Localized projective measurement of a quantum field in non-inertial frames

2013 Class. Quantum Grav. 30 235006

(http://iopscience.iop.org/0264-9381/30/23/235006)

IOP PUBLISHING CLASSICAL AND QUANTUM GRAVITY

Class. Quantum Grav. **30** (2013) 235006 (17pp) doi:10.1088/0264-9381/30/23/235006

Localized projective measurement of a quantum field in non-inertial frames

,

Andrzej Dragan[1,2], **Jason Doukas**[3,6], **Eduardo Martín-Martínez**[4,5] **and David Edward Bruschi**[1]

[1] School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK
[2] Institute of Theoretical Physics, University of Warsaw, Hoża 69, 00-049 Warsaw, Poland
[3] National Institute for Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
[4] Institute for Quantum Computing, Department of Physics and Astronomy and Department of Applied Mathematics, University of Waterloo, 200 University Avenue W, Waterloo, Ontario N2 L 3G1, Canada
[5] Perimeter Institute for Theoretical Physics, 31 Caroline St N, Waterloo, ON, N2L 2Y5, Canada
[6] School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK

E-mail: dragan@fuw.edu.pl

Received 29 December 2012, in final form 3 July 2013
Published 7 October 2013
Online at stacks.iop.org/CQG/30/235006

Abstract
We propose a projective operator formalism that is well-suited to study the correlations of quantum fields in non-inertial frames and curved spacetimes. We generalize a Glauber model of detection of a single localized field mode to allow for making measurements in an arbitrary reference frame. We show that the model correctly reproduces the Unruh temperature formula of a single accelerated detector, and use it to extract vacuum entanglement by a pair of counter-accelerating detectors. This latter example is a proof of principle that this approach will be appropriate to further studies on the nature of entanglement in curved spacetimes and, in general, to model experimentally feasible scenarios in quantum field theory in non-inertial frames. Finally, as further confirmation of the validity of our approach, we introduce an explicit perturbative matter–radiation interaction model and show that under uniform acceleration it reproduces both the generalized Glauber model and the projective measurement results in the weak coupling regime.

PACS numbers: 04.62.+v, 03.67.−a

(Some figures may appear in colour only in the online journal)

1. Introduction

One of the most fascinating features of relativistic quantum fields is that the notion of a particle is frame dependent. Even if a given observer describes the field state as free from particles, the state of the same field described by another observer can be in fact populated by particles. This relativistic effect is a direct consequence of the Bogolyubov transformation between inertial and non-inertial reference frames and has inspired the development of the whole new field of relativistic quantum information. The main objective of this study is to find the consequences of relativistic effects on protocols describing the storage, processing and transmission of quantum information. Perhaps most striking of these investigations has been the realization that entanglement, which is a key resource in quantum information protocols, depends on the state of motion of the observers.

Seminal works on this topic [1] studied the effects of acceleration on the entanglement of global states of free fields. However, several issues remained unsettled, including how to properly describe a possible experimental setting [2] and how to correctly address the technical solutions beyond the single-mode approximation [3]. The recent approach [3] reanalysed the general setting of [1] beyond the single-mode approximation. However, it required the preparation of a whole family of orthogonal states of global Unruh modes, one for each accelerated observer, instead of a single, fixed state given to all observers. It was also not clear how such global modes could be physically prepared. For these reasons the physical effects of acceleration on entanglement remained in disrepute.

In this paper we describe a framework for which these issues can be resolved. We introduce a model of a detector that performs a projective measurement on a localized single mode of the quantum field. Our detector can either move inertially or accelerate. This framework allows a fixed state of the field defined in an inertial frame to be probed for any acceleration. Thereby avoiding having to use a different initial state for each acceleration and a non-local initial mode.

To motivate the projective measurement formalism we introduce a relativistic generalization of the Glauber model [4] and show the relationship between the quantities calculated in each approach. One particularly nice feature of the Glauber detector is that the detector only clicks when particles are present in the mode that the device measures. This lends itself to a *particle* detector interpretation rather than a detector of fluctuations like that of the Unruh–DeWitt detector [5].

The model can be applied to a variety of scenarios in quantum field theory in curved space times. However, in order to show its applicability, in this paper the model is applied to the simplest case of the Minkowski vacuum state and we use it to reproduce the well-known Unruh effect for a single accelerated detector [7–9]. We then study how a pair of such detectors can extract non-local correlations present in the vacuum [10], showing that our model is suitable for studies of entanglement in non-inertial frames. Our analysis can be extended to arbitrary states of the field, which is the subject of another work [11].

We first outline, in section 2, the conceptual ideas behind the detector model presented here. Then, in section 3 we introduce the formalism to work with uniformly accelerated observers of a quantum field and discuss the process of field quantization in the Rindler basis. We then justify a natural choice of physically motivated localized detector modes in section 4. In section 5 we calculate the average number of particles populating the field mode probed by a localized accelerated detector when the field state is the Minkowski vacuum state; and in section 6 we introduce the formalism for the projective measurements on the field mode probed by the detector. This will allow us to estimate the temperature of the photo-count statistics (in section 7) and to study the extraction of field entanglement of spatially separated spacetime regions (in section 8). Our conclusions are presented in section 9 where we briefly discuss the possibility of application of these methods to curved spacetime scenarios. Additionally, a detailed analysis on the photo-detector interaction dynamics is given in the appendix.

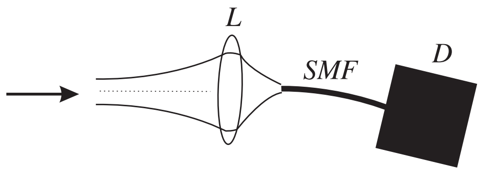

Figure 1. Detection of a quantum state of a single light-mode by means of a lens *L*, single-mode fibre SMF and a photon-counting detector *D*.

2. Model of the detector

We consider a model of measurement carried out locally on a quantum field with a device that is only sensitive to a single localized wave-packet mode. The model is inspired by a quantum-optical setting depicted in figure 1 that has been considered both in theory [12] and realized experimentally [13]. The original scheme consists of a lens, *L*, focusing incident light onto one end of a single-mode optical fibre SMF that transmits only one particular transverse mode of light. The other end of the fibre is attached to a single-photon detector, *D*, utilizing the photo-electric effect to detect and count single light quanta arriving in the fibre-coupled mode of light. In the idealized version of this scheme the state of the selected wave-packet is projected onto the Fock basis and all the other wave-packet modes that are orthogonal to the one transmitted through the fibre are not affected by the detection process. This scheme can also be generalized to allow the detection of an arbitrary wavelet by placing optical elements in front of the lens, *L*, which transform the given input mode into the mode transmitted by the fibre.

Notice that the state of the measured system is completely destroyed during the measurement process (photons are absorbed) and the physical measurement corresponding to this model is not instantaneous. The minimum time of such a detection is equal to the time it takes light collected by the lens to reach the SMF plus the time it takes for the device that captures the light to make the read-out. Therefore the bigger the wavelet mode, the longer it takes to extract it from all the other modes. In many practical scenarios, such as the one considered above, the time of detection can be safely neglected. However, one has to bear in mind that such idealization has its limits and not taking measurement time into account can lead to problems. It is not surprising that failing to take these limits into account and assuming that the state of the quantum field survives the measurement leads directly to the possibility of superluminal signalling. In the present study, however, no such problems appear, as we do not assume that the state of the field collapses instantaneously [16].

In this work we consider an abstract 'black-box' device that measures a wave-packet mode of the field in the same way: the process consists of filtering out the relevant mode and then measuring its state. For simplicity we consider a 1+1-dimensional version of the above setting. The process we describe disturbs the field only in the proximity of the measurement device; we assume that the detector's response is not affected by the state of other modes orthogonal to the one it measures, and that the measurements performed on this wavelet mode do not affect the state of the orthogonal modes. In this sense the process is localized in space.

