\nabla}\times\vec{\vec{j}}) \\
&+ C_1^{\nabla s}(\vec{\nabla}\cdot\vec{\vec{s}})^2 + C_1^F\vec{\vec{s}}\cdot\circ\vec{\vec{F}},
\end{aligned}$$

Pairing

$$\begin{aligned}
\breve{\chi}_0(\vec{r}) &= \breve{C}_0^s|\breve{\vec{s}}_0|^2 + \breve{C}_0^{\Delta s}\Re(\breve{\vec{s}}_0^*\cdot\Delta\breve{\vec{s}}_0) \\
&+ \breve{C}_0^T\Re(\breve{\vec{s}}_0^*\cdot\breve{\vec{T}}_0) + \breve{C}_0^j|\breve{\vec{j}}_0|^2 \\
&+ \breve{C}_0^{\nabla j}\Re(\breve{\vec{s}}_0^*\cdot(\vec{\nabla}\times\breve{\vec{j}}_0)) \\
&+ \breve{C}_0^{\nabla s}|\vec{\nabla}\cdot\breve{\vec{s}}_0|^2 \\
&+ \breve{C}_0^F\Re(\breve{\vec{s}}_0^*\cdot\breve{\vec{F}}_0), \\
\breve{\chi}_1(\vec{r}) &= \breve{C}_1^\rho|\breve{\vec{\rho}}|^2 + \breve{C}_1^{\Delta\rho}\Re(\breve{\vec{\rho}}^*\circ\Delta\breve{\vec{\rho}}) \\
&+ \breve{C}_1^\tau\Re(\breve{\vec{\rho}}^*\circ\breve{\vec{\tau}}) \\
&+ \breve{C}_1^{J0}|\breve{\vec{J}}|^2 + \breve{C}_1^{J1}|\breve{\vec{\vec{J}}}|^2 \\
&+ \breve{C}_1^{J2}|\breve{\vec{\underline{\mathsf{J}}}}|^2 \\
&+ \breve{C}_1^{\nabla J}\Re(\breve{\vec{\rho}}^*\circ\vec{\nabla}\cdot\breve{\vec{\vec{J}}}).
\end{aligned}$$

where × stands for the vector product

Mean-field equations

Mean-field potentials:

$$\Gamma_t^{\text{even}} = -\vec{\nabla}\cdot M_t(\vec{r})\vec{\nabla} + U_t(\vec{r}) + \tfrac{1}{2i}\left(\overleftrightarrow{\nabla}\sigma\cdot\overleftrightarrow{B}_t(\vec{r}) + \overleftrightarrow{B}_t(\vec{r})\cdot\overleftrightarrow{\nabla}\sigma\right)$$
$$\Gamma_t^{\text{odd}} = -\vec{\nabla}\cdot(\vec{\sigma}\cdot\vec{C}_t(\vec{r}))\vec{\nabla} + \vec{\sigma}\cdot\vec{\Sigma}_t(\vec{r}) + \tfrac{1}{2i}(\vec{\nabla}\cdot\vec{I}_t(\vec{r}) + \vec{I}_t(\vec{r})\cdot\vec{\nabla}) - \vec{\nabla}\cdot\vec{D}_t(\vec{r})\vec{\sigma}\cdot\vec{\nabla}$$

where

$$U_t = 2C_t^{\rho}\rho_t + 2C_t^{\Delta\rho}\Delta\rho_t + C_t^{\tau}\tau_t + C_t^{\nabla J}\vec{\nabla}\cdot\vec{J}_t,$$
$$\vec{\Sigma}_t = 2C_t^{s}\vec{s}_t + 2C_t^{\Delta s}\Delta\vec{s}_t + C_t^{T}\vec{T}_t + C_t^{\nabla j}\vec{\nabla}\times\vec{j}_t, -2C_t^{\nabla s}\Delta\vec{s}_t + C_t^{F}\vec{F}_t - 2C_t^{\nabla s}\vec{\nabla}\times(\vec{\nabla}\times\vec{s}_t)$$
$$M_t = C_t^{\tau}\rho_t,$$
$$\vec{C}_t = C_t^{T}\vec{s}_t,$$
$$\overleftrightarrow{B}_t = 2C_t^{J}\overleftrightarrow{J}_t - C_t^{\nabla J}\overleftrightarrow{\nabla}\rho_t,$$
$$\vec{I}_t = 2C_t^{j}\vec{j}_t + C_t^{\nabla j}\vec{\nabla}\times\vec{s}_t,$$
$$\vec{D}_t = C_t^{F}\vec{s}_t,$$

