Tetrahedral Symmetry in Actinide and New Forms of Nuclear Shape Coexistence

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Group-theory arguments lead to predictions of stabilizing effects of tetrahedral symmetry in terms of new classes of nuclear shapes. The latter imply an existence of negative-parity rotational bands with vanishing $E2$ transitions while preserving the rotor relation $E_I \sim I(I+1)$. This is predicted to occur, among others, in heavy nuclei with $Z = 90 - 94$ and $N \geq 136$. We examine these predictions using the experimental results existing in the literature. Both theoretical and experimental arguments suggest that the lighter Th and U isotopes manifest 'traditional' pear-shape (Y_{30}) octupole geometry while heavier Th and U and several nuclides with $Z > 92$ are presumably tetrahedral.

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Unlike in molecular and condensed matter physics - where the use of point groups constitutes a standard tool - in sub-atomic physics the point-group theory has been seldom employed. An exception is provided by discussions of the D_{2h}-symmetry, which is usually masked behind the term "triaxiality." This *implicit* use of the D_{2h}-symmetry has been motivated by the properties of the early prototype of the nuclear mean field: the D_{2h}-symmetric triaxial harmonic oscillator.

Applications of point groups to sub-atomic physics focusing on T_d^D (tetrahedral) and O_h^D (octahedral) double point-group symmetries have been recently discussed in Ref. [1], where the references to the previous publications can be found. By convention these two symmetries are sometimes referred to as 'high-rank' because they both lead to the appearance of four-fold degenerate nucleonic levels in the corresponding deformed mean-fields, rather than the usual two-fold Kramers degeneracy. It is worth emphasizing at this point that the four-fold degeneracies are the highest mathematically permitted in the case of the *deformed mean-field Hamiltonians of Fermions.*

One of the consequences of the exact T_d^D and/or O_h^D mean-field symmetries is the vanishing of the nuclear quadrupole moments, cf. Ref. [1] and the following discussion. Tetrahedral-symmetric nuclei have a well-defined spatial orientation as they are obviously non-spherical. This implies an existence of collective rotational bands. However, these bands differ from those known in the literature *since in the exact symmetry limit the associated nuclear quadrupole moments vanish, implying vanishing of collective E2 transitions.* As discussed below there exist at least three well recognized quantum mechanisms that may induce small symmetry-breaking effects. The latter may occasionally be sufficiently strong to allow for extraordinarily weak $E2$ transitions.

Mean-field calculations focusing on tetrahedral symmetry for actinide nuclei show a huge proton gap of over 3 MeV at a significant tetrahedral deformation - cf. Fig. 4 in Ref. [2]. However, a four-fold degenerate level lies in the middle of this gap giving rise to two ~2 MeV gaps at $Z = 90$ and 94. This suggests the existence of strong tetrahedral symmetry driving shell effects in nuclei with $Z = 90 - 94$. The same calculations predict a significant gap of tetrahedral symmetry at $N = 136$. (For details about 'magic numbers' generated by tetrahedral symmetry see Refs. [2–4].) It is well known that large spherical gaps not only have a strong effect on nuclei with magic numbers, but they also influence the neighboring nuclei to have nearly spherical shapes. A similar symmetry-driving effect is expected in nuclei near those with the strong tetrahedral 'magic' numbers. Consequently it will be instructive to examine possible effects of tetrahedral symmetry in nuclei with Z around $Z_o = 92$, i.e. thorium, uranium, and plutonium with neutron numbers at and in excess of $N_o = 136$. (For $N < 136$ the impact of the very strong spherical gap at $N = 126$ is strengthened, counteracting tetrahedral symmetry effects sought here.) Our analysis shows that bands with vanishing $E2$ transitions, a possible signal of tetrahedral symmetry, are numerous in actinides with $Z = 90$, 92, and 94 and $N \geq 136$, but also present in at least three Pu- and two Cm-isotopes. In the following we focus on uranium nuclei for illustra-

