

Shortest path optimizations

From graphs to ants

Problem

- I'm visiting Rome for only a few days, and I want to make the most of it. How I can plan my sightseeing in a way that minimizes the walking between attractions?

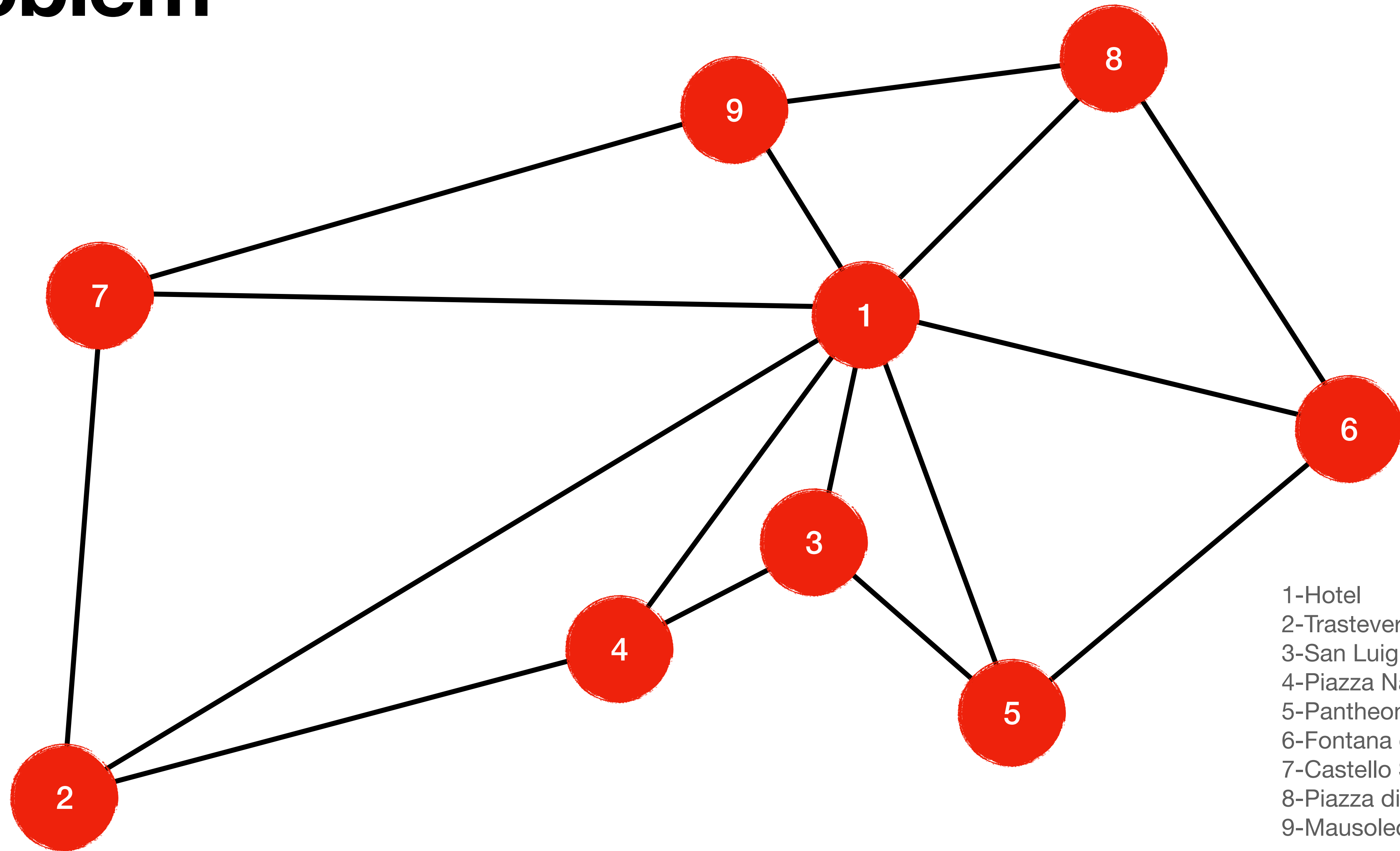


Problem

ROMA

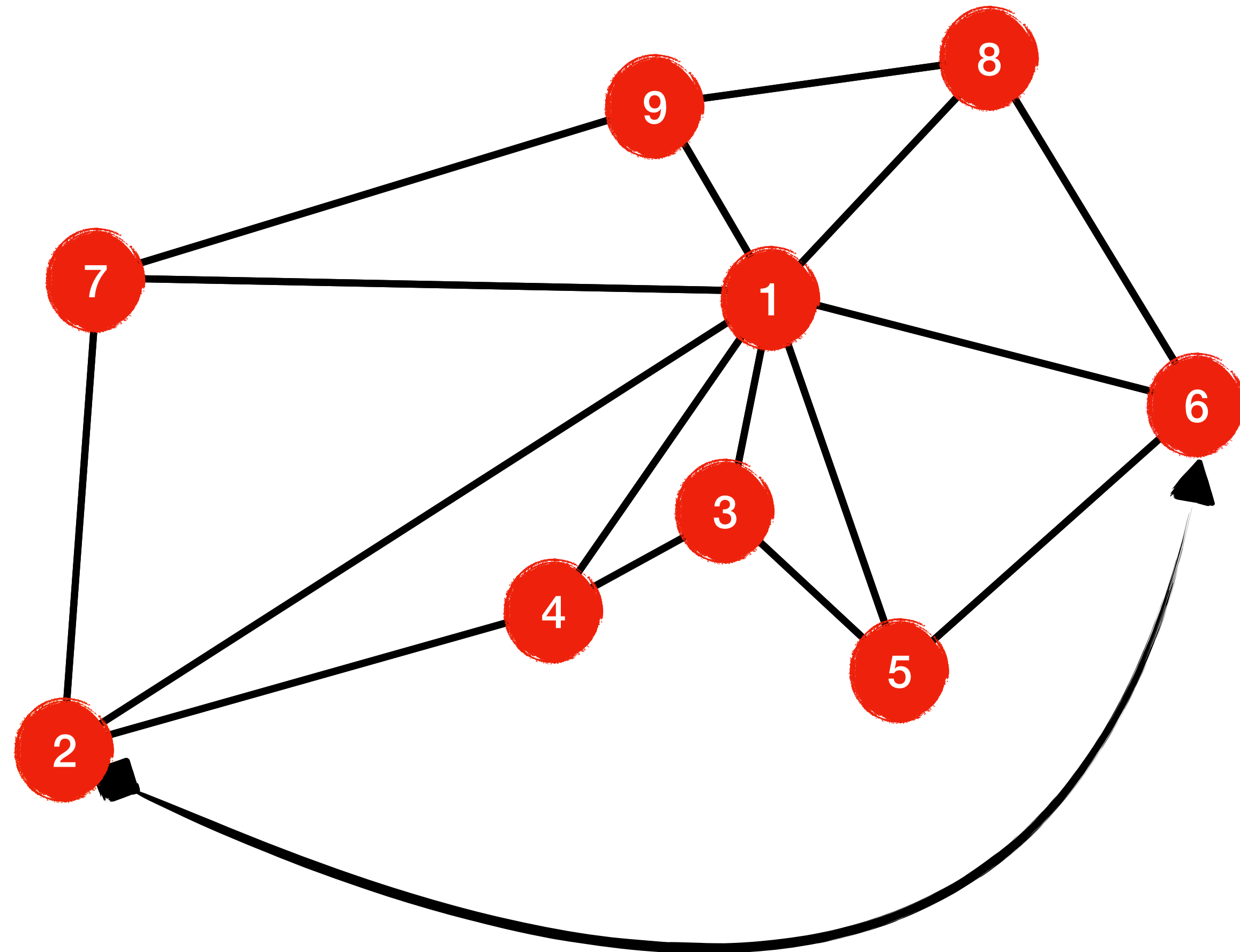