Neutron and proton mean-field Hamiltonians:

$$h_n = -\frac{\hbar^2}{2m}\Delta + \Gamma_0^{\text{even}} + \Gamma_0^{\text{odd}} + \Gamma_1^{\text{even}} + \Gamma_1^{\text{odd}},$$
$$h_p = -\frac{\hbar^2}{2m}\Delta + \Gamma_0^{\text{even}} + \Gamma_0^{\text{odd}} - \Gamma_1^{\text{even}} - \Gamma_1^{\text{odd}}.$$

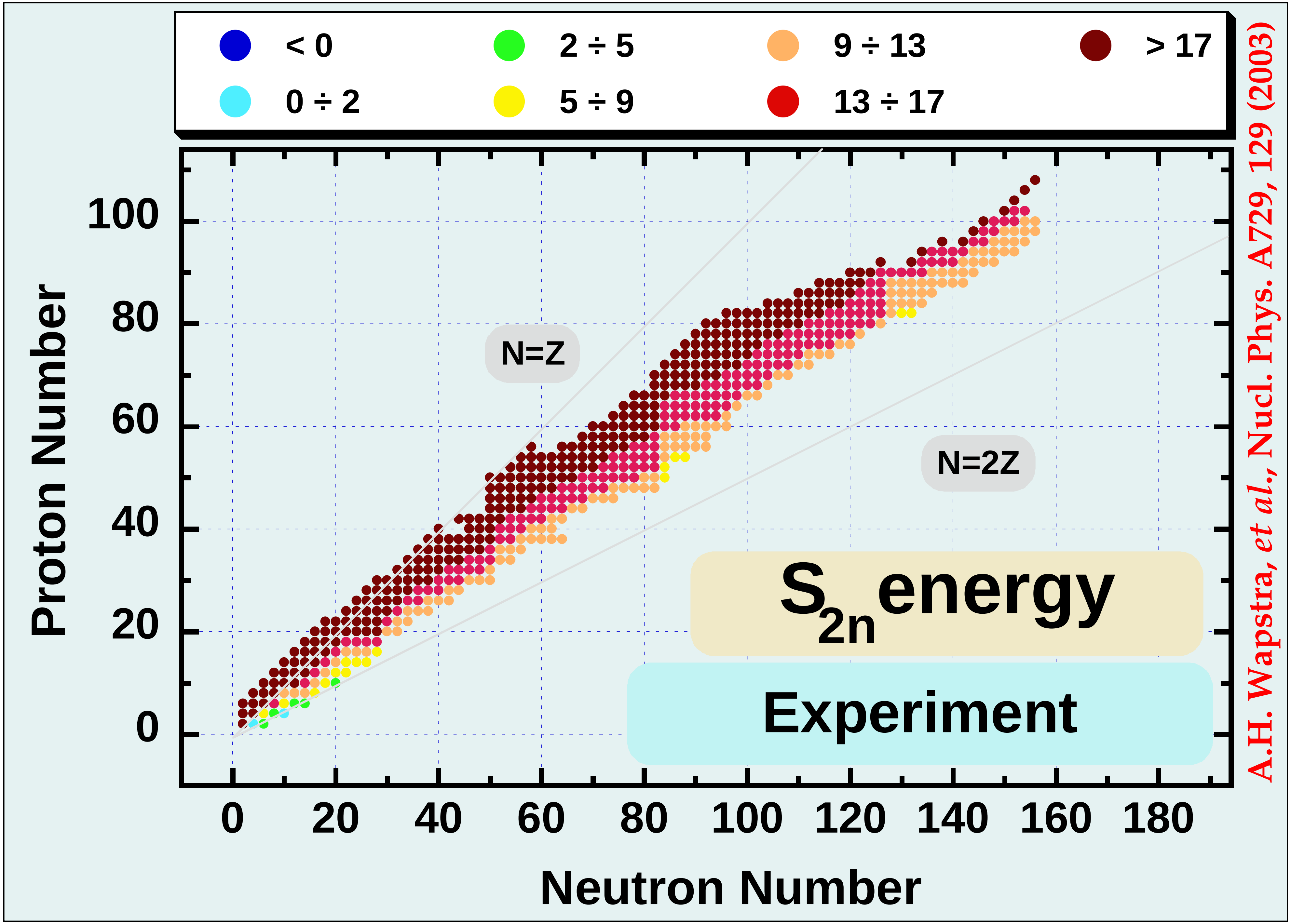
HF equation for single-particle wave functions:

$$h_\alpha\psi_{i,\alpha}(\vec{r}\sigma) = \epsilon_{i,\alpha}\psi_{i,\alpha}(\vec{r}\sigma),$$

where i numbers the neutron (α=n) and proton (α=p) eigenstates.

Nuclear densities as composite fields

Density distributions of **matter, spin, and current** in a nucleus can be used as fields defining new degrees of freedom that describe nucleus as a composite particle. In terms of these fields, the most general **energy density functional** can be constructed by using symmetry arguments. The functional depends on a number of **coupling constants**, which have to be either adjusted to the experimental data or determined from a higher-level theory. In the spirit of the **Effective Field Theory**, the energy density can be supplemented by higher-order contact terms, which amount to a density dependence of the coupling constants. These terms take into account **high-energy phenomena** that are not resolved when one looks at nuclear phenomena within the scale of the nucleon binding (~10 MeV). On the other hand, all effects within the **low-energy scale** of collective excitations (~1 MeV) have to be treated explicitly (deformations, zero-point motion, pairing corrections, symmetry restoration, etc.).



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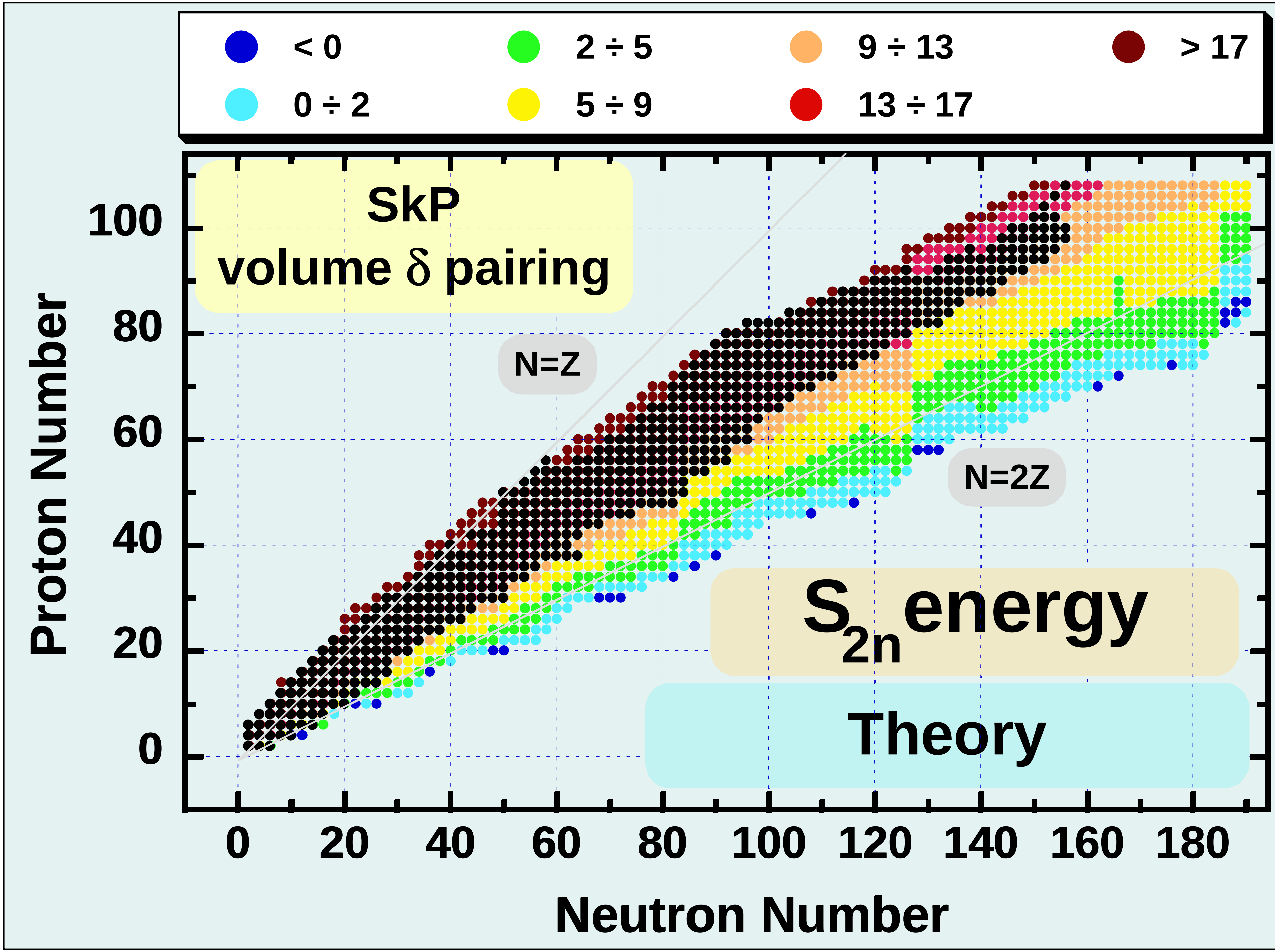
Rare Isotope Accelerator (RIA)

