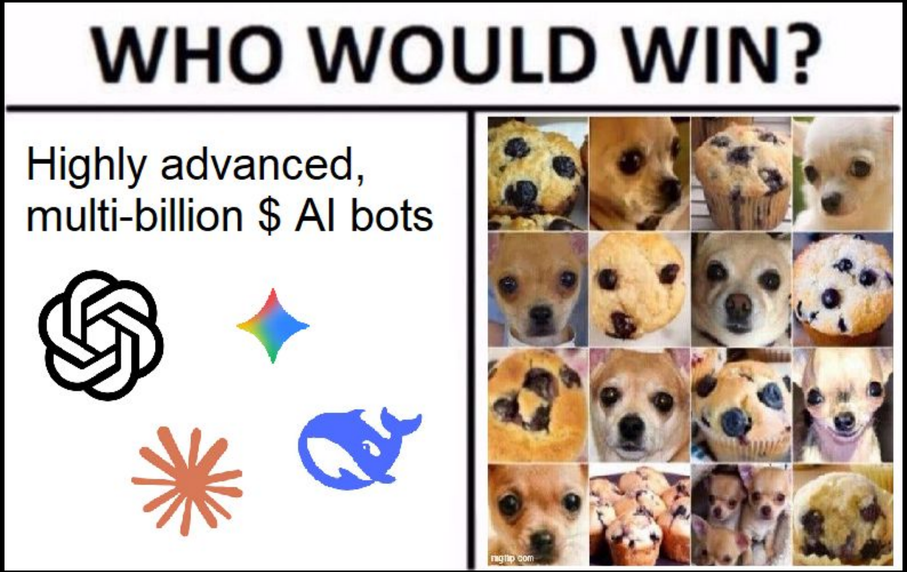
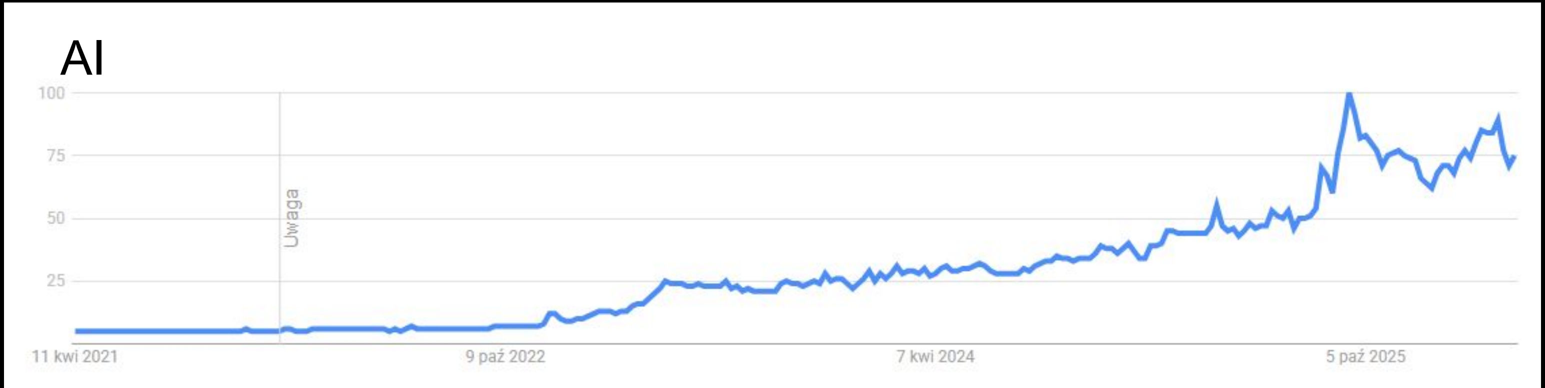
Computer modeling of physical phenomena