Problem



- 1-Hotel
- 2-Trastevere
- 3-San Luigi
- 4-Piazza Navona
- 5-Pantheon
- 6-Fontana de Trevi
- 7-Castello San Angelo
- 8-Piazza di Spagna
- 9-Mausoleo di Augusto

Problem



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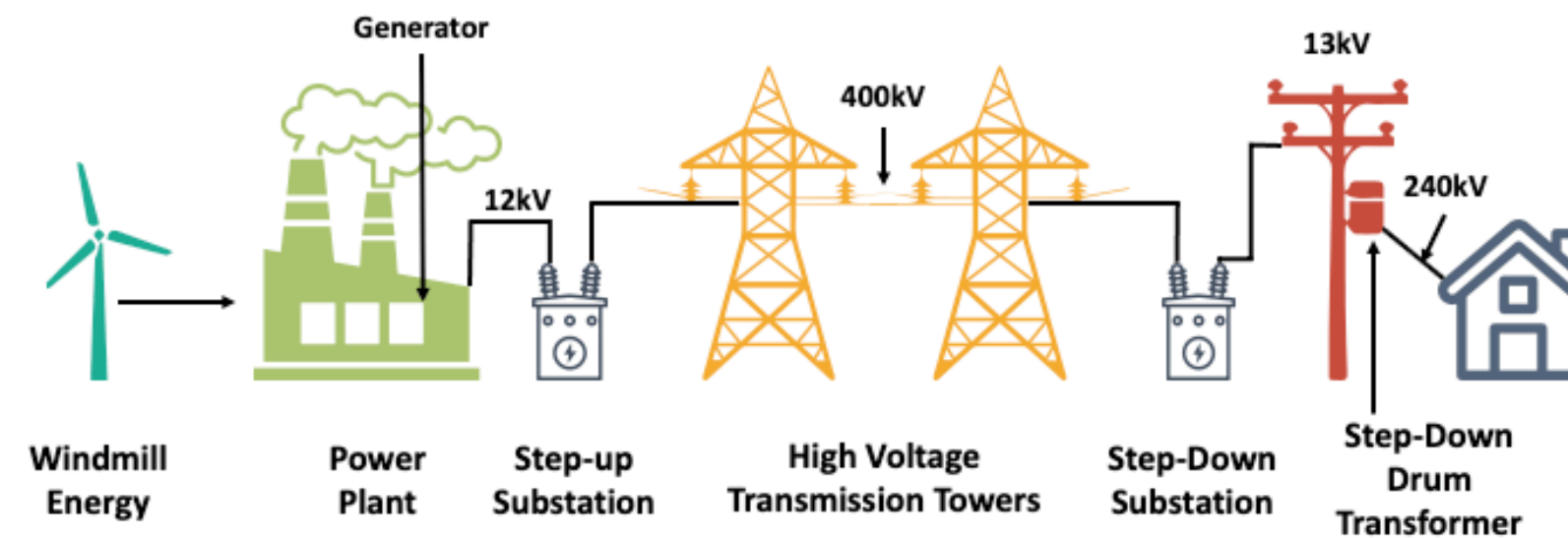
8-Piazza di Spagna