The quantum field under consideration is taken to be a real non-interacting massless scalar field, $\hat{\phi}$, described by a Klein–Gordon equation, $\Box\hat{\phi} = 0$, in 1+1 Minkowski spacetime. Let the wave-packet measured by the detector be denoted by $\psi_{\mathrm{D}}(\xi, \tau)$, where ξ and τ, are coordinates of the reference frame co-moving with the detector. We assume that $\psi_{\mathrm{D}}(\xi, \tau)$ consists only of positive frequency waves with respect to τ. Although the splitting into positive and negative frequencies is frame dependent, the linear combination $\psi_{\rm D}(\xi,\tau)+\psi^{\star}_{\rm D}(\xi,\tau)$ is independent of the splitting. This real field solution describes the classical wave which couples to the detector, however we will be interested in the quantum effects. Using the positive frequency part it is possible to define an operator $\hat{d}=(\psi_{\rm D},\hat{\phi})$, associated with the wave-packet that the detector measures. Here $(\cdot,\cdot)$ is the Klein–Gordon scalar product [6]. If $\psi_{\rm D}$ is unit normalized with respect to this scalar product, then $\hat{d}$ satisfies the usual commutation relations $[\hat{d},\hat{d}^{\dagger}]=1$. If the motion of the detector is inertial (uniformly accelerated) then $\hat{d}$ annihilates the Minkowski (Rindler) vacuum state. This leads to the identification of $\hat{d}^{(\dagger)}$ as an annihilation (creation) operator. Any state of the field that is formed by the linear superposition of the basis states $\{\frac{\hat{d}^{\dagger n}}{\sqrt{n!}}|0\rangle\}$ or a mixture of them is said to be a state in the mode ψ_D. The operator $\hat{d}^{\dagger}\hat{d}$ is a number operator for the number of particles in the mode $\psi_{\rm D}$. While these excitations are identified as particles, they are not energy eigenstates of the free-field Hamiltonian, rather they are superpositions of such energy eigenstates. The state accessible to the detector is the total field state $|\Psi\rangle$ projected onto the $\psi_{\rm D}$ subspace:

$$\mathrm{Tr}_{\perp\psi_{\rm D}}|\Psi\rangle\langle\Psi|, \tag{1}$$

where the trace is to be taken over the subspace of all wave-packet modes orthogonal to $\psi_{\rm D}$. In this work we assume that $\psi_{\rm D}$ is localized[7], which will allow us to attribute the detector a certain position in space. For example if the measurement on $\psi_{\rm D}$ takes place at times close to $\tau=0$ then the detector has to be placed in the region where $\psi_{\rm D}(\xi,0)$ is non-zero.

One of the greatest advancements of the field of quantum optics occurred with the introduction of the Glauber model [4]. The detection process described in the Minkowski frame by the Glauber model for the scalar field results in a particle absorption occurring at the point (ct,x), described in the weak coupling regime by the following absorption amplitude:

$$A_{if}(x,t)=\langle f|\hat{\phi}^{+}(x,t)|i\rangle, \tag{2}$$

where $|i\rangle$ and $|f\rangle$ are initial and final states of the field and $\hat{\phi}^{+}(x,t)$ is the positive frequency part of the field operator in Minkowski coordinates. However, the final state of the field remains unknown after the measurement, so the total probability $\mathcal{P}$ of observing a particle is given by taking the squared modulus of the amplitude and summing over all final states. Generalizing to mixed states and substituting (1) for the initial state results in

$$\mathcal{P}(x,t)=\mathrm{Tr}\{\hat{\phi}^{-}(x,t)\hat{\phi}^{+}(x,t)\mathrm{Tr}_{\perp\psi_{\rm D}}|\Psi\rangle\langle\Psi|\}. \tag{3}$$

In this work we generalize the Glauber theory of detection to the case of detectors moving with relativistic accelerations. The crucial step in this generalization is to replace the positive and negative frequency parts in the above equations with the positive and negative frequency parts with respect to the proper time of the frame co-moving with the detector $(c\tau,\xi)$. Since the positive and negative frequency parts of the field are different in the accelerating and inertial frames, the intensity of particle clicks in the photo-counter will depend on the detector's motion. We prove in the appendix that our generalized Glauber model agrees to first order in perturbation theory with the predictions of a field-apparatus dynamical model that utilizes an Unruh–DeWitt interaction, analogously to the inertial case [19]. We find that even in the non-inertial case the measurement outcomes can be completely described by a set of projective operators, as in the standard quantum-optical non-relativistic setups.

[7] The position-space representation of the single particle state $\hat{d}^{\dagger}|0\rangle$ is $\langle 0|\hat{\phi}d^{\dagger}|0\rangle=\psi_{\rm D}$. It is well-known, however, that a relativistic particle cannot be localized within its characteristic wavelength without superimposing negative frequencies in its wave-packet (i.e., a massive particle cannot be localized any smaller than its Compton wavelength). However, in the massless case, the characteristic frequency of the mode is arbitrary and can be specified at will. Therefore, any desired degree of localization can be achieved by choosing a sufficiently large characteristic frequency. One should not confuse the word *localized*, as we are using it, with the strict criteria suggested in [21]. Indeed the wave-packets are not eigenstates of a single particle position operator.

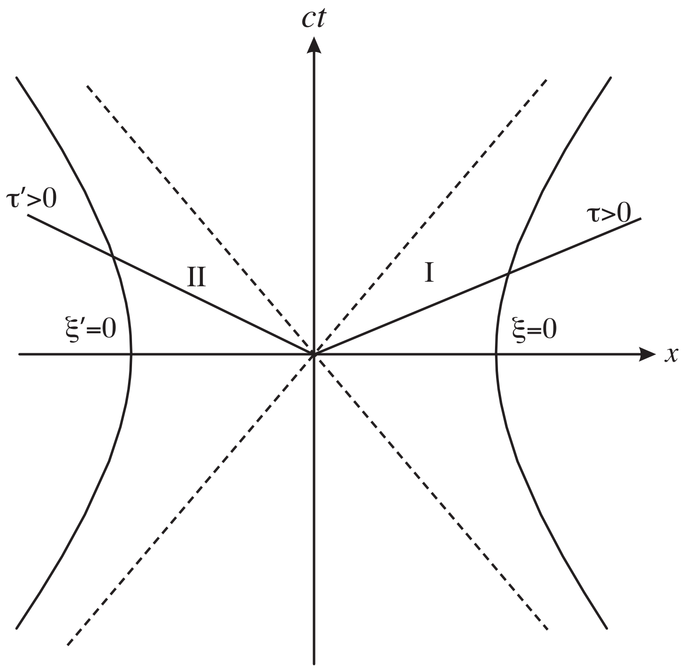

Figure 2. Minkowski diagram, with Rindler regions I ($x > c|t|$) and II ($x < -c|t|$) covered with conformal Rindler coordinates (τ, ξ) and (τ', ξ') respectively.

3. The accelerated frame of reference

In this paper we focus exclusively on measurements made by the uniformly and relativistically accelerated detectors. A natural choice of coordinates for the accelerated observers are the Rindler coordinates:

$$\begin{aligned} ct &= \frac{c^2}{a}\,\mathrm{e}^{a\xi/c^2}\sinh\frac{a\tau}{c}, \quad x = \frac{c^2}{a}\,\mathrm{e}^{a\xi/c^2}\cosh\frac{a\tau}{c}, \quad (I) \\ ct &= \frac{c^2}{a}\,\mathrm{e}^{a\xi'/c^2}\sinh\frac{a\tau'}{c}, \quad x = -\frac{c^2}{a}\,\mathrm{e}^{a\xi'/c^2}\cosh\frac{a\tau'}{c}, \quad (II), \end{aligned} \tag{4}$$

where $-\infty < \xi, \xi', \tau, \tau' < \infty$, a is an arbitrary parameter, and I, II represent the two distinct regions of spacetime relevant to accelerated observers; Rindler region I ($x > c|t|$) and Rindler region II ($x < -c|t|$) shown in figure 2.