Neural networks - introduction



15-17 April 2026


AI
100
75
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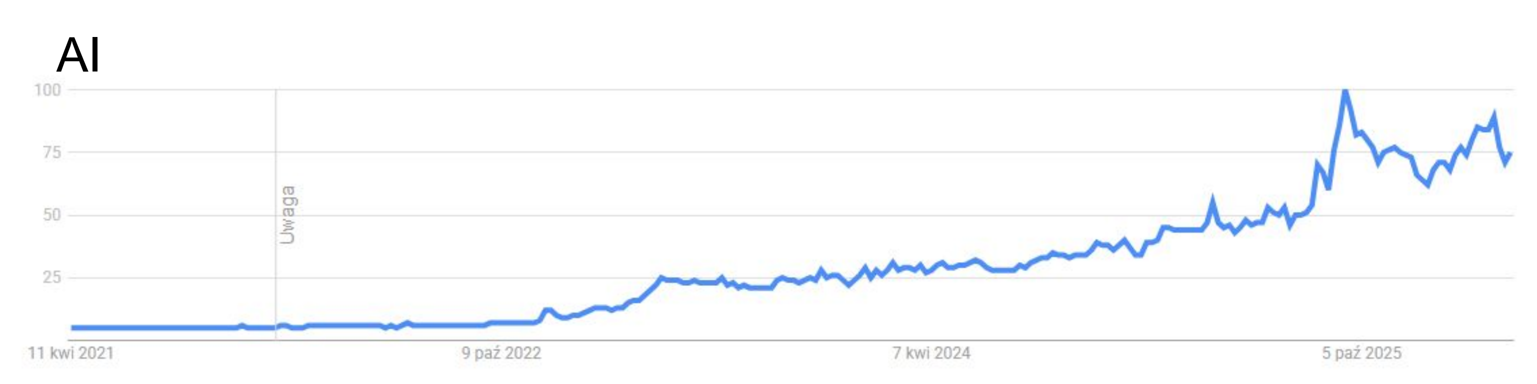
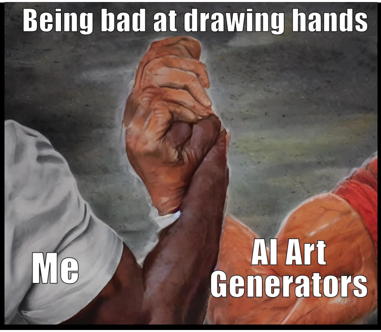
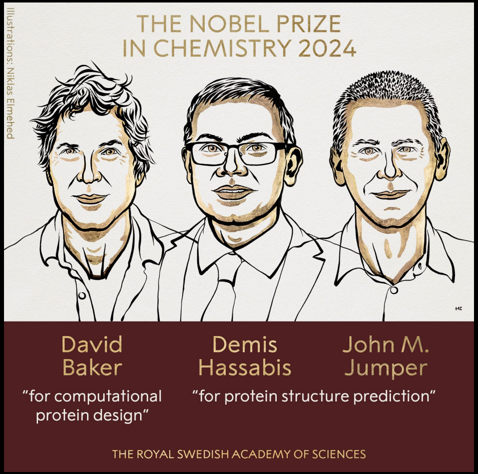
Uwaga
11 kwi 2021
9 paź 2022
7 kwi 2024
5 paź 2025







ARTIFICIAL INTELLIGENCE

How Will AI Affect the Global Workforce?

www.fuw.edu.pl/~dwos/cmpp2026/

The use of AI-based tools is strictly prohibited during classes.

The main culprit

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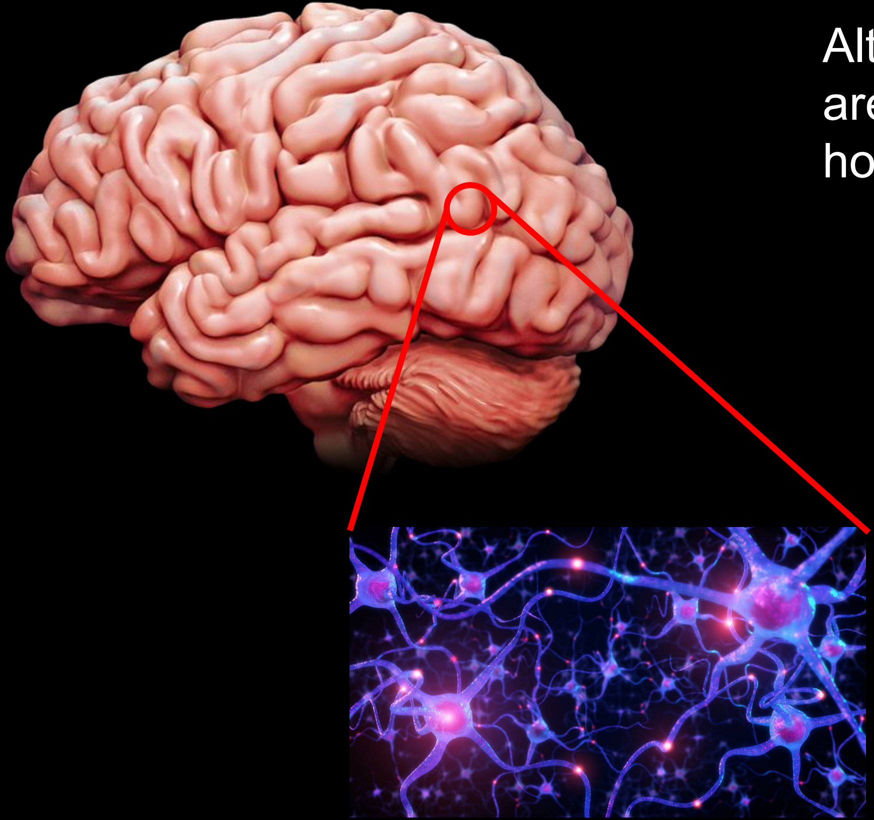