RIA: CONNECTING NUCLEI with the UNIVERSE



M.V. Stoitsov, *et al.*, Phys. Rev. C68, 054312 (2003)

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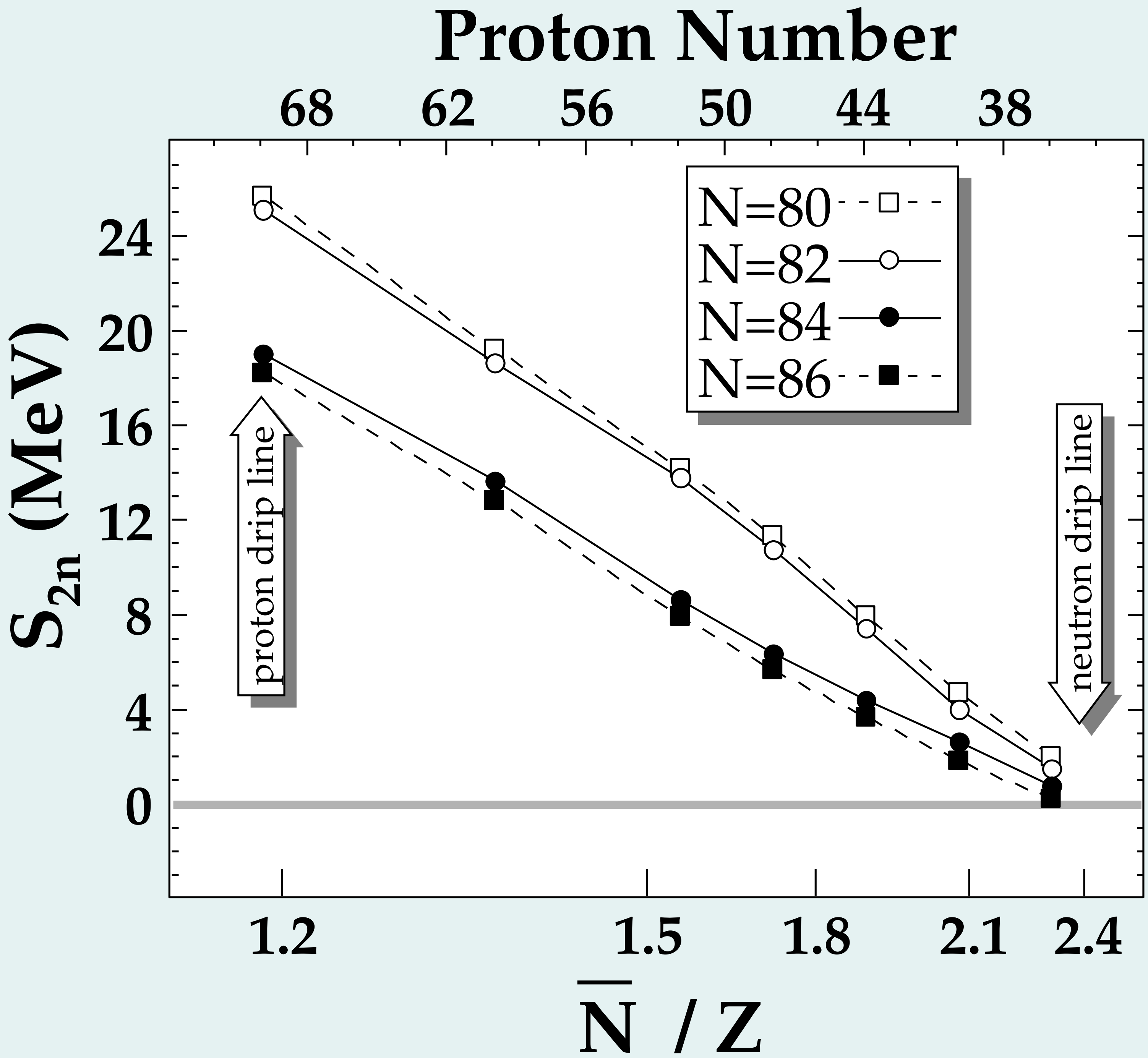