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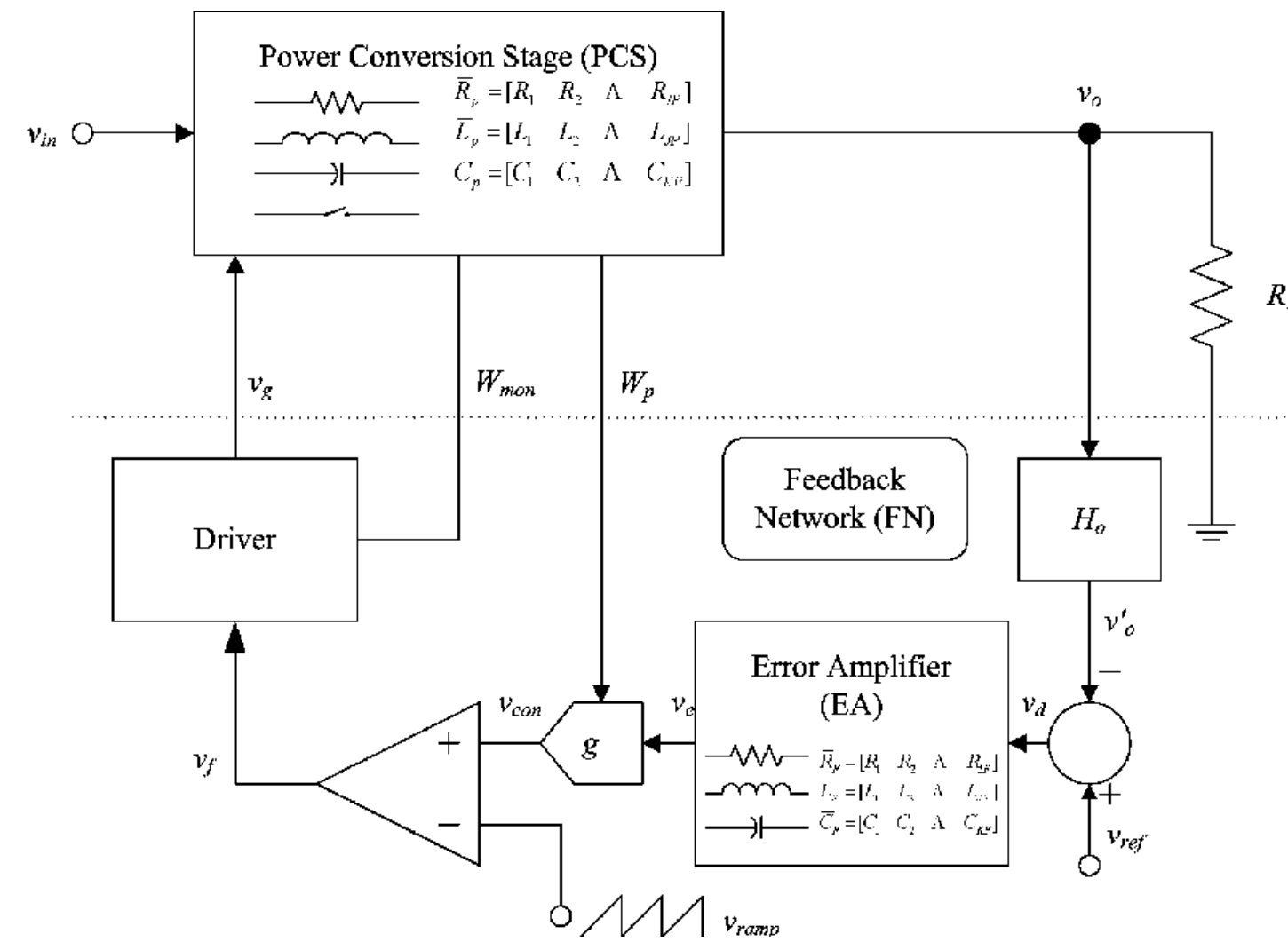
Why this problem matters to you?