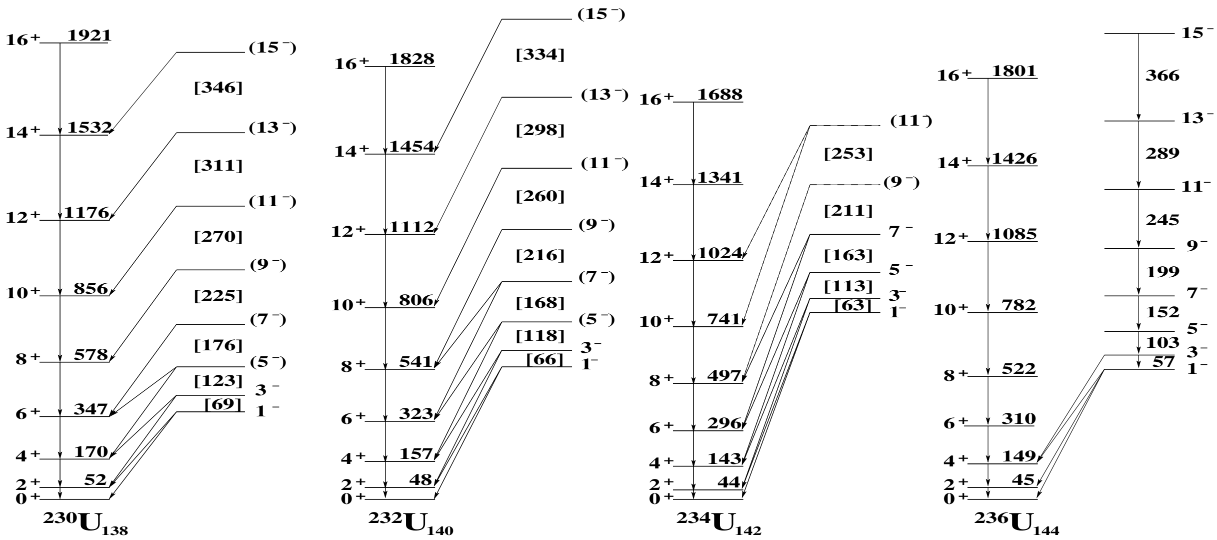

FIG. 1: Experimental ground-state and negative-parity bands in $^{230-236}$U even-even nuclei - all studied under the same experimental conditions in Ref. [5] using e-γ and *e*-*e* spectroscopy. These electron measurements play an important role due to the high values of the electron-conversion coefficients for the $E2$ transitions; they also help to reduce the fission background considerably. In $^{230-234}$U, the presence of the negative-parity bands has been established only through the observation of the $E1$ transitions to the ground-state bands. The results for $^{230-232}$U have been confirmed in Ref. [6] where two E2-transitions depopulating states 13^- and 11^- in ^{230}U are mentioned but considered uncertain. In contrast, in ^{236}U no $E1$ transitions for states above $I = 3$ have been observed although this isotope has been studied with 17 different types of nuclear reactions, cf. Ref. [7]. The numbers in square brackets give the energies of unobserved $E2$ transitions.

tion. Their relevant partial decay schemes are shown in Fig. 1.

Let us emphasize that measurements for nuclei in Fig. 1 have been performed using a common detection system. In $^{230-234}$U, no inband $E2$ transitions were observed and the sequences were based solely on the observed $E1$ transition feeding the ground-state structure. In contrast, the inband $E2$ transitions were found in ^{236}U and the only $E1$ interband transitions are observed for the $I = 1$ and 3 states. Comparison of these results obtained *under the same experimental conditions* suggests that the vanishing $E2$ transitions are a manifestation of a physical mechanism rather than a result of a limitation of the experiment. This situation resembles the case in $^{156}_{64}$Gd$_{92}$ whose $Z = 64$ coincides with one of the tetrahedral magic numbers and $N = 92$ coincides with the number of protons in uranium nuclei. Indeed, in ^{156}Gd over a dozen different experiments have been performed (cf. Ref. [9]) using various reactions and detection systems. None of them has succeeded in measuring the $E2$ transitions below $I^\pi = 9^-$ in the negative-parity band which was interpreted in Ref. [1] as being based upon a tetrahedral shape.

