EPJ manuscript No.
(will be inserted by the editor)

Skyrme-QRPA Calculations of Multipole Strength in Exotic Nuclei

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Received: date / Revised version: date

Abstract. We present test calculations of the quasiparticle random-phase approximation with Skyrme and delta-pairing forces. We examine the convergence of solutions in the isoscalar 0^+ channel as we increase the number of single-quasiparticle states, and the separation of spurious states from physical excited states in the isoscalar 1^- channel. Our calculation is fully self-consistent as it neglects no component of the interaction. We focus on Sn isotopes near the two-neutron drip line.

PACS. 21.30.Fe – 21.60.Jz – 24.30.Cz

1 Introduction

One of the most important subjects in nuclear structure is the nature of nuclei near the particle drip lines. In these regions, some of the basic concepts developed in studies of stable nuclei may need to be modified. An urgent task for theoreticians is to develop methods which are reliable enough to investigate those unstable nuclei. The Quasiparticle Random-Phase Approximation (QRPA) is one of the general and well-developed methods for calculating excited states, and we expect it to work well in all nuclei. (The approximation is reliable only in the small-amplitude limit [1].) A fully self-consistent QRPA calculation with a Skyrme force, however, is not easy because of technical difficulties, and in typical applications self-consistency is broken and/or some components of the interaction neglected (see [2]). In this paper, we exhibit a self-consistent QRPA calculation and examine its accuracy. This is important preparation for an investigation of exotic nuclei that is free from theoretical ambiguities.

In the next section, we explain our procedure for solving the QRPA equations. Results of calculations are illustrated in Sec. 3, while Sec. 4 contains conclusions.

2 Calculation

Our QRPA calculation is based on the "matrix formulation" (see e.g [1]), with spherical symmetry assumed. The explicit expressions for the dynamical equations and matrix elements of the interactions, as well as the definitions of the transition operators, are given in [2]. First, we solve the Hartree-Fock-Bogoliubov (HFB) equation with the method of Ref. [3], in which discretized continuum-energy quasiparticle wave functions are obtained in a spherical box. Then we obtain canonical-basis wave functions by diagonalizing the nuclear one-body density matrix. We calculate the quasiparticle energies, matrix elements of the interaction, and uv-factors of the special Bogoliubov transformation within this basis, and then use those quantities to obtain the Hamiltonian matrix of the QRPA, which we diagonalize to get QRPA wave functions. Because the box makes our continuum solutions discrete, we introduce a width parameter[1] when we display strength functions.

3 Results

To test the applicability of our method near the neutron drip line, we performed calculations for ^{174}Sn, which the Skyrme parameter set SkM* with volume-type delta pairing interaction places very close to the two-neutron drip line. The results in the isoscalar 0^+ channel are shown in Fig. 1, which displays three curves calculated within

[1] This parameter depends on the eigenvalue E_k of the QRPA solution; it is constant (100 keV) if E_k is lower than the neutron threshold energy, and increases above threshold (to 3 MeV around $E_k = 20$ MeV) [2].

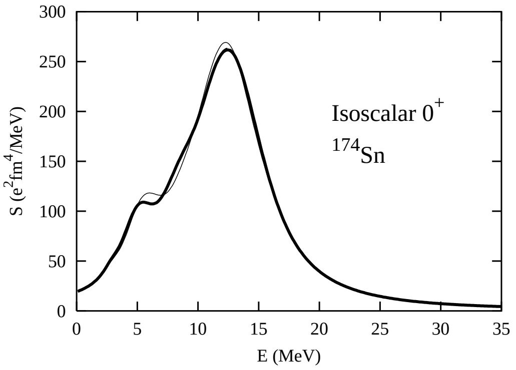

Fig. 1. Strength functions in the isoscalar 0^+ channel of ^{174}Sn. The thick (thin) line was obtained with $v^2_{\rm crit} = 10^{-16}$ (10^{-8}), for the neutrons, and $\varepsilon_{\rm crit} = 200$ (100) MeV, for the protons. The result with $v^2_{\rm crit} = 10^{-12}$ and $\varepsilon_{\rm crit} = 150$ MeV is almost identical to the thick line.