ELECTRIC POWER DISTRIBUTION

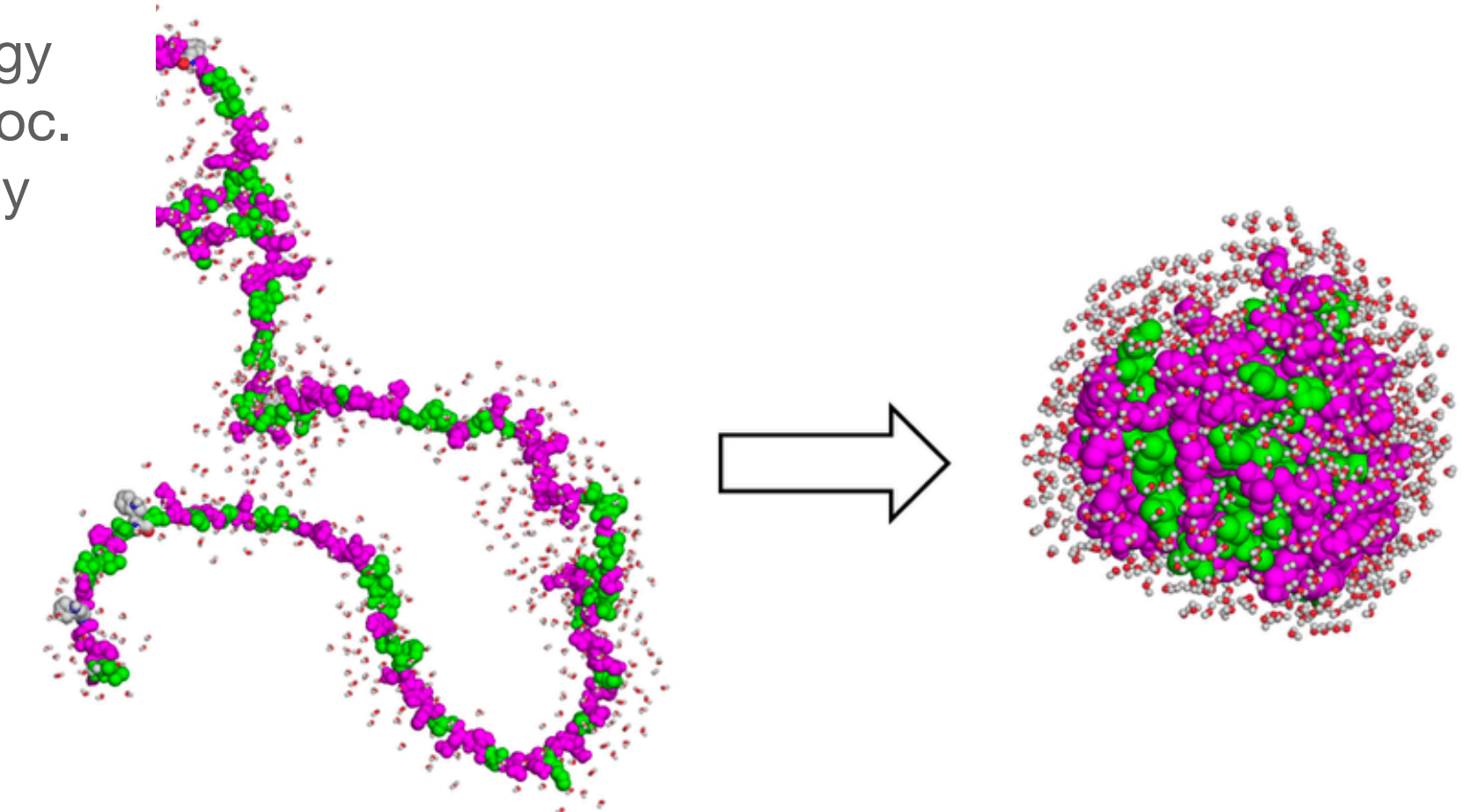
Enter your sub headline here



Warner L and Vogel U 2008 Optimization of energy supply networks using ant colony optimization Proc. Environmental Informatics and Industrial Ecology



Zhang J, Chung H S-H, Lo A W-L and Huang T
2009 Extended Ant Colony Optimization
Algorithm for Power Electronic Circuit Design
IEEE Trans. Power Electron. 24 147-62