At this point let us first present a *heuristic argument* that will be followed by ones based on the microscopic calculations and experimental results. We attribute octupole deformation to the negative-parity bands in question, but since the $E2$ transitions are missing we deduce that quadrupole deformation may have vanished. Fig. 2 shows quadrupole moments induced by $\alpha_{3\mu}$ octupole deformations, calculated using the realistic Woods-Saxon Hamiltonian. The results show that all but α_{32} (tetrahedral) deformation give non-zero quadrupole moments and thus the tetrahedral symmetry can give a possible explanation for the origin of the vanishing $E2$ transitions.

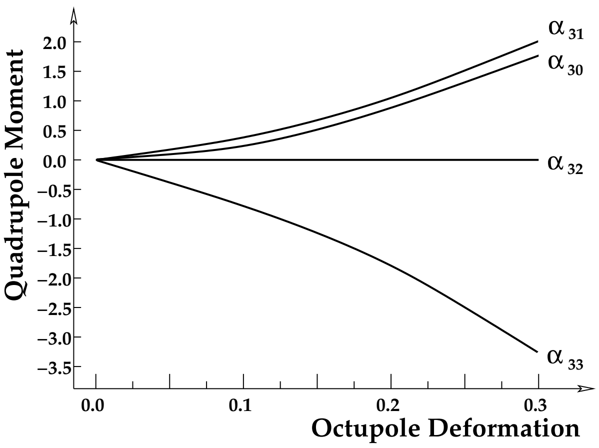

FIG. 2: Example of charge Q_{20} moments [e$\cdot b$] as functions of the $\alpha_{3\mu}$ multipole deformation parameters for ^{230}U. For $\alpha_{31} \neq 0$ there is also Q_{22} moment that is non-vanishing (but of no further interest here). In all cases $\alpha_{2\mu}$=0.

Let us first address an important mathematical aspect related to point-group symmetries of the mean-field solutions - here in the case of the tetrahedral symmetry. As it is well known in group theory, the double tetrahedral point group T_d^D is a sub-group of the octahedral double point-group O_h^D. It then follows that the Hamiltonian invariant under O_h^D is at the same time invariant under T_d^D. A practical consequence for the total energy calculations like those in Fig. 2 (top) is that we need to take into account, in general, both the tetrahedral deformation, α_{32}, and the octahedral ones at the same time. It can be demonstrated that the two lowest-order octahedral deformations are obtained by combining the expansion coefficients $\alpha_{44} = \alpha_{40} * \sqrt{5/14}$ and $\alpha_{64} = \alpha_{60} * \sqrt{7/2}$ in terms of standard spherical harmonics, all other $\alpha_{4\mu}$

TABLE I: Test for octahedral deformations compatible with tetrahedral deformation in the nucleus ^{226}Th with two Skyrme parameterizations. At the exact tetrahedral symmetry the results in columns 3 and 4 should coincide (similarly those in columns 5 and 6). Moments in units $[(10\text{ Fermi})^\lambda]$.

Force	Q_{32}	$Q_{40}\sqrt{5/14}$	Q_{44}	$Q_{60}\sqrt{7/2}$	Q_{64}
SkM*	3.4166	0.5582	0.5583	0.1537	0.1538
SLy4	3.3353	0.5471	0.5617	0.1306	0.1341

and $\alpha_{6\mu}$ being set to zero. In Table I the results of the Hartree-Fock unconstrained calculations just giving the tetrahedral local minima are given for the tetrahedral symmetry solution in ^{226}Th. These results show that indeed the superposition of the tetrahedral and the octahedral symmetries ('summing up' to the final tetrahedral symmetry as argued above) are to be expected in the realistic calculations.