different single-particle spaces. We include single-particle states for which occupation probabilities are larger than a cutoff parameter $v^2_{\rm crit}$ (which is set to a very small value so that we omit little of physical significance) if the system is paired in the HFB ground state, or those for which the Hartree-Fock (HF) energies are lower than a cutoff parameter $\varepsilon_{\rm crit}$, if the system is unpaired. In ^{174}Sn, the neutrons are paired, and protons are unpaired in the HFB calculation. Figure 1 demonstrates that our solution converges when we make $v^2_{\rm crit}$ small enough and $\varepsilon_{\rm crit}$ large enough. Since the neutrons of ^{174}Sn are paired, there is a spurious state associated with particle-number nonconservation. We checked that the transition strengths for the particle-number operator are smaller than 10^{-5} to the real excited states.

The isoscalar 1^- mode is challenging technically because of spurious center-of-mass motion; a careful calculation is necessary to accurately separate the spurious state from real excited states. In calculations that are not fully self-consistent, the strength is often corrected by including a term $-\eta r Y_{1M}(\Omega)$ (where $\eta = (5/3)\langle r^2 \rangle$ with $\langle r^2 \rangle$ the mean value in the HFB ground state) in the isoscalar-dipole transition operator [4]. We performed calculations of the strength functions with and without the correction term and obtained identical results for real excited states; our 1^- solutions are therefore essentially free from contamination. In a perfect calculation, the spurious state would have zero energy and the correction term would remove strength only from this state. In our calculation of ^{174}Sn (^{120}Sn), even though the spurious state energy is 0.319 (0.713) MeV, the correction removes almost no strength except from this spurious state. This check is important for proving that the strong enhancement of strength at low energy in nuclei near the neutron drip line, illustrated in Fig. 2, is not an artifact of the calculation.

Finally we mention that the energy-weighted sum rules of the 0^+, 1^-, and 2^+ modes of ^{120,174}Sn are satisfied with errors of ± 1 % at most.

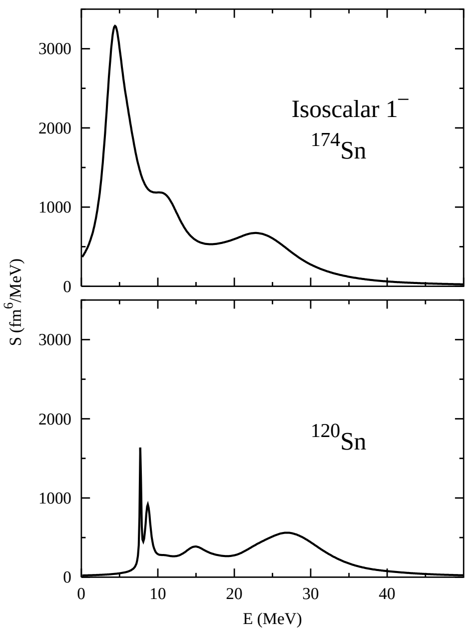

Fig. 2. Strength function in the isoscalar 1^- channel of ^{174}Sn (upper) and ^{120}Sn (lower). The low-energy strength is greatly enhanced in the drip-line nucleus.

4 Conclusion

We have performed fully self-consistent QRPA calculations with Skyrme and delta-pairing interactions, without neglecting any of their components, and showed them to be numerically accurate. Investigations of the structure of excited states near the neutron drip line, as well as more systematic calculations, are in progress.

This work was supported in part by the U.S. Department of Energy, Contract Nos. DE-FG02-97ER41019 (University of North Carolina), DE-FG02-96ER40963 (University of Tennessee), DE-AC05-00OR22725 with UT-Battelle, LLC (Oak Ridge National Laboratory), DE-FG05-87ER40361 (Joint Institute for Heavy Ion Research), and W-31-109-ENG-38 (Argonne National Laboratory); by the National Science Foundation, Contract No. 0124053 (U.S.-Japan Cooperative Science Award); by the Polish Committee for Scientific Research (KBN), Contract No. 1 P03B 059 27; and by the Foundation for Polish Science (FNP).

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