The Rindler coordinate system is constructed in such a way that the proper distance between two hyperbolic trajectories of constant ξ is constant on every (proper) time slice labelled by τ. This happens because the Rindler frame is a special case of the Fermi–Walker coordinate system for uniformly accelerated motion [14, 15]. We assume that our detector accelerated by the action of a constant external force (in the co-moving frame) remains static (in that frame) due to the presence of binding forces that keep the body coherent preventing it from falling apart. Consequently, individual points of the accelerated body will move along different hyperbolas of the Rindler frame. The body can even be spatially deformed by the external force, since no perfectly rigid body exists. However, as long as the deformation does not vary in (proper) time in the Rindler frame of reference, that frame still provides the appropriate description of motion of the uniformly accelerated body. We take into account certain deformations due to acceleration, which we will discuss in detail in the next section.

To interpret a appearing in (4) as the proper acceleration of the detector, we must ensure that the photo-counter is spatially localized at $t = 0$ around $\xi = 0$ when $a > 0$, or $\xi' = 0$ when $a < 0$. Then all of its components will approximatively experience the same proper time, which coincides with Rindler time τ. Assuming that the measurement device is internally coherent (having constant proper length in time but possibly deformed from its rest configuration), it will be naturally coupled to the family of normal modes in the Rindler reference frame associated with the Killing field of that frame, which are called Rindler modes and are given by

$$
\begin{aligned}
w_{k,I}(\xi,\tau) &\equiv \frac{1}{\sqrt{4\pi|k|}}\,\mathrm{e}^{\mathrm{i}(k\xi-|k|c\tau)}\theta(x-c|t|),\\
w_{k,II}(\xi',\tau') &\equiv \frac{1}{\sqrt{4\pi|k|}}\,\mathrm{e}^{\mathrm{i}(-k\xi'-|k|c\tau')}\theta(-x-c|t|).
\end{aligned}
\tag{5}
$$

These modes form a complete basis of solutions of the field equation and therefore can be used to decompose the field operator:

$$\hat{\phi} = \sum_k \hat{b}_{k,I}w_{k,I}(\xi,\tau) + \hat{b}_{k,II}w_{k,II}(\xi',\tau') + \text{h.c.} \tag{6}$$

The role of the deformations due to acceleration has been studied in some detail for the spatially extended generalization of the Unruh–DeWitt model of detector and in certain acceleration regimes the role of the deformations was shown to be negligible [14, 15]. In this work we take into account static spatial deformations of the detector due to its acceleration in analogy with mode deformations of accelerated cavity, as we discuss in detail in the next section.

It is a known fact [9] that the Minkowski vacuum state can be expressed as a multi-frequency two-mode-squeezing of the Rindler vacuum state:

$$|0\rangle_{\mathrm{M}} = \hat{S}_{I,II}|0\rangle_{\mathrm{R}}, \tag{7}$$

where the squeezing operator, $\hat{S}_{I,II}$, is characterized by the squeezing parameter, $r_k = \operatorname{arctanh}(\mathrm{e}^{-\pi|k|c^2/a})$, and fulfils the following relations:

$$\hat{S}^\dagger_{I,II}\hat{b}_{l,I}\hat{S}_{I,II} = \cosh r_l\hat{b}_{l,I} + \sinh r_l\hat{b}^\dagger_{l,II}, \tag{8}$$

where $\hat{b}_{l,I}$ and $\hat{b}_{l,II}$ are the annihilation operators associated with the Rindler modes (5). We will find these commutation relations extremely useful when performing the algebraic manipulations of the following sections.

4. Choice of the detector mode

The wave-packet corresponding to the localized field mode probed by the detector $\psi_{\mathrm{D}}(\xi,\tau)$ must be spatially localized around $\xi = 0$ at the measurement time around $\tau = 0$. In principle, any positive frequency localized wave-packet satisfying this condition will be sufficient, but our choice will be dictated by the convenience of further calculations. By 'localized' we mean that the variance of the mode must be significantly smaller that the distance of the event horizon from the average position of the mode. We do not demand that the mode is compactly supported, it is enough that its tails are Gaussian-like (see footnote 5).

Consider a resting detector that selects $\psi_{\mathrm{D}}(x,t)$ such that it has a central frequency coinciding with the Nth eigenmode of a cavity. We ask what wave-packet $\psi_{\mathrm{D}}(\xi,\tau)$ would a uniformly accelerated detector select if it was to operate on this same principle? In order to answer this question we compare the eigenmodes of an ideal cavity at rest and the eigenmodes of the same cavity in uniform acceleration [20]. One infers that the central frequency of the mode $\psi_{\mathrm{D}}(\xi,\tau)$ should be $\frac{Nc}{\sigma}$, where, $\sigma = \frac{2c^2}{a}\operatorname{asinh}(\frac{aL}{2c^2})$, L is the proper length of the cavity and

N is a natural number corresponding to the chosen cavity eigenmode. We choose a Gaussian envelope of that mode, characterized by the width σ, to make it localized, however all our main results will not depend on the specific choice of that envelope:

$$\psi_{\mathrm{D}}(\xi,0)=\frac{1}{\sqrt{N\sqrt{2\pi}}}\exp\left[-\frac{\xi^2}{\sigma^2}+\mathrm{i}\frac{N}{\sigma}\xi\right],$$
$$\partial_\tau\psi_{\mathrm{D}}(\xi,0)=-\mathrm{i}\frac{Nc}{\sigma}\psi_{\mathrm{D}}(\xi,0). \tag{9}$$

The above expressions always have some contribution from negative frequency components, however these are negligible for $N\gg 1$ (actually $N>3$ is already enough). To get rid of them completely we introduce an infra-red cut-off Λ to the considered mode

$$\psi_{\mathrm{D}}(\xi,\tau)\to\frac{\sum_{k>\Lambda}(w_{I,k},\psi_{\mathrm{D}})w_{I,k}(\xi,\tau)}{\sqrt{\sum_{k>\Lambda}|(w_{I,k},\psi_{\mathrm{D}})|^2}}. \tag{10}$$

Note this then also ensures that the single particle states exist for this degree of localization (see footnote 5). The device is defined to be only sensitive to the right-moving modes in region I but this choice is arbitrary and can be changed if necessary. In the limit of small accelerations, the mode (9) written in Minkowski coordinates reduces to $\psi_{\mathrm{D}}(x,0)=\frac{1}{\sqrt{N\sqrt{2\pi}}}\exp[-\frac{x^2}{L^2}+\mathrm{i}\frac{N}{L}x]$ up to a translation along x. It reproduces a regularized Nth mode of a resting cavity of a length L.

Note that the mode shape (9) does implicitly depend on the acceleration and undergoes a deformation due to acceleration exactly the same way that the modes of an accelerated cavity do. One can consider other types of deformations for specific types of materials used to build the measurement device. The deformation would then depend on solid-state properties of the detector. Other types of dependence of the mode shape on acceleration have been considered in [11], where the authors have studied how the choice of that dependence affects the measurement outcomes.

5. Detection of particles from the Vacuum state

Consider measuring the occupation number operator of the transmitted localized mode, $\hat{d}^\dagger\hat{d}$. If at $\tau=0$ the measured field is in a Fock state $|n\rangle$ of a wavelet mode $\phi(\xi,\tau)$ that coincides at $\tau=0$ with ψ_{D}, i.e., $\phi(\xi,0)=\psi_{\mathrm{D}}(\xi,0)$, $\partial_\tau\phi(\xi,0)=\partial_\tau\psi_{\mathrm{D}}(\xi,0)$ then the outcome of the detection will be a classical variable n.

To generalize the result to an arbitrary state of the field it is useful to introduce the following decomposition of the field operator:

$$\hat{\phi}(\xi,0)=\hat{d}\psi_{\mathrm{D}}(\xi,0)+\hat{d}^\dagger\psi_{\mathrm{D}}^\star(\xi,0)+\hat{\phi}', \tag{11}$$

where $\hat{\phi}'$ is the remaining part of the mode decomposition containing all the modes orthogonal to the wavelet ψ_{D}. In principle, a complete and orthonormal basis could be found for the Hilbert space by using Gram–Schmidt orthonormalization on the positive frequency subspace of complex solutions beginning with the vector ψ_{D}. However, it will not be required to calculate such a basis for anything that follows.