J.Dobaczewski and W.Nazarewicz
Phil. Trans. R. Soc. Lond. A356, 2007 (1998)

Two-neutron separation energy S_{2n}

$$S_{2n}(N,Z) = E(N,Z) - E(N-2,Z)$$

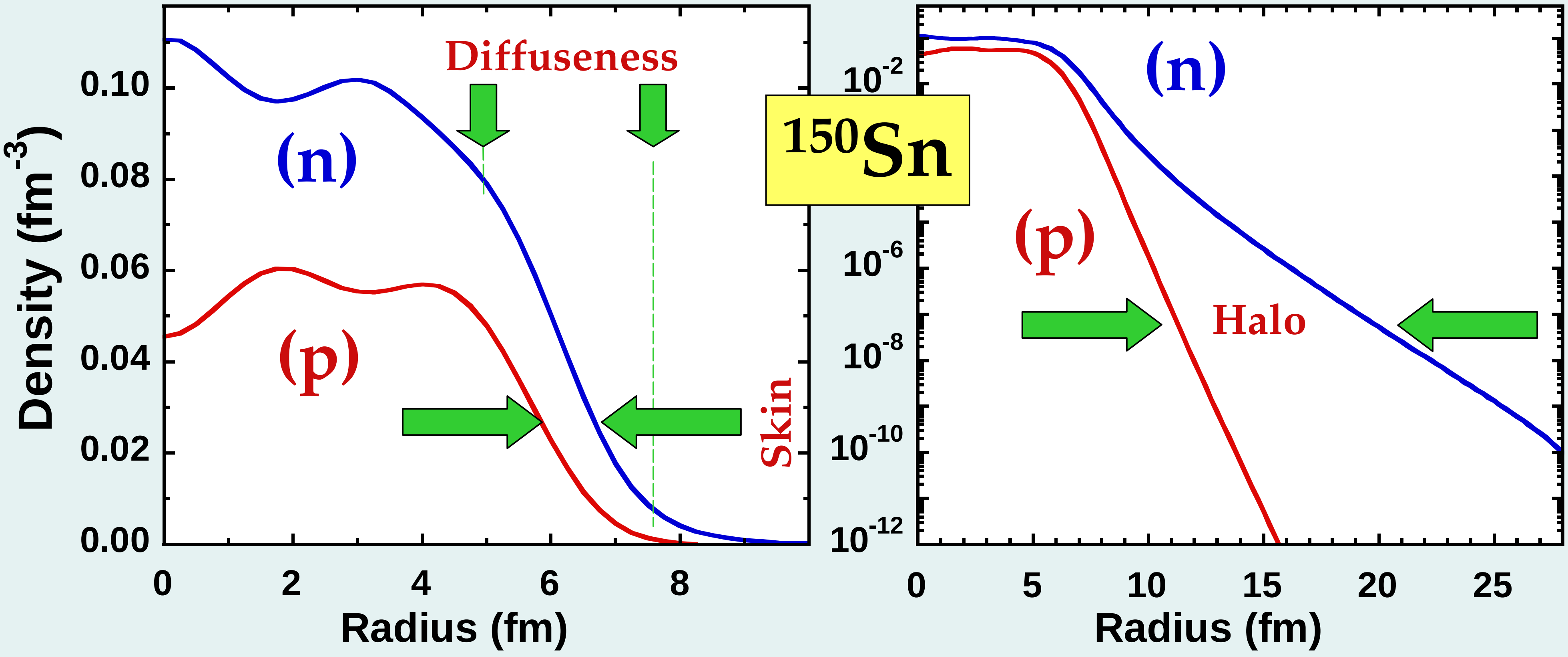
where E(N,Z) is the ground-state energy (negative) of the nuclide with N neutrons and Z protons.