Although divided into highly specialized areas, on a micro level stays relatively homogeneous and self-similar – neurons

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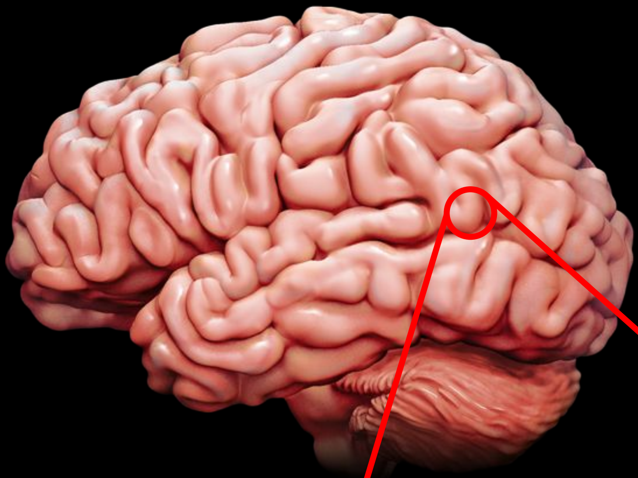

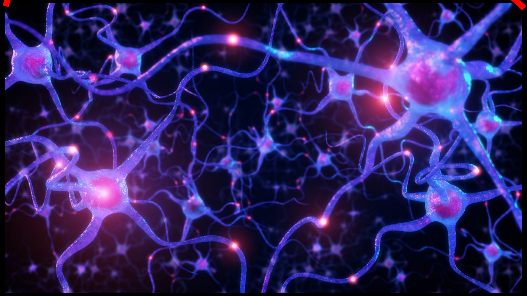

Although divided into highly specialized areas, on a micro level stays relatively homogeneous and self-similar – neurons

Neurons receive input from other neurons through synapses

Based on all inputs decides on the output (e.g. sends signal if the sum of all inputs exceeds a certain threshold)

Is capable of learning!
(unless it’s pre-exam session student)

Neural networks - history

Early XX century – human learning should be represented as strengthening of *some property* of neurons

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‘When an axon of cell A is neat enough to excite cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased.’ ~ D. Hebb, The Organization of Behavior, 1949

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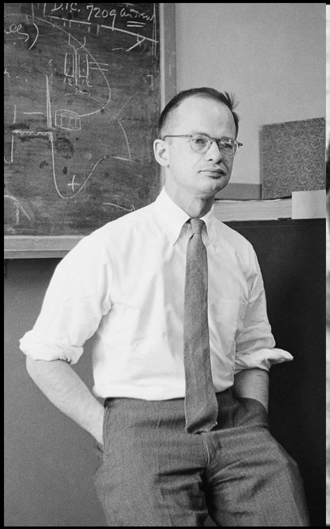
‘Neurons that fire together, wire together.’

Neural networks - history

1948 (Walter Pitts and Warren McCulloch) - artificial neuron

each neuron in the brain is a simple digital processor and the brain as a whole is a form of computing machine

„A logical calculus of the ideas immanent in nervous activity” Bulletin of Mathematical Biophysics, 5:115-133.



1923 - 1969

1898 - 1969

Neural networks - history

1948 (Alan Turing) - Intelligent Machinery

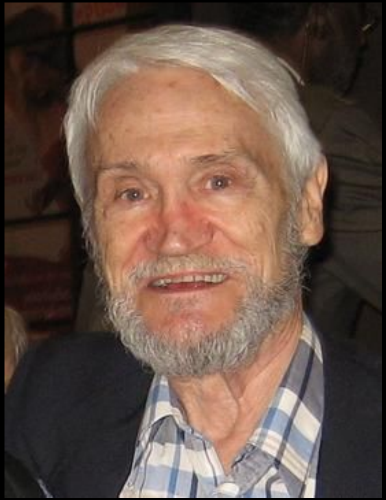
Network of randomly connected artificial neurons can be trained to perform a certain task.