The probability $\mathcal{P}$ of the detector clicking at time $\tau=0$ and position ξ according to the generalized Glauber model (3) is:

$$\mathcal{P}(\xi,0)=|\psi_{\mathrm{D}}(\xi,0)|^2\langle\Psi|\hat{d}^\dagger\hat{d}|\Psi\rangle. \tag{12}$$

Of course, the Glauber model describes an idealized situation in which the measurement is performed instantaneously and at a localized position. It also ignores the fact that the apparatus itself is a complex (classical and quantum) system. Some of these considerations are taken into account in the appendix where a model of field-apparatus interaction is considered. It is nevertheless found that up to an apparatus dependent proportionality constant (i.e., dependent on collection efficiencies etc) the single click probability is still proportional to the expectation of the operator $\hat{d}^\dagger\hat{d}$, i.e., the number of particles in the mode.

Let us calculate the expectation value of the number of particles seen by an accelerating detector when the field is initialized into the Minkowski vacuum state, $|\Psi\rangle = |0\rangle_{\mathrm{M}}$:

$$\langle\hat{d}^\dagger\hat{d}\rangle = -\langle(\psi_{\mathrm{D}}^\star, \hat{\phi})(\psi_{\mathrm{D}}, \hat{\phi})\rangle. \tag{13}$$

From here on the angular brackets represent taking expectation values with respect to the Minkowski vacuum state. Using (7) and (8) one finds:

$$\begin{aligned}\langle\hat{d}^\dagger\hat{d}\rangle &= \sum_{k,l}(\psi_{\mathrm{D}}, w_{k,I})^\star(\psi_{\mathrm{D}}, w_{l,I})_{\mathrm{R}}\langle 0|\hat{S}_{I,II}^\dagger\hat{b}_{k,I}^\dagger\hat{b}_{l,I}\hat{S}_{I,II}|0\rangle_{\mathrm{R}}\\ &= \sum_{k}\langle\hat{n}_k\rangle|(\psi_{\mathrm{D}}, w_{k,I})|^2,\end{aligned} \tag{14}$$

where $\langle\hat{n}_k\rangle = \sinh^2 r_k$ is the average occupation number of a two-mode squeezed state of the plane-wave mode k. For the mode choice (9), we find:

$$\langle\hat{d}^\dagger\hat{d}\rangle = \frac{\mathrm{e}^{-\frac{2\pi c^2}{aL}(N-\frac{c^2\pi}{aL})}}{4N\sqrt{2\pi}}\int_1^\infty\frac{\mathrm{d}k'}{k'}\frac{(N+k')^2\mathrm{e}^{-\frac{1}{2}(k'-N-\frac{2\pi c^2}{aL})^2}}{1-\ \mathrm{e}^{-\frac{2\pi k' c^2}{aL}}}, \tag{15}$$

where we completed the square in the exponent, and took out the k'-independent factor. The denominator of the second factor in the integrand is approximately unity on the whole domain, furthermore in the limit $N \gg \frac{2\pi c^2}{aL}$ the integral is dominated by the contribution from $k' \sim N$. Thus, we find up to an order 1 proportionality constant:

$$\langle\hat{d}^\dagger\hat{d}\rangle \sim \mathrm{e}^{-\frac{2\pi c^2}{aL}(N-\frac{c^2\pi}{aL})}. \tag{16}$$

This equation is only valid for the range of parameters satisfying the conditions above, namely $\frac{N}{2\pi} \gg \frac{c^2}{aL} \gg 1$. As we expect, the number of particles that can be observed by the detector is exponentially suppressed by the factor $\mathrm{e}^{-2\pi c\omega_c/a}$ where $\omega_c = \frac{cN}{L}$ is the characteristic frequency of the detected mode.

6. Projective measurement formalism

Notice that calculating the expectation value of the number of particles in the mode can be done in a purely abstract way without any reference to the detection process. In fact, working with the field theory at this abstract level of description can often be simpler and more illuminating than getting lost in the details of a specific detector model. The aim of this paper is to show that the usual phenomena of fields in non-inertial frames can be re-derived at this abstract level.

The notion of a quantum state makes sense, because in principle there could exist a device that measures it acquiring the complete information about the system encoded in the state. The appendix shows that the abstract calculations of the previous section can also be obtained with a dynamical model of interaction between a small detector and a quantum field. In a similar way all the information extractable from a device that resolves the number of particles in a state must be describable by a set of projective operators $|n\rangle\langle n|$ associated with possible measured outcomes. This is imposed by the Hilbert space structure of quantum states.

To calculate the probability $\mathcal{P}(n)$ of detecting n field quanta we use the projector onto the nth Fock state of the mode $\hat{d}$, that can be written in the following manner [22]:[8]

$$\mathcal{P}(n) = \left\langle :\mathrm{e}^{-\hat{d}^\dagger\hat{d}}\frac{(\hat{d}^\dagger\hat{d})^n}{n!}:\right\rangle. \tag{17}$$

In the weakly coupled regime the above expression can be derived from a dynamical model described in the appendix , with an extra modification of $\hat{d}^\dagger\hat{d}$ being replaced with $\eta\hat{d}^\dagger\hat{d}$, where $\eta \leqslant 1$ [19]. The new parameter η describes the efficiency of the detection and is related to the coupling strength between the systems.

The probability distribution (17) can be related to the characteristic function $Z(\lambda)$ via the Fourier transform:

$$\mathcal{P}(n) = \int_0^{2\pi}\frac{\mathrm{d}\lambda}{2\pi}\mathrm{e}^{-\mathrm{i}\lambda n}Z(\lambda). \tag{18}$$

Here

$$\begin{aligned}
Z(\lambda) &= \sum_{n=0}^{\infty}\mathrm{e}^{\mathrm{i}\lambda n}\mathcal{P}(n) = \langle :\mathrm{e}^{(\mathrm{e}^{\mathrm{i}\lambda}-1)\hat{d}^\dagger\hat{d}}:\rangle \\
&= \sum_n \frac{(\mathrm{e}^{\mathrm{i}\lambda}-1)^n}{n!}\,{}_{\mathrm{R}}\langle 0|\hat{S}^\dagger_{I,II}\hat{d}^{\dagger n}\hat{d}^n\hat{S}_{I,II}|0\rangle_{\mathrm{R}} \\
&= \sum_n \frac{(\mathrm{e}^{\mathrm{i}\lambda}-1)^n}{n!}\,{}_{\mathrm{R}}\langle 0|\left[\sum_k \sinh r_k(\psi_{\mathrm{D}}, w_{k,I})^\star\hat{b}_{k,II}\right]^n \\
&\quad\times\left[\sum_l \sinh r_l(\psi_{\mathrm{D}}, w_{l,I})\hat{b}^\dagger_{l,II}\right]^n|0\rangle_{\mathrm{R}} \\
&= \sum_n(\mathrm{e}^{\mathrm{i}\lambda}-1)^n\langle\hat{d}^\dagger\hat{d}\rangle^n = \frac{1}{1-(\mathrm{e}^{\mathrm{i}\lambda}-1)\langle\hat{d}^\dagger\hat{d}\rangle},
\end{aligned} \tag{19}$$

where the multinomial expansion was used to obtain the last line and the mean particle number $\langle\hat{d}^\dagger\hat{d}\rangle$ is given by equation (14). Through (18) we find the excitation statistics:

$$\mathcal{P}(n) = \frac{\langle\hat{d}^\dagger\hat{d}\rangle^n}{(1+\langle\hat{d}^\dagger\hat{d}\rangle)^{1+n}} \tag{20}$$

for the Minkowski vacuum state $|0\rangle_{\mathrm{M}}$.