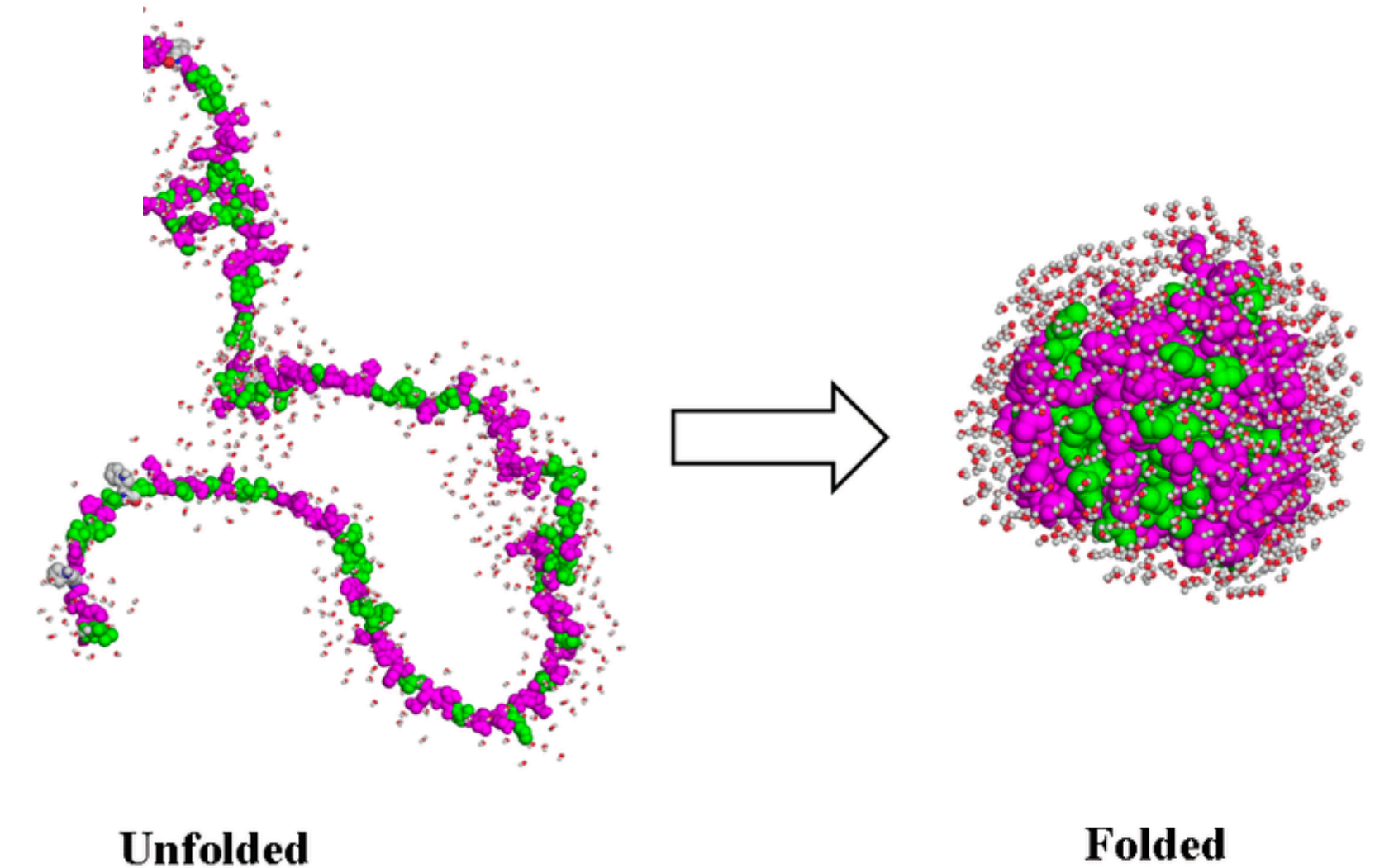
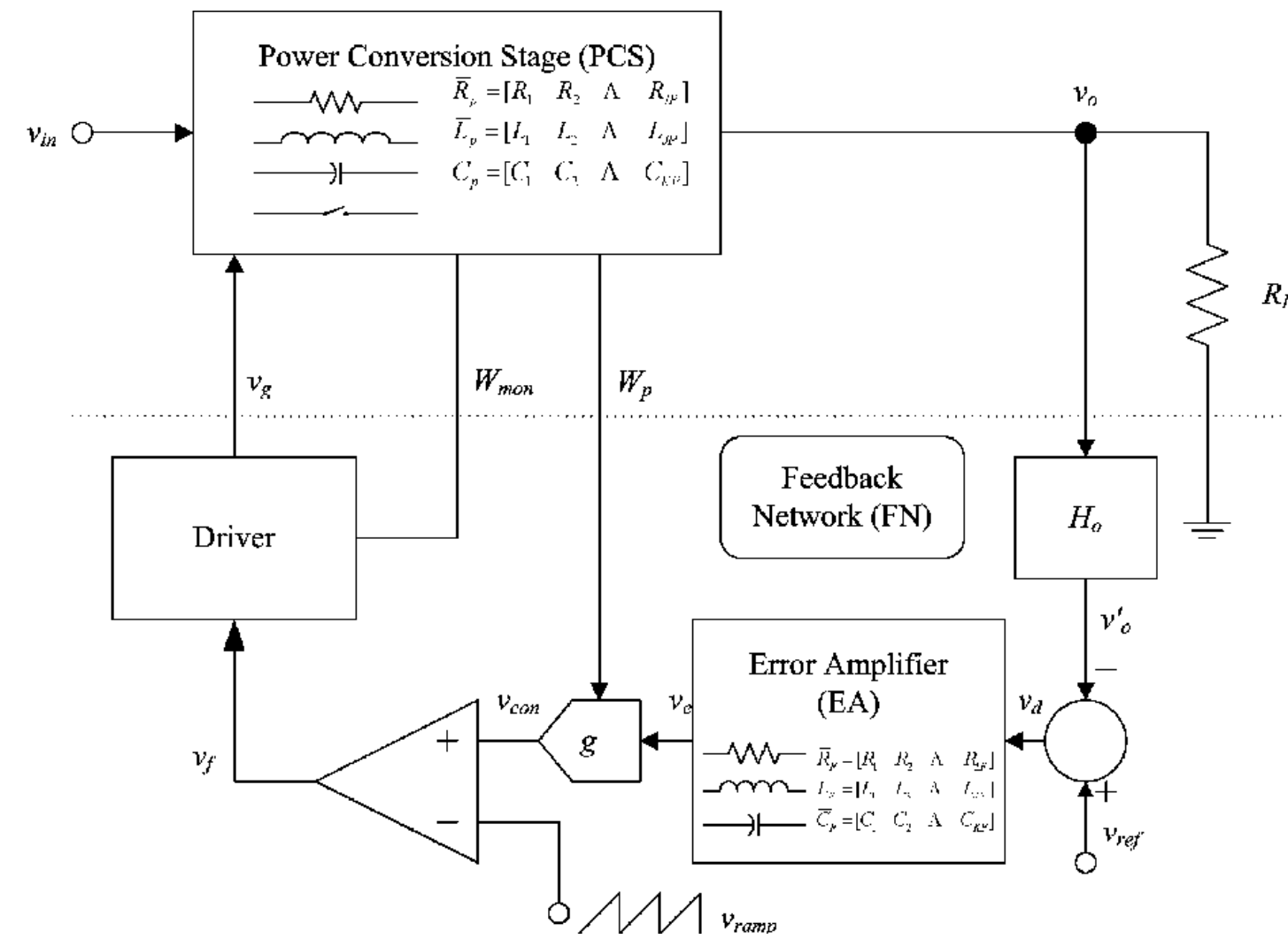