Training can be done by means of genetic algorithm.

‘There is the genetical or evolutionary search by which a combination of genes is looked for, the criterion being the survival value. The remarkable success of this search confirms to some extent the idea that intellectual activity consists mainly of various kinds of search’

Neural networks - history

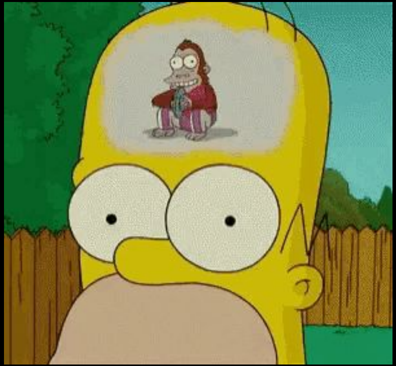
1953 (Belmont Farley and Wesley Clark) - first computer simulation of neural network



128 neurons – enough to recognize simple patterns

They discovered, that randomly destroying 10% of neurons does not affect the network's performance significantly

Brain vs computer



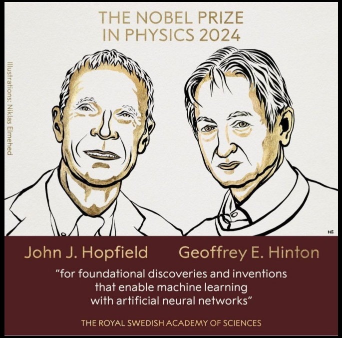
2*10^11 neurons, 3*10^13 synapses
Element size: 10^-6 m
Energy use: 25 W
Processing speed: 100 Hz
Parallel, distributed
Learns: Yes
Intelligent/Conscious: Sometimes

10^10 bytes RAM, 10^12 bytes on disk
Element size: 10^-9 m
Energy use: 100 W
Processing speed: 10^9 Hz
Serial, centralized
Learns: sometimes
Intelligent/Conscious: ???

Spin glass model – Hopfield network

Model hugely inspired by Ising model of ferromagnetism

John Hopfield
b. 1933



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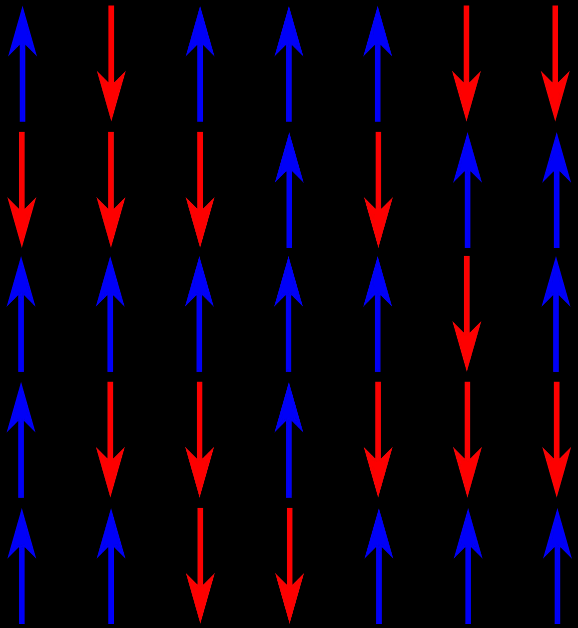
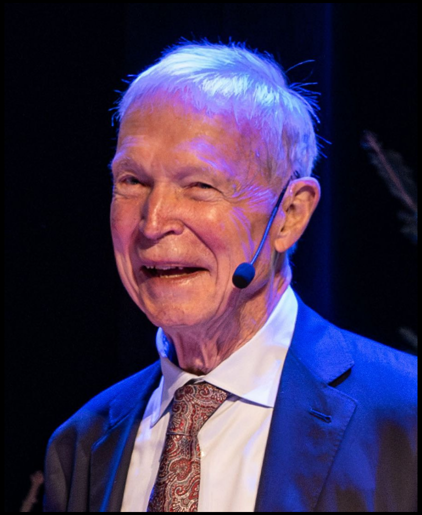