7. Determining the Unruh temperature

One might wonder what temperature is associated with the above statistics. For a thermalized state of a harmonic oscillator the temperature is defined by the relation $\langle\hat{d}^\dagger\hat{d}\rangle = (\mathrm{e}^{E/kT}-1)^{-1}$, where E is the energy of a single excitation of the field mode. However in general the mode ψ_{D} has a frequency spread so it does not have well defined energy, therefore the

[8] The derivation is the following. Let $|n\rangle_{\mathrm{D}} = \frac{\hat{d}^{\dagger n}}{\sqrt{n!}}|0\rangle_{\mathrm{D}}$ define a Fock basis of the mode $\hat{d}$. Consider the following expression:

$${}_{\mathrm{D}}\langle n| :\mathrm{e}^{-\hat{d}^\dagger\hat{d}}: |m\rangle_{\mathrm{D}} = \delta_{n,m}\sum_{k=0}^{n}(-1)^k\binom{n}{k} = \delta_{n,0}\,\delta_{m,0}.$$

This proves that the operator $:\mathrm{e}^{-\hat{d}^\dagger\hat{d}}:$ is a projector onto the ground state of the mode $\hat{d}$. Consequently, a projector onto the nth Fock state has the form $:\mathrm{e}^{-\hat{d}^\dagger\hat{d}}\frac{\hat{d}^{\dagger n}\hat{d}^n}{n!}:$. Note that the projection is restricted only to the mode associated with $\hat{d}$ and does not affect any other modes.

temperature is also not well defined. Nevertheless, one can define a temperature estimator T_{est} by replacing E with the expectation value of the energy of a single excitation of the field mode, $E \to {}_{\mathrm{R}}\langle 0|\hat{d}\hat{H}_{\mathrm{R}}\hat{d}^{\dagger}|0\rangle_{\mathrm{R}} = \sum_k \hbar\omega_k |(\psi_{\mathrm{D}}, w_{k,I})|^2$, where $\hat{H}_{\mathrm{R}} = \sum_k \hbar\omega_k \hat{b}^{\dagger}_{k,I}\hat{b}_{k,I} + (I \leftrightarrow II)$ is the generator of proper time translations in the Rindler frame and ω_k are Rindler frequencies. For a detector moving with proper acceleration, a, this temperature estimator is equal to:

$$kT_{\mathrm{est}} = \frac{\sum_k \hbar\omega_k |(\psi_{\mathrm{D}}, w_{k,I})|^2}{\log(1 + \langle \hat{d}^{\dagger}\hat{d}\rangle^{-1})}. \tag{21}$$

Here we find that the temperature observed depends not only on the proper acceleration of the detector, a, but also on the shape of the mode ψ_{D}. This is a new and quite surprising result, as the standard Unruh temperature formula had no dependence on frequency of the mode and only depended on the proper acceleration.

For the choice of mode (9), we find that the temperature estimator (21) reduces approximately (when $\frac{N}{2\pi} \gg \frac{c^2}{aL} \gg 1$) to:

$$kT_{\mathrm{est}} \approx \frac{\hbar a}{2\pi c} \frac{1}{1 - \pi c^2/aLN}. \tag{22}$$

Note that the temperature estimator deviates from the Unruh temperature formula [9] for finite N. This stems from the fact that the energy of the mode is not well defined. However, for large N, which corresponds to a peaked energy spectrum, we recover the celebrated Unruh result [9].

It is interesting to realize that the idealized concept of temperature only makes sense when modes of fixed energy are analysed. Here, we study the properties of more localized wavelets and it turns out that the notion of temperature and related expressions known in the literature are only approximated.

8. Non-locality of the vacuum state

In order to extract entanglement from the vacuum state we need more than just one detector. Consider two identical detectors, one moving with proper acceleration $a > 0$ in region I and coupled to a mode $\psi_{\mathrm{D},I}$, and the other moving with acceleration $-a$ in region II and coupled to $\psi_{\mathrm{D},II}$. The corresponding annihilation operators are $\hat{d}_I$ and $\hat{d}_{II}$ respectively and both detectors are causally disconnected. One can generalize the calculation (14) to determine the average product of the particle content detected in the two modes and obtain the result:

$$\langle \hat{d}^{\dagger}_I \hat{d}_I \hat{d}^{\dagger}_{II} \hat{d}_{II}\rangle = \langle \hat{d}^{\dagger}_I \hat{d}_I\rangle\langle \hat{d}^{\dagger}_{II}\hat{d}_{II}\rangle + |\langle \hat{d}_I \hat{d}_{II}\rangle|^2, \tag{23}$$

where the first two terms on the rhs of the equation are given by (14) and the last term is equal to:

$$\langle \hat{d}_I \hat{d}_{II}\rangle = \sum_k \sqrt{\langle \hat{n}_k\rangle(1 + \langle \hat{n}_k\rangle)}(\psi_{\mathrm{D},I}, w_{k,I})(\psi_{\mathrm{D},II}, w_{k,II}). \tag{24}$$

We find that the measurement outcomes carried out by the detectors are correlated and the correlations are determined by the magnitude of expression (24). In order to prove the non-locality of these correlations we note that a two party symmetric Gaussian state is entangled if [23]:

$$\langle(\hat{x}_I - \hat{x}_{II})^2\rangle\langle(\hat{p}_I + \hat{p}_{II})^2\rangle < 1, \tag{25}$$

where $\sqrt{2}\hat{x}_\sigma = \hat{d}_\sigma + \hat{d}^{\dagger}_\sigma$ and $\sqrt{2}\mathrm{i}\hat{p}_\sigma = \hat{d}_\sigma - \hat{d}^{\dagger}_\sigma$ for $\sigma \in \{I, II\}$.

In order to detect entanglement we need to re-program our detectors to perform projective measurements corresponding to the Hermitian quadrature operators $\hat{x}_\sigma$ or $\hat{p}_\sigma$ instead of

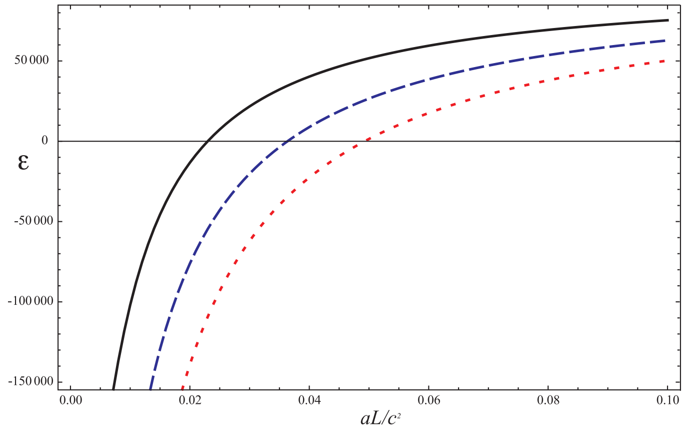

Figure 3. Entanglement estimator $\mathcal{E}$ as a function of the dimensionless parameter $\frac{aL}{c^2}$ for $N = 800$ (solid line), $N = 1200$ (dashed line) and $N = 1600$ (dotted line).

projecting in the Fock basis. Typically, such a measurement is realized by means of homodyne detection with the use of an auxiliary beam of light [24]. In the present work we do not study the details of such setup [25] assuming that our device carries out an ideal measurement of the quadratures. We find that the left-hand side of the inequality (25) is equal to $(1 + \langle\hat{d}_I^\dagger\hat{d}_I\rangle + \langle\hat{d}_{II}^\dagger\hat{d}_{II}\rangle - 2\mathrm{Re}\langle\hat{d}_I\hat{d}_{II}\rangle)^2$ and again it turns out that the presence of the entanglement is dictated by the same term (24) that was responsible for the existence of correlations between the detectors' counts.