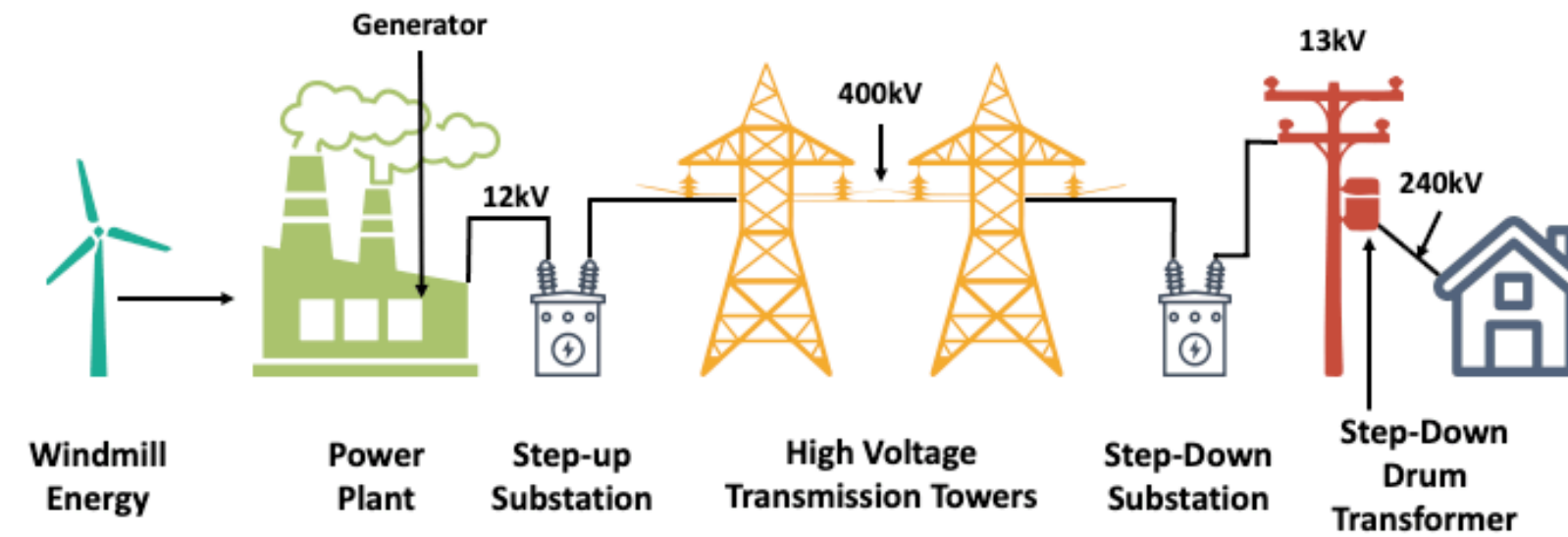
Unfolded