Two-neutron separation energies are insensitive to pairing correlations, because complete pairs are simultaneously removed.

Two-neutron separation energies exhibit jumps when crossing magic neutron numbers. The magnitude of the jump is a measure of the neutron magic shell gap for a given proton number.

Two-neutron unbound nuclei may exist beyond the neutron drip line due to the shape coexistence phenomenon.

Neutron & proton density distributions



The Helm model

Helm density distribution:

$$\rho^{(\mathrm{H})}(\vec{r})=\int d^3\vec{r}'\,\rho_0\Theta(R_0-|\vec{r}'|)\left(\frac{1}{(2\pi\sigma^2)^{-3/2}}e^{-\frac{|\vec{r}-\vec{r}'|^2}{2\sigma^2}}\right)$$

The Fourier transform of the Helm density is a product of the Fourier transforms of the Gaussian profile and step function, i.e.,

$$F^{(\mathrm{H})}(q)=\frac{3}{R_0q}j_1(qR_0)e^{-\frac{\sigma^2q^2}{2}}.$$

Since the central density $\rho_0=\frac{3N}{4\pi R_0^3}$ is fixed by the particle-number condition, the Helm density has 2 parameters: R_0 and σ. These parameters can be determined from the first zero q_1 and the first maximum q_m of the Fourier transform $F^{(\mathrm{H})}(q)$:

$$R_0=4.49341/q_1,$$

$$\sigma^2=\frac{2}{q_m^2}\ln\frac{3R_0^2j_1(q_mR_0)}{R_0q_mF(q_m)}.$$

By comparing the root-mean-squared (rms) radius of the real density distribution:

$$R_{\mathrm{geom}}=\sqrt{\frac{5}{3}}R_{\mathrm{rms}}=\sqrt{\frac{5}{3}}\sqrt{\frac{\int d^3\vec{r}\,r^2\rho(\vec{r})}{\int d^3\vec{r}\,\rho(\vec{r})}}$$

with that of the Helm density distribution:

$$R_{\mathrm{Helm}}=\sqrt{\frac{5}{3}}R_{\mathrm{rms}}^{(\mathrm{H})}=\sqrt{(R_0^2+5\sigma^2)}$$

we obtain a simple quantitative measure of the halo size:

$$\delta R_{\mathrm{halo}}\equiv R_{\mathrm{geom}}-R_{\mathrm{Helm}}$$

By comparing the neutron and proton Helm radii, we obtain a simple quantitative measure of the skin:

$$\delta R_{\mathrm{skin}}\equiv R_{\mathrm{Helm}}(n)-R_{\mathrm{Helm}}(p)$$

S. Mizutori, et al., Phys. Rev. C61, 044326 (2000)

Neutron & proton density distributions

Diffuseness of the density distribution is equal to the difference of radii where the density has values of 10% and 90% of the average central density. Better quantitative measure of the diffuseness can be formulated within the Helm model (step-like distribution folded with a Gaussian).