For a pair of identical, counter-accelerating detectors we have $\langle\hat{d}_I^\dagger\hat{d}_I\rangle = \langle\hat{d}_{II}^\dagger\hat{d}_{II}\rangle$. By this criterion we have checked that for any finite acceleration the state measured by the detectors is entangled. Knowing this, we can now quantify this entanglement extracted by the detectors by introducing the entanglement estimator $\mathcal{E}$:

$$\mathcal{E} = \log\left|\langle\hat{d}_I^\dagger\hat{d}_I\rangle - \mathrm{Re}\langle\hat{d}_I\hat{d}_{II}\rangle\right| + C \tag{26}$$

where C is an arbitrary real constant factor. Here $\mathcal{E} \to -\infty$ when the detectors state is separable and grows monotonically as the degree of quantum entanglement is increased. For $\psi_{\mathrm{D},I}$ given by (9) and $\psi_{\mathrm{D},II}$ given by the transformation $\xi' = -\xi$ of (9) we have $(\psi_{\mathrm{D},II}, w_{k,II}) = (\psi_{\mathrm{D},I}, w_{k,I})$. Taking the limit of $\frac{N}{2\pi} \gg \frac{c^2}{aL} \gg 1$ the estimator can be approximated by:

$$\mathcal{E} \approx -\frac{\pi c^2}{aL}\left(N - \frac{\pi c^2}{2aL}\right) + C. \tag{27}$$

We know that the extracted entanglement must vanish ($\mathcal{E} \to -\infty$) as $a \to 0$, although for fixed N our approximations are not valid in this limit. In figure 3 we plot the dependence of the entanglement $\mathcal{E}$ (26) on the dimensionless quantity $\frac{aL}{c^2}$ for several values of N, finding good agreement with the approximated result (27). We find that the largest amount of entanglement can be extracted from the Minkowski vacuum state when the proper lengths and proper accelerations of the detectors are large and the frequency numbers N are low.

9. Discussion and conclusions

Our results may help to understand the nature of the correlations between localized measurements of the quantum vacuum studied in the context of algebraic QFT [26]. It is also interesting to compare our findings with the results of [27], where entanglement between two localized, orthogonal modes was observed in the inertial frame. This entanglement was shown to decay with the spatial separation of the modes. On the other hand, we find that no entanglement can be extracted from inertial detectors. This difference can be attributed to the fact that our detectors are probing modes that consist only of positive frequencies and the authors of [27] consider modes that have no such restriction.

The main idea of this paper is to emphasize that for single-mode measurements the outcomes of photo-count experiments in the accelerating frame can be described by projective operators operating only on the single-mode subspace. In this way calculations can be performed at an abstract level without making any reference to the actual detection processes. The versatility and power of this approach is demonstrated by the analytic expressions we were able to re-derive for the standard phenomena in non-inertial frames, i.e., the Unruh effect and extraction of vacuum entanglement. Although here we restricted our attention to measurements of the occupation number of the mode (see section 5) and quadrature measurements (see section 8), in principle any observable in question can be chosen at will by the experimentalist.

It is both mathematically and conceptually advantageous to work at the abstract level of projective operators, but one must keep in mind the underlying physical processes. For example, the state of the measured system is completely destroyed during the measurement process (photons are absorbed) and the physical measurement is not instantaneous and not taking them into account can lead to problems [16]. In the appendix we showed by taking the interaction time into account that under reasonable conditions the results of the generalized Glauber model can be validated.

The model presented here can be readily applied to other types of motion, as well as curved spacetimes in regions where time-like Killing vectors exist. Indeed, one can readily apply the tools developed in this paper to all the regimes where Unruh modes have been employed in the past to study entanglement in quantum fields in the vicinity of the event horizon of a stationary black hole [17] or in dynamical gravitational collapse scenarios [18]. Further research is currently being undertaken to use this approach to investigate localized entangled states, instead of the Minkowski vacuum. The principles laid out in this work can be generalized to other types of fields, and it is expected that new insights into the nature of entanglement in accelerated frames are to be found in this approach.

Acknowledgments

We thank P Alsing, J Louko, M Montero, T Ralph, and M Del Rey for very useful discussions. AD thanks for the financial support to EPSRC (CAF grant EP/G00496X/2), Polish Ministry of Education and National Science Center, Sonata BIS grant 2012/07/E/ST2/01402. EM-M thanks the Banting Postdoctoral Fellowship programme of NSERC for his financial support.

Appendix

In this appendix we consider a dynamical interaction model for the photon absorption processes occurring at D in figure 1 and show that the model reproduces the general behaviour of a generalized Glauber model (12).

In idealized photo-detectors electrons in bound potential wells are excited into free states by the absorption of photons, see for example [19]. In this approach the interaction of a photon with a single electron is calculated using a non-relativistic Schrödinger equation for the electron and using the minimal coupling substitution $p \to p - eA(x, t)$ resulting in an interaction term,

$$H_I = \frac{e}{m}\hat{p}\cdot\hat{A}(r_0,t). \tag{A.1}$$

Although this Hamiltonian only describes the interaction between a single electron and single-photon system, many such interactions are assumed to take place inside the detector, allowing the detector to operate as a multi-photon-counting device. Since the transformation of this setup into the accelerated frame is complicated by many factors (including relativistic considerations, and the effects of acceleration on the binding material properties etc) it is more practical to consider a detector with subsystems interacting with the field via the interaction Lagrangian:

$$L[x(\tau)] = \mu\hat{m}(\tau)\hat{\phi}[x(\tau)] \tag{A.2}$$

where $x(\tau)$ is the classical path followed by the detector, τ is the proper time along the path and $\hat{m}$ is a monopole moment having discrete energy levels. The time evolution of the monopole moment can be written, $\hat{m}(\tau) = \mathrm{e}^{\mathrm{i}\hat{H}_0\tau/\hbar}\hat{m}(0)\mathrm{e}^{-\mathrm{i}\hat{H}_0\tau/\hbar}$ where $\hat{H}_0$ is the free Hamiltonian for the monopole. This is of course just an Unruh–DeWitt interaction [6]. For an inertial path equations (A.1) and (A.2) are remarkably similar. We can even approximate the continuum of energy states in the photo-electric case (above the binding energy $> \hbar\omega_0$) by discrete energies of the monopole. For example, we could assume that the energies of the monopole have a large gap from ground to first excited state representing the binding energy of the electron potential then smaller perhaps equally spaced gaps thereafter. One would then be able to reproduce the standard theory of photo-detection using this model [19]. Furthermore, since the interaction transforms simply as a scalar under coordinate transformations, one would also expect to be able to perform the equivalent calculations in the accelerated frame. It is this direction we will now pursue.

We assume the transition to a higher energy state of the monopole, as in the photo-electric model, results in a current or cascade process that magnifies the occurrence of the quantum transition into a classical measurement. However, we do not model the quantum to classical process here. It is important to emphasize that only the excitation transitions of the monopole will be relevant to photo-detection. After the classical click in the detector occurs the detector is supposed to be reset into its initial ground state configuration. This is unlike the Unruh–DeWitt model where (in the usual presentation) both up and down transitions contribute to the final state of the monopole which in the long time limit is found to be in thermal equilibrium with the field.

We assume the 'binding energy' for this model is of the typical energy scale of the mode of the field we measure. For the Gaussian in (9) we would have $E_1 - E_0 \sim \frac{N\hbar c}{L}$.

The initial field state is taken to be the mixed state (1) and the monopole is initialized into the ground state $\rho_m(\tau_0) = |E_0\rangle\langle E_0|$, so that the initial state of the system (monopole plus single field mode) is given by the product state $\hat{\rho}(\tau_0) = \hat{\rho}_m(\tau_0)\otimes \mathrm{Tr}_{\perp\psi_\mathrm{D}}|\Psi\rangle\langle\Psi|$. From the outset we work in the accelerated frame with the detector positioned at $\xi = \xi_0$. In this frame the system will be assumed to undergo evolution according to the operator of infinitesimal displacements of proper time. The different inertial and accelerating photo-count statistics will then be seen to arise because of the different proper time parameters under which the Hamiltonian systems evolve respectively. The probability of finding the system in some orthogonal state $\hat{\rho}_f = |E,\chi\rangle\langle E,\chi|$ after an interaction time T can be calculated using a perturbation expansion of the equations of motion for the density matrix. The second order approximation is given by [19]:

$$\mathcal{P}(\xi_0,\tau_0;T) = \frac{\mu^2}{\hbar^2}\int_{\tau_0}^{\tau_0+T}\mathrm{d}\eta_1\int_{\tau_0}^{\eta_1}\mathrm{d}\eta_2$$
$$\times\langle E,\chi|m(\eta_1)\hat{\phi}(\xi_0,\eta_1)\hat{\rho}(\tau_0)m(\eta_2)\hat{\phi}(\xi_0,\eta_2)|E,\chi\rangle + \mathrm{c.c}, \tag{A.3}$$