Folded

Hu X, Zhang J, Xiao J and Li Y 2008 Protein folding in hydrophobic-polar lattice model: a flexible ant-colony optimization approach
Protein Pept. Lett. 15 469-77

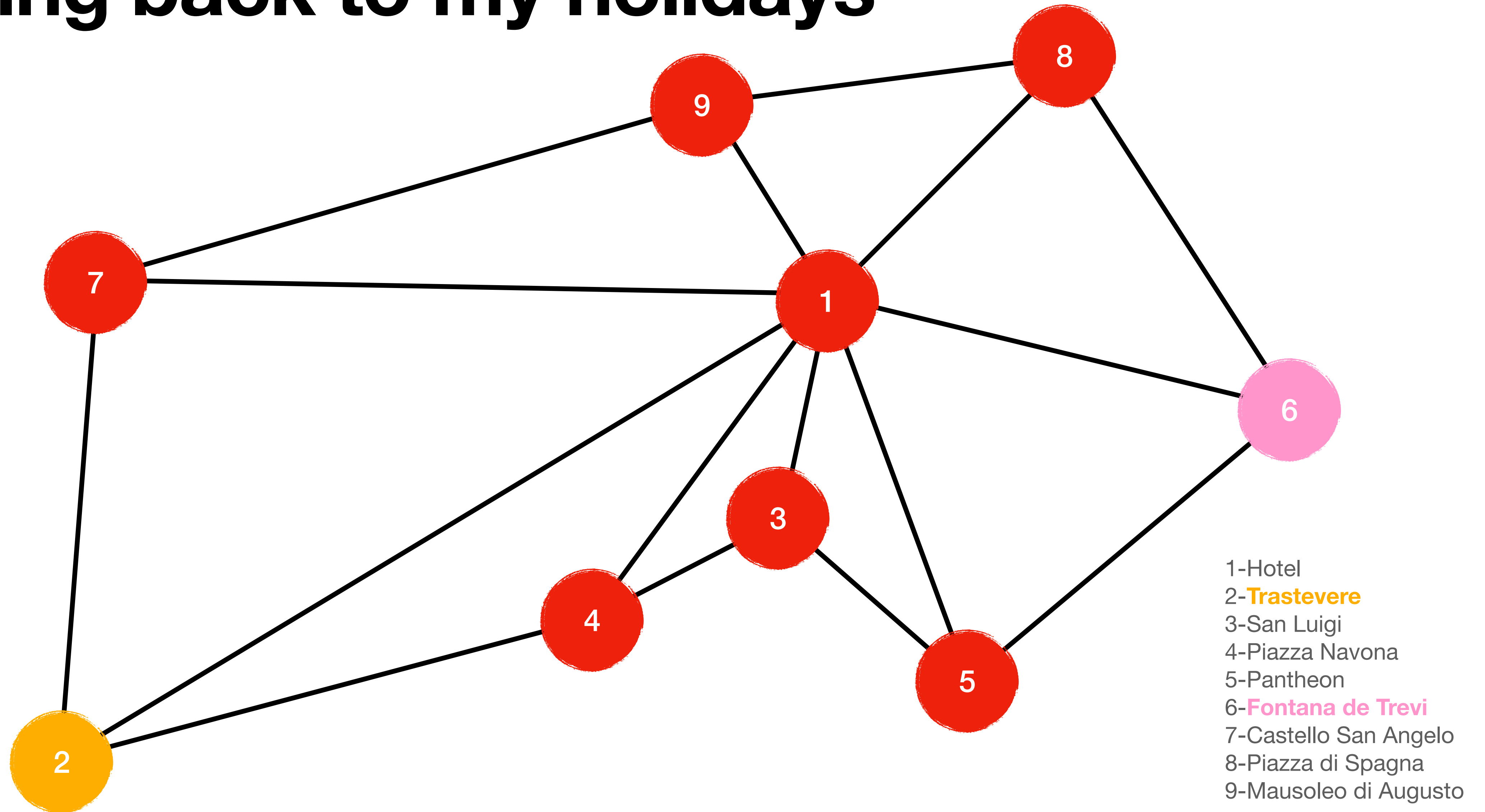
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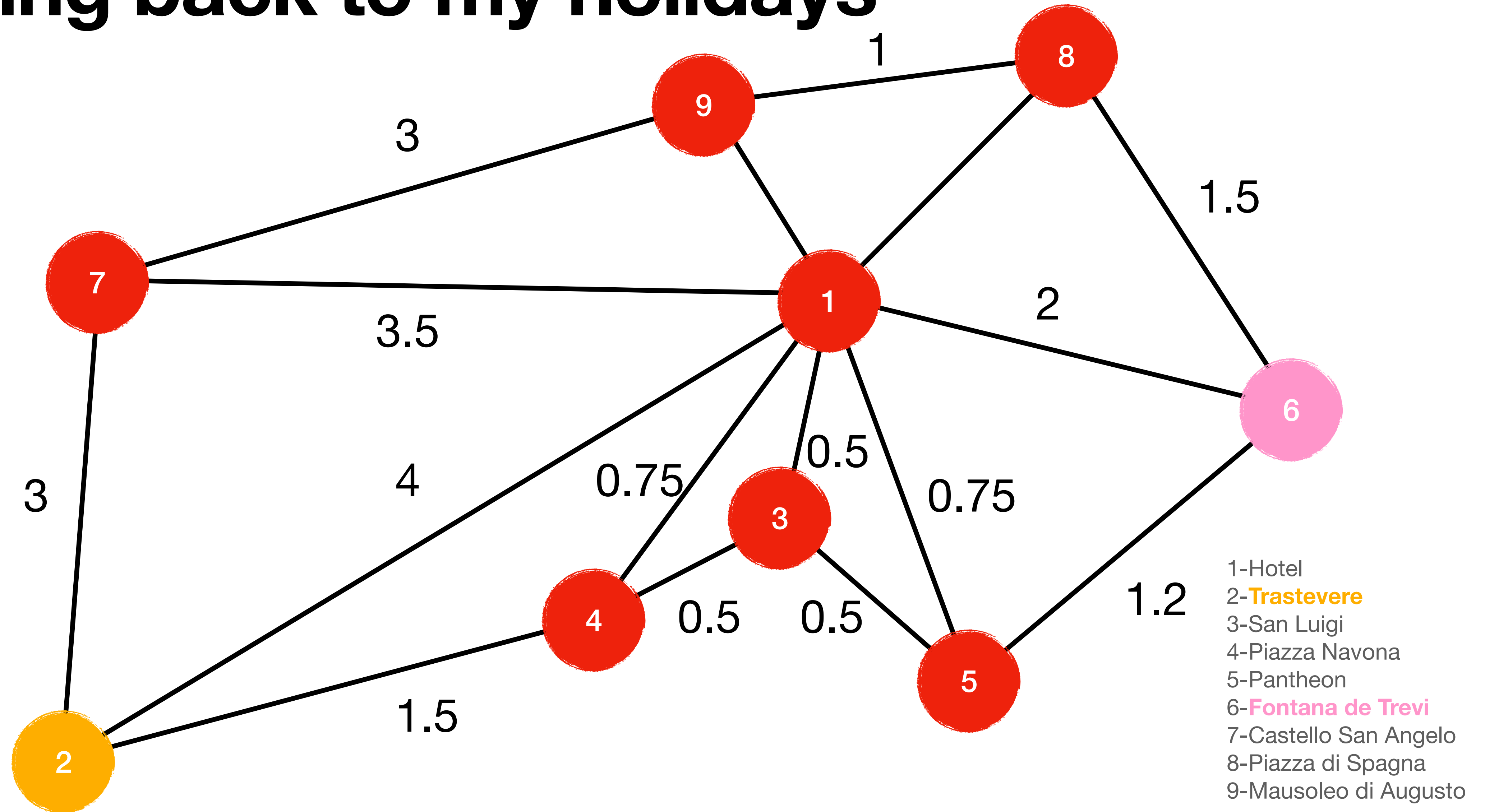