Neutron skin size is equal to the difference of radii where the neutron and proton densities have values of 50% of their respective average central densities Better quantitative measure of the skin can be formulated within the Helm model as the difference of neutron and proton diffraction radii.

Neutron halo size is the difference between the neutron root-mean-squared and diffraction radii. Properties of the neutron halo are governed by the asymptotic features of tails of quantal wave functions.

Asymptotic properties of mean-field single-particle wave functions

Single-particle mean-field wave function $\phi(r)$ is a solution of the Schrödinger equation:

$$\left(-\frac{\hbar^2}{2m}\Delta + V(r)\right)\phi(r) = \epsilon\phi(r).$$

For $\ell=0$ and potential $V(r)$ vanishing at infinity the asymptotic solution reads:

$$\phi(r) \simeq \exp(-\kappa r)/r \quad \text{for} \quad r \to \infty$$

where

$$\kappa = \sqrt{-2m\epsilon}/\hbar$$

For small binding energy ϵ, the root-mean-squared radius

$$R^2_{\rm rms} = \frac{\int 4\pi r^2 dr r^2 \phi^2(r)}{\int 4\pi r^2 dr \phi^2(r)}$$

is determined by the asymptotic solution, and hence:

$$R^2_{\rm rms} \simeq \frac{\int dr r^2 \exp(-2\kappa r)}{\int dr \exp(-2\kappa r)}$$

The integrals are elementary:

$$\int dr \exp(-2\kappa r) \sim \frac{1}{2\kappa}$$

$$\int dr r^2 \exp(-2\kappa r) \sim \frac{1}{4}\frac{d^2}{d\kappa^2}\frac{1}{2\kappa} = \frac{1}{16\kappa^3},$$

and hence

$$R_{\rm rms} \simeq \frac{1}{\sqrt{8}\kappa} = \frac{\hbar}{4\sqrt{m}}(-\epsilon)^{-1/2}$$

while for $\ell \neq 0$

$$R_{\rm rms} \simeq \frac{\hbar}{4\sqrt{m}}(-\epsilon)^{-1/2+\ell/4}$$

Asymptotic properties of paired quasiparticle wave functions

Quasiparticle two-component wave function $(\phi_1(r),\phi_2(r))$ is a solution of the HFB equation:

$$\begin{aligned}\left(-\frac{\hbar^2}{2m}\Delta + V(r) - \lambda\right)\phi_1(r) + \Delta(r)\phi_2(r) &= E\phi_1(r)\\ \left(-\frac{\hbar^2}{2m}\Delta + V(r) - \lambda\right)\phi_2(r) - \Delta(r)\phi_1(r) &= -E\phi_2(r)\end{aligned}.$$

For $\ell=0$ and potentials $V(r)$ and $\Delta(r)$ vanishing at infinity the asymptotic solution for the lower component $\phi_2(r)$ (which defines the particle density) reads:

$$\phi_2(r) \simeq \exp(-\kappa r)/r \quad \text{for} \quad r \to \infty$$

where

$$\kappa = \sqrt{2m(E-\lambda)}/\hbar$$

For small Fermi (λ) and quasiparticle (E) energies, the root-mean-squared radius behaves as

$$R_{\text{rms}} \simeq \frac{\hbar}{4\sqrt{m}}(E-\lambda)^{-1/2+\ell/4}$$

i.e., even at the drip line ($\lambda = 0$) the radius stays finite for paired orbitals (pairing gap in the quasiparticle spectrum requires $E>\Delta$).

Asymptotic properties

Asymptotic properties of nuclear wave functions determine the size and characteristics of **nuclear halos**. By assuming that neutrons are bound only by the mean-field potential, one concludes that **weakly-bound s and p waves** induce halos of infinite size. Nuclear correlations induce additional binding, which qualitatively modifies this conclusion. In particular, **pairing correlations** lead to additional binding of a weakly-bound neutron within the pair, and hence always lead to finite halos (**the pairing anti-halo effect**). Nevertheless, halos obtained by occupying weakly-bound s and p states are larger than those corresponding to higher orbital momenta. Since the pairing correlations always induce some **non-zero occupation of weakly-bound s and p states**, the halo phenomenon may, in fact, occur more often than one might have guessed by using the pure mean-field picture without correlations.