To this order the only processes that occur are those in which a single photon is either created or annihilated. In analogy with electrons excited in photo-diodes we assume a classical click of the detector occurs when the monopole makes a transition into any of the excited states. Therefore, to calculate the probability of a click we sum over all possible final energy states of the monopole. Furthermore, since when we obtain a click we do not know which state the field has transitioned into the total single click probability is found by summing over all possible final states of the field. We then arrive at the probability for a single detector click in a time T given by:

$$\mathcal{P}(\xi_0, \tau_0; T) = \frac{\mu^2}{\hbar^2} \sum_E |\langle E|m(0)|E_0\rangle|^2 \mathcal{F}(E - E_0) \tag{A.4}$$

where

$$\mathcal{F}(E - E_0) = \int_{\tau_0}^{\tau_0+T} \mathrm{d}\eta_1 \int_{\tau_0}^{\eta_1} \mathrm{d}\eta_2\, \mathrm{e}^{\mathrm{i}(E-E_0)(\eta_1-\eta_2)/\hbar} \mathrm{Tr}[\hat{\phi}(\xi_0, \eta_2)\hat{\phi}(\xi_0, \eta_1) Tr_{\perp\psi_{\mathrm{D}}}|\Psi\rangle\langle\Psi|] + \mathrm{c.c.} \tag{A.5}$$

Aside from a slightly different interpretation, until this point we have effectively just done a normal Unruh–DeWitt calculation for an excitation transition of the monopole [6] but over a finite time and with a different initial field state. Usually the initial state is assumed to be the Minkowski vacuum state, which is a state with excitations over all Rindler frequencies. Since all frequencies are relevant for this state (including arbitrarily low frequencies), it is necessary to perform the experiment over an infinite period of time. However, here we have the mixed state $\hat{\rho}_F(\tau_0)$ which is a state in a single mode with a peaked frequency, therefore as in usual optical experiments (i.e., in the inertial frame) there exists a finite optical timescale over which the measurement will be performed. From here we study the implications of this timescale in the intensity of single particle clicks. Consider the terms in equation (6). Since the accelerated worldline $x_0(\tau) = (\xi_0, \tau)$ lies completely in region I, the region II operators do not contribute to the monopole interactions (A.2). Therefore,

$$\hat{\phi}[x(\tau)] = \int \mathrm{d}k \hat{b}_{k,I} w_{k,I}(\xi_0, \tau) + \hat{b}^{\dagger}_{k,I} w_{k,I}(\xi_0, \tau)^{\star} \equiv \hat{\psi}_I(\xi_0, \tau), \tag{A.6}$$

and using the expansion:

$$\hat{\psi}_I(\xi_0, \eta_2)\hat{\psi}_I(\xi_0, \eta_1) =: \hat{\psi}_I(\xi_0, \eta_2)\hat{\psi}_I(\xi_0, \eta_1) : + \int \frac{\mathrm{d}k}{4\pi|k|} \mathrm{e}^{-\mathrm{i}|k|c(\eta_2-\eta_1)}, \tag{A.7}$$

equation (A.5) can be rewritten:

$$\begin{aligned}\mathcal{F}(E - E_0) = &\int_{\tau_0}^{\tau_0+T} \mathrm{d}\eta_1 \int_{\tau_0}^{\eta_1} \mathrm{d}\eta_2\, \mathrm{e}^{\mathrm{i}(E-E_0)(\eta_1-\eta_2)/\hbar} \Big(\mathrm{Tr}\{: \hat{\psi}_I(\xi_0, \eta_2)\hat{\psi}_I(\xi_0, \eta_1) : \mathrm{Tr}_{\perp\psi_{\mathrm{D}}}|\Psi\rangle\langle\Psi|\} \\ &+ \int \frac{\mathrm{d}k}{4\pi|k|} \mathrm{e}^{-\mathrm{i}|k|c(\eta_2-\eta_1)}\Big) + c.c.\end{aligned} \tag{A.8}$$

It is useful to separate $\mathcal{F}(E - E_0) = \mathcal{F}_{n.o}(E - E_0) + \mathcal{F}_{vac}(E - E_0)$ into a normally ordered part and a vacuum fluctuation contribution (that is independent of the state). First we focus on the vacuum fluctuation contribution. Defining $\eta = \eta_1 - \eta_2$, we find:

$$\mathcal{F}_{vac}(E - E_0) = \int \frac{\mathrm{d}k}{4\pi|k|} \int_{-T}^{T} \mathrm{d}\eta (T - |\eta|)\, \mathrm{e}^{\mathrm{i}((E-E_0)/\hbar+|k|c)\eta}. \tag{A.9}$$

Consider the innermost integrand. Since the time of measurement is much larger than the typical timescale of a transition, $T \gg \frac{\hbar}{E_1-E_0}$, for all wave numbers k the phase oscillates rapidly over the η integration domain. The inner integral is then approximately equal to $2\pi\delta((E - E_0)/\hbar + |k|c)$. Since this is zero for all k the vacuum contribution is negligible.

Now to evaluate the normally ordered field contribution we use the expansion:

$$\hat{\phi}(\xi,\tau) = \hat{d}\psi_{\mathrm{D}}(\xi,\tau) + \hat{d}^{\dagger}\psi_{\mathrm{D}}^{\star}(\xi,\tau) + \hat{\phi}'. \tag{A.10}$$

In normal order all the annihilation operators are to the right of the creation operators, however, only the mode ψ_D is occupied in the state accessible to the detector, $\mathrm{Tr}_{\perp\psi_{\mathrm{D}}}|\Psi\rangle\langle\Psi|$. Therefore, no contributions to (A.5) can arise from the $\hat{\phi}'$ part of the field operator since those terms will be zeroed by acting on the orthogonal subspace vacuum state, therefore:

$$\begin{aligned}\mathrm{Tr}\{: \hat{\psi}_I(\xi_0,\eta_2)\hat{\psi}_I(\xi_0,\eta_1) : \mathrm{Tr}_{\perp\psi_{\mathrm{D}}}|\Psi\rangle\langle\Psi|\} &= \psi_{\mathrm{D}}(\xi_0,\eta_2)\psi_{\mathrm{D}}(\xi_0,\eta_1)\langle\Psi|\hat{d}^2|\Psi\rangle \\ &+\psi_{\mathrm{D}}^{\star}(\xi_0,\eta_2)\psi_{\mathrm{D}}(\xi_0,\eta_1)\langle\Psi|\hat{d}^{\dagger}\hat{d}|\Psi\rangle + \mathrm{c.c.}\end{aligned} \tag{A.11}$$

To analyse $\mathcal{F}_{n.o}(E - E_0)$ further, it is useful to decompose ψ_{D} into Rindler frequencies:

$$\psi_{\mathrm{D}}(\xi,\tau) = \int(\psi_{\mathrm{D}}, w_{k,I})w_{k,I}(\xi,\tau)\,\mathrm{d}k. \tag{A.12}$$

Consider the contribution to (A.8) from the first term in (A.11), let $\Delta\omega = \frac{E-E_0}{\hbar}$ then:

$$\begin{aligned}&\int_{\tau_0}^{\tau_0+T}\mathrm{d}\eta_1\int_{\tau_0}^{\eta_1}\mathrm{d}\eta_2\,\mathrm{e}^{\mathrm{i}(E-E_0)(\eta_1-\eta_2)/\hbar}\psi_{\mathrm{D}}(\xi_0,\eta_2)\psi_{\mathrm{D}}(\xi_0,\eta_1) \\ &\quad= \int\frac{\mathrm{d}k\,\mathrm{d}k'(\psi_{\mathrm{D}}, w_{k,I})(\psi_{\mathrm{D}}, w_{k',I})}{4\pi\sqrt{|k||k'|}}\mathrm{e}^{\mathrm{i}(\xi_0(k+k')-\tau_0(|k|+|k'|)} \\ &\quad\times\int_0^T\mathrm{d}\eta_1\,\mathrm{e}^{\mathrm{i}(\Delta\omega-|k'|)\eta_1}\int_0^{\eta_1}\mathrm{d}\eta_2\,\mathrm{e}^{-\mathrm{i}(\Delta\omega+|k|)\eta_2}.\end{aligned} \tag{A.13}$$