We have performed realistic calculations of the potential energy surfaces by using the macroscopic-microscopic method with the universal Woods-Saxon potential parametrization, which we used in Ref. [1]. Results that are shown in Fig. 3 (top) were obtained by minimizing the total energy at each $\{\alpha_{20}, \alpha_{32}\}$ point over both octahedral deformations introduced above. The effect of the second order octahedral deformation is to lower the tetrahedral minimum energy by over 500 keV. The potential energy surfaces shown in Fig. 3 (bottom) were minimized over the hexadecapole deformation, conforming with the 'traditional' calculations of this kind published in the past.

These results confirm the expectation of the presence of the static tetrahedral symmetry minimum and the absence of the static ('traditional') pear-shape minimum. Indeed, a well defined tetrahedral symmetry minimum corresponding to $\alpha_{32} \approx 0.15$ and vanishing quadrupole deformation deserves attention. This minimum is separated from the ground-state minimum with the barrier of the order of 1 MeV. In view of these results we suggest the interpretation of the negative-parity bands with vanishing $E2$ transitions (like those in Fig. 1 for uranium as well as in other nuclei of the actinide region) as a possible manifestation of the nuclear tetrahedral symmetry.

Our microscopic calculations similar to those in Fig. 3 show that the character of the negative-parity bands in actinide nuclei changes when the neutron number increases. At low neutron numbers these calculations confirm the earlier interpretation of the band structure in terms of the Y_{30} 'pear-shape' octupole configurations (see Ref. [8] and references therein), while for heavier isotopes the Y_{32} 'pyramid shape' tetrahedral interpretation seems the correct one.

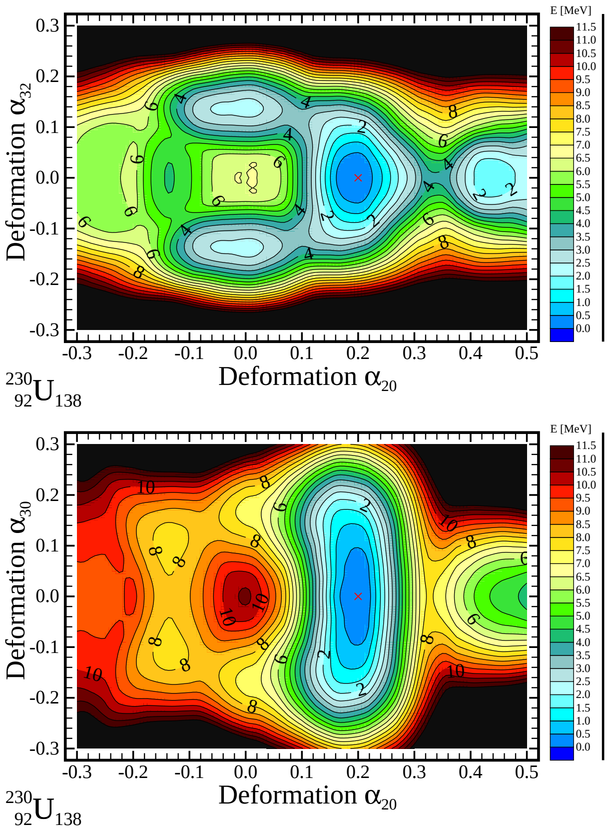

FIG. 3: (color on line) Potential energy surfaces for ^{230}U as a function of the quadrupole $[\alpha_{20}]$ and tetrahedral octupole $[\alpha_{32}]$ (top) and pear-shape octupole $[\alpha_{30}]$ (bottom) deformations.

Our calculations also show the presence of tetrahedral minima for thorium ($Z = 90$) nuclei. In Table II, we collect what is known from the literature on the measured $B(E2)/B(E1)$ intra-band/inter-band reduced transition strength ratios for the negative-parity bands in Th isotopes and compare them to the corresponding values for two Gd isotopes that we have earlier associated with tetrahedral minima [1]. The small and rather constant ratio of the order of 0.4×10^6 for Th isotopes with $N \leq 134$ can be attributed to the simultaneous presence of a static quadrupole deformation combined with the collective pear-shape octupole effect. As seen in Table II, this ratio increases by a factor of 10 or more with increasing neutron number for Th nuclei. We attribute this increase to a difference in the underlying intrinsic structure of the negative-parity bands between light ($N < 136$) and heavy ($N \geq 136$) thorium nuclei, involving tetrahedral shapes for the heavier isotopes.