Assume N x N spin lattice



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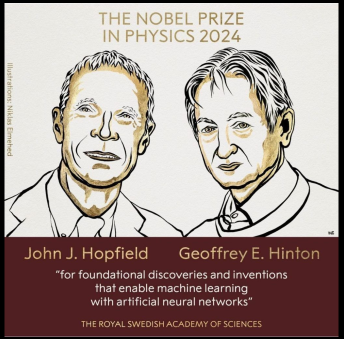

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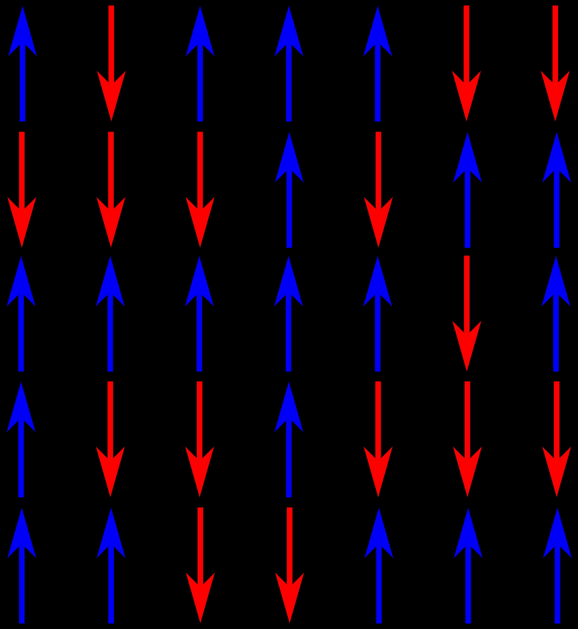

Assume N x N spin lattice with a hamiltonian given as

$$H = -\frac{1}{2}\sum_{i,j} w_{ij} s_i s_j$$

where $s_i = \pm 1$ is a spin and w_{ij} is a coupling constant. All spins can be coupled!

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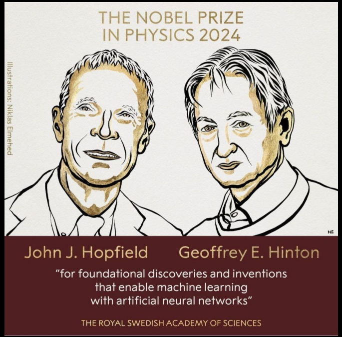
b. 1933



Training – Hebb’s rule

‘Neurons that fire together, wire together.
Neurons that fire out of sync, fail to link.’

J. J. Hopfield, "Neural networks and physical systems with emergent collective computational abilities", PNAS, 79,. 2554, 1982

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Given set of training data containing n elements, spin coupling can be calculated as

$$w = \frac{1}{n}\sum_{i=1}^{n} x_i \otimes x^i - \mathbb{1}$$

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If the bits corresponding to neurons i and j are equal, then their product will be positive. On the contrary, if they are unequal, their product will result in a negative.

J. J. Hopfield, "Neural networks and physical systems with emergent collective computational abilities", PNAS, 79,. 2554, 1982

Using the trained network

Energy of i-th spin $E_i = -\frac{1}{2} s_i \sum_j w_{ij} s_j = -\frac{1}{2} s_i h(i)$

where $h(i)$ is a field in which i-th spin resides.

If the product $s_i h(i)$ is negative, spin flip would result in a decrease of systems total energy.

Thus, given state S, we can calculate a state with lower total energy as

$$S' = \mathrm{sgn}(w \cdot S)$$

Using the trained network

- multiple iterations should transform input state into some state, which is a local energy minimum
- due to coupling matrix construction, training set becomes attractors of the system
- however, it is possible for other local minima to emerge
- capacity of such network is ~0.138 minimum per node

Tasks

1. Model a simple Hopfield network which detects whether a given 5x5 image (for simplicity, consider numpy array) is either an A or a Z. (0.3 p.)

2. Plot energy evolution of this system. (0.3 p.)

3. Try to expand the system up to 10x10 images with at least 5 recognized letters (try to use letters as different from each other as you can, distinguishing I from l is not advised) (0.4 p.)

4. (Bonus task, training might take a long time, be warned!)
   Try to expand the system even further so it can reliably distinguish images of all the letters from the English alphabet. (0.2 p.)
   If you want to go all out, here is a dataset with a lot of handwritten letters:
   https://www.kaggle.com/datasets/sankalpsrivastava26/capital-alphabets-28x28?resource=download

```
A = “””
XOOOX
OXXXO
OOOOO
OXXXO
OXXXO
“””
```

```
Z = “””
OOOOO
XXXOX
XXOXX
XOXXX
OOOOO
“””
```