Since the ψ_D mode has a peaked frequency approximately equal to the typical transitional frequency, $\omega_c \sim \Delta\omega$, the innermost integral oscillates at double the optical frequency. But the measurement time is assumed to be many optical periods long (i.e., $T \sim L/c = \frac{N}{\omega_c}$), therefore this term only makes a small contribution to the total integral. By analysing all the other terms in a similar way, it can be seen that the dominant contribution comes from the second term in (A.11). One may recognize this step as the rotating wave approximation, however this is a different approximation to the usual RWA in the inertial frame because the frequencies here are Rindler frequencies, (in contrast see [28] for a different interpretation of the Glauber model when the RWA is made w.r.t Minkowski frequencies). One then obtains:

$$\begin{aligned}\mathcal{F}(E - E_0) = \langle\Psi|\hat{d}^{\dagger}\hat{d}|\Psi\rangle&\int_0^T\mathrm{d}\eta_1\int_0^{\eta_1}\mathrm{d}\eta_2\,\mathrm{e}^{\mathrm{i}(E-E_0)(\eta_1-\eta_2)/\hbar} \\ &\times\psi_{\mathrm{D}}^{\star}(\xi_0,\eta_2+\tau_0)\psi_{\mathrm{D}}(\xi_0,\eta_1+\tau_0) + \mathrm{c.c.}\end{aligned} \tag{A.14}$$

Note that all the dependence on the state of the field, $|\Psi\rangle$, has completely factored out of the integral. The probability of a single click at the spacetime point (ξ_0,τ_0) (over a measurement duration T) then takes the form:

$$\mathcal{P}(\xi_0,\tau_0;T) = \alpha(\xi_0,\tau_0;T)\langle\Psi|\hat{d}^{\dagger}\hat{d}|\Psi\rangle, \tag{A.15}$$

where

$$\begin{aligned}\alpha(\xi_0,\tau_0,T) = \frac{\mu^2}{\hbar^2}\sum_E|\langle E|m(0)|E_0\rangle|^2&\int_0^T\mathrm{d}\eta_1\int_0^{\eta_1}\mathrm{d}\eta_2\,\mathrm{e}^{\mathrm{i}(E-E_0)(\eta_1-\eta_2)/\hbar} \\ &\times\psi_{\mathrm{D}}^{\star}(\xi_0,\eta_2+\tau_0)\psi_{\mathrm{D}}(\xi_0,\eta_1+\tau_0) + c.c.\end{aligned} \tag{A.16}$$

We now consider two measurement scenarios. Consider first the case when the measurement time T is much shorter than the period of the entire pulse but still longer than the binding period, $\frac{\hbar}{E_1 - E_0} \ll T \ll L/c$. In this regime, the measurement time is too short to resolve the spectral line-width of the wave-packet and so the quasi-monochromatic approximation can be used:

$$\psi_{\mathrm{D}}(\xi_0, \eta_1 + \tau_0) \sim \psi_{\mathrm{D}}(\xi_0, \tau_0)\, \mathrm{e}^{-\mathrm{i}\omega_c \eta_1} \tag{A.17}$$

where $\omega_c = \frac{cN}{L}$. In this case, $\alpha(\xi_0, \tau_0, T) \propto T$. Therefore, the single particle absorption probability scales linearly with the measurement duration.

The second case we consider is when the measurement time is taken to be greater than the time it takes the single mode to traverse the photo-detector, $T > L/c$. Then frequencies other than the peak frequency will contribute to the detection probability. In this case equation (A.15) must be interpreted as the probability of a single count per shot (unit time T). One can build up photo-count statistics by preparing an ensemble of identical systems and performing a single shot measurement (of time T) on each system in the ensemble.

We conclude this appendix by noting that taking $|\Psi\rangle$ to be the Minkowski vacuum state then agrees up to a device-dependent proportionality constant with the same expression arrived at using the generalized Glauber mode of detection (12).

References

[1] Alsing P M and Milburn G J 2003 *Phys. Rev. Lett.* **91** 180404
Fuentes I and Mann R B 2005 *Phys. Rev. Lett.* **95** 120404
[2] Schützhold R and Unruh W G 2005 quant-ph/0506028
[3] Bruschi D, Louko J, Martín-Martínez E, Dragan A and Fuentes I 2010 *Phys. Rev* A **82** 042332
[4] Glauber R J 1963 *Phys. Rev.* **130** 2529
[5] Candelas P 1980 *Phys. Rev.* D **21** 2185–2202
Letaw J R and Pfautsch J D 1980 *Phys. Rev.* D **22** 1345
Sriramkumar L and Padmanabhan T 2002 *Int. J. Mod. Phys.* D **11** 1
[6] Birrell N D and Davies P C W 1982 *Quantum Fields In Curved Space* (Cambridge: Cambridge University Press)
[7] Hawking S W 1975 *Commun. Math. Phys.* **43** 199–220
[8] Takagi S 1986 *Prog. Theor. Phys. Suppl.* **88** 1–142
[9] Unruh W 1976 *Phys. Rev.* D **14** 870
[10] Reznik B 2003 *Found. Phys.* **33** 167
Reznik B, Retzker A and Silman J 2005 *Phys. Rev.* A **71** 042104
[11] Dragan A, Doukas J and Martín-Martínez E 2013 *Phys. Rev.* A **87** 052326
[12] Dragan A 2004 *Phys. Rev.* A **70** 053814
Dragan A and Zin P 2007 *Phys. Rev.* A **76** 042124
[13] Banaszek K, Dragan A, Wasilewski W and Radzewicz C 2004 *Phys. Rev. Lett.* **92** 257901
[14] Schlicht S 2004 *Class. Quantum Grav.* **21** 4647
[15] Martín-Martínez E, Montero M and del Rey M 2013 *Phys. Rev.* D **87** 064038
[16] Sorkin R 1993 *Proc. 1993 Int. Symp. Directions in General Relativity (Maryland)* vol 2 (Cambridge: Cambridge University Press) p 293 (arXiv:gr-qc/9302018)
Lin S-Y 2012 See also arXiv:1104.0772v2
[17] Martín-Martínez E, Garay L J and León J 2010 *Phys. Rev.* D **82** 064006
[18] Martín-Martínez E, Garay L J and León J 2010 *Phys. Rev.* D **82** 064028
Martín-Martínez E, Garay L J and León J 2012 *Class. Quantum Grav.* **29** 224006
[19] Mandel L and Wolf E 1995 *Optical coherence and quantum optics* (Cambridge: Cambridge University Press)
[20] Downes T D, Fuentes I and Ralph T 2011 *Phys. Rev. Lett.* **106** 210502
[21] Newton T D and Wigner E P 1949 *Rev. Mod. Phys.* **21** 400
[22] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: Wiley) chapter III
[23] Duan L-M, Giedke G, Cirac J I and Zoller P 2000 *Phys. Rev. Lett.* **84** 2722
Giovannetti V, Mancini S, Vitali D and Tombesi P 2003 *Phys. Rev.* A **67** 022320

[24] See for example Dragan A and Banaszek K 2001 *Phys. Rev.* A **63** 062102
[25] Downes T G, Ralph T C and Walk N 2013 *Phys. Rev.* A **87** 012327
[26] Summers S J and Werner R F 1985 *Phys. Lett.* A **110** 257
Summers S J and Werner R F 1987 *J. Math. Phys.* **28** 2448
[27] Zych M, Costa F, Kofler J and Brukner C 2010 *Phys. Rev.* D **81** 125019
[28] Buscemi F and Compagno G 2009 *Phys. Rev.* A **80** 022117