TABLE II: Experimental ratios $B(E2)/B(E1) \times 10^6 [fm^2]$, for intra-band $E2$ transitions vs. inter-band $E1$ transitions. Meaning of symbols: "-" - state has not been observed; "?" - intensities to calculate the branching ratios not available; "(?)" - known information insufficient to obtain error bars. Sources: ^{220}Th [10]; ^{222}Th [11]; $^{224-228}$Th and ^{152,156}Gd [12].

State	^{220}Th	^{222}Th	^{224}Th	^{226}Th	^{228}Th	^{152}Gd	^{156}Gd
21^-	no E1	0.2(?)	-	-	-	-	no E1
19^-	no E1	0.3(?)	-	2.0(5)	-	-	50(10)
17^-	no E1	0.4(2)	0.3(1)	2.3(4)	no E1	no E1	16(3)
15^-	0.9(2)	0.4(2)	0.4(1)	2.0(4)	no E1	no E1	6(2)
13^-	0.2(1)	0.3(2)	0.5(1)	?	16(2)	14(?)	7(2)
11^-	0.5(1)	0.4(2)	0.4(1)	2.0(2)	13(1)	4(?)	15(7)
9^-	0.4(1)	0.4(2)	?	2.0(2)	14(1)	4(?)	no E2
7^-	0.4(1)	0.4(3)	?	?	no E2	no E2	no E2

Both ^{228}Th and ^{156}Gd exhibit a trend of $B(E2)/B(E1)$ values that is understandable in terms of a tetrahedral shape - small or zero for the lowest spins and increasing to values of $(5-50) \times 10^6$ for medium spins. At the exact tetrahedral symmetry limit both the quadrupole and the dipole moments within the mean-field solutions tend to zero. However, there are at least three mechanisms leading to *dynamical shape polarizations* [13] and thus small non-zero values of the $B(E1)$ and $B(E2)$ probabilities: a) zero-point motion in the quadrupole degrees of freedom; b) gradual Coriolis alignment of individual nucleonic angular momenta with the axis of collective rotation; c) presence of 'valence nucleons' occupying single-particle states above the magic tetrahedral gaps. Due to these effects, some $E2$ and $E1$ transitions are non-zero in nuclei exhibiting tetrahedral shapes and lead, according to our interpretation, to the trend of $B(E2)/B(E1)$ values seen in Table II for ^{228}Th and ^{156}Gd.

Even larger $B(E2)/B(E1)$ values are expected for negative-parity two-quasiparticle bands. For example, the lowest-lying negative-parity band in the even-even W isotopes interpreted in the literature as of two-quasiparticle structure has $B(E2)/B(E1)$ values in the range of $(40-3000) \times 10^6$ [12], reflecting enhanced $E2$ transition rates for these well deformed nuclei and $E1$ rates that are not enhanced, because of non-collective (single-quasi-particle) origin. This leads us to the suggestion that $B(E2)/B(E1)$ ratios provide a valuable test of the nature of the underlying symmetries, geometrical shapes and/or quasiparticle-excited structure of the band in question, with values falling in one of three ranges: around ~0.5 for the traditional pear-shape octupole deformations, 1-to-2 orders of magnitude higher for medium spins in bands where no $E2$ transitions are found at low-spins (the suggested tetrahedral structures), and finally 1-to-2 orders of magnitude higher still for two-quasiparticle negative-parity bands for normal shapes. Our calculations of these exact ratios for bands of different tetrahedral deformations are continuing and will be presented elsewhere.

In summary, following theoretical predictions and the experimental results we suggest that tetrahedral symmetry is very likely present in heavy actinide isotopes leading to a new form of nuclear shape coexistence.

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