Optimization problems!

Going back to my holidays



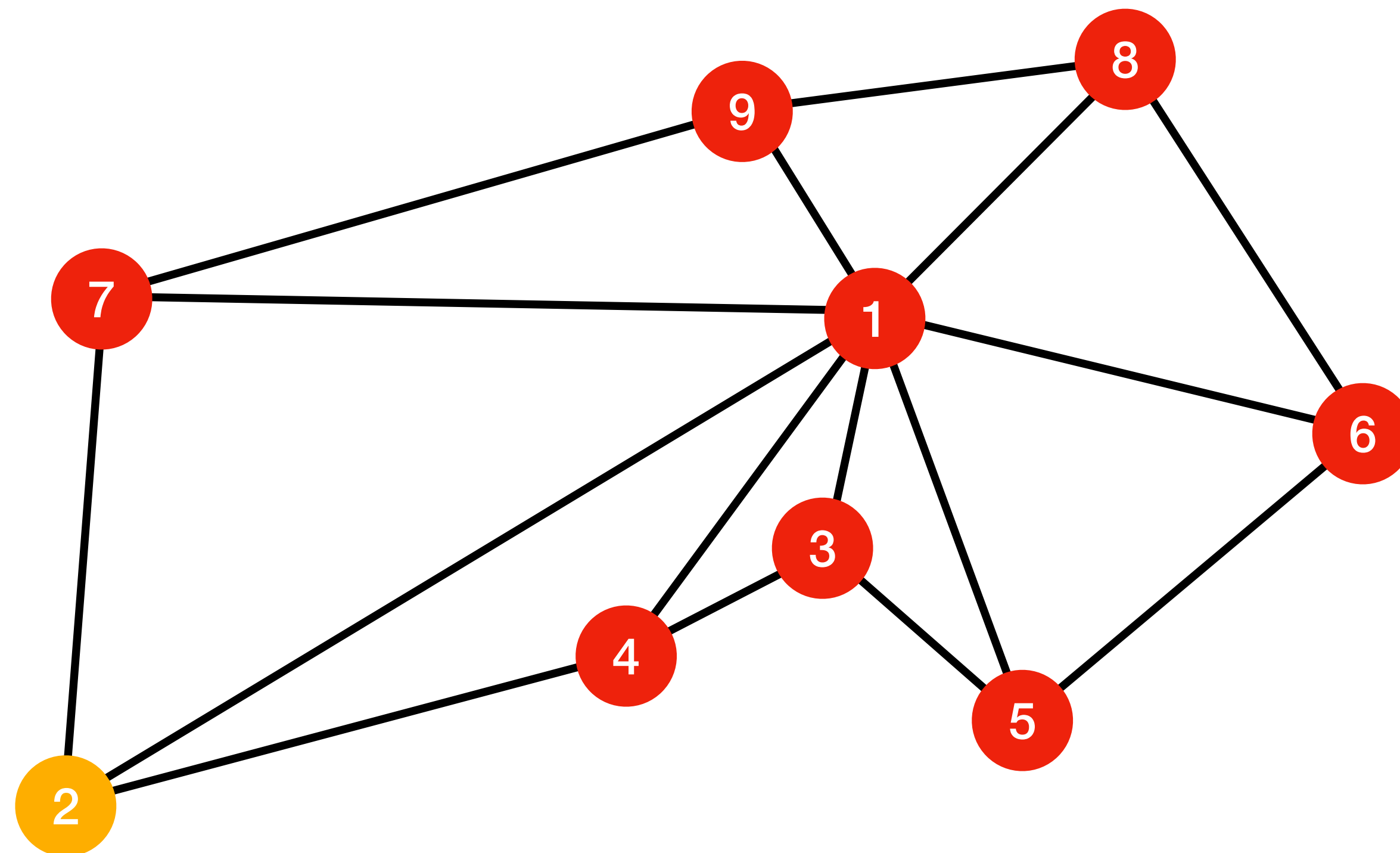
Going back to my holidays



Brute force

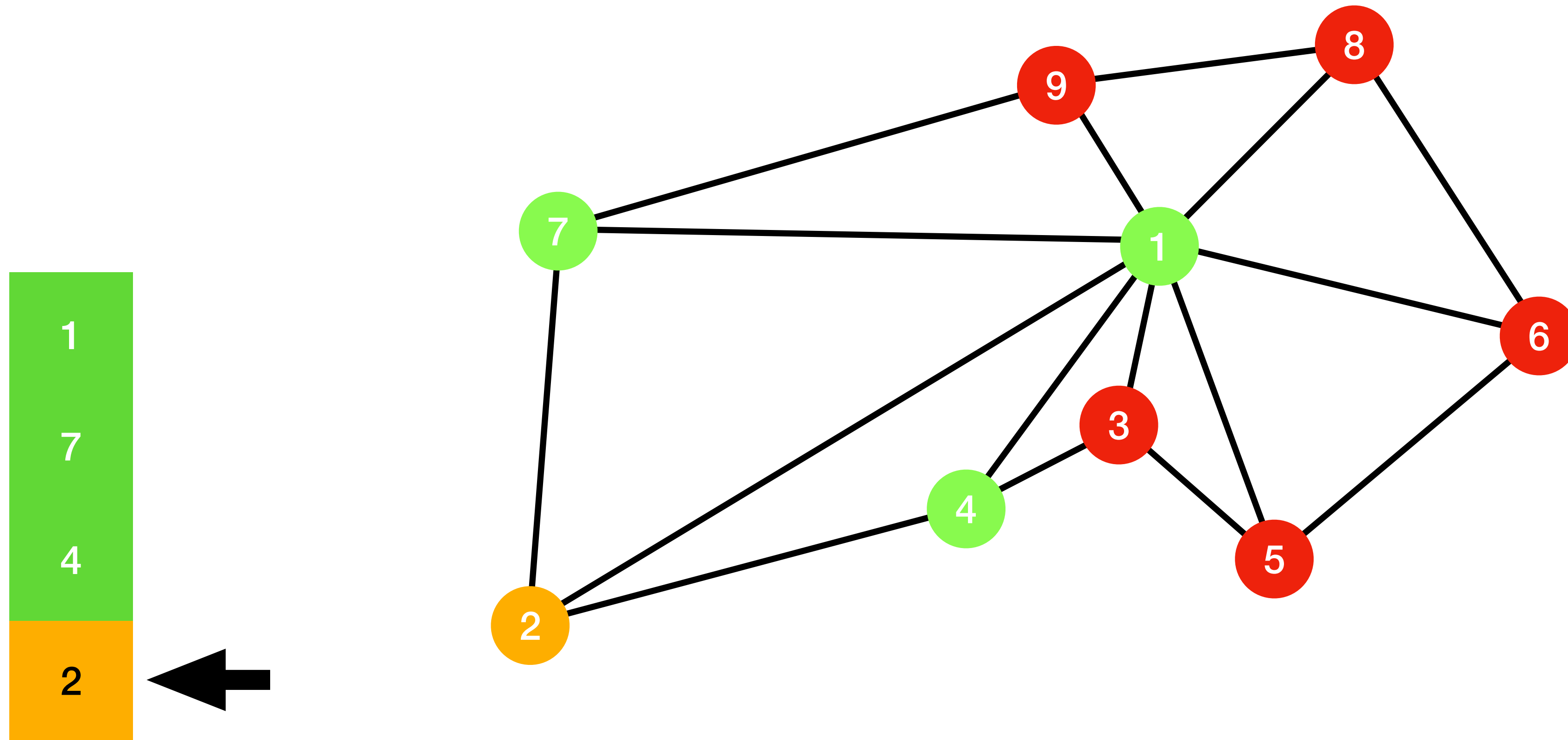
- Starting at the node 2 we can calculate the shortest path to the next node connected to it. Then re run the procedure for the next until we find a solution, this is called *Breadth-first search*.

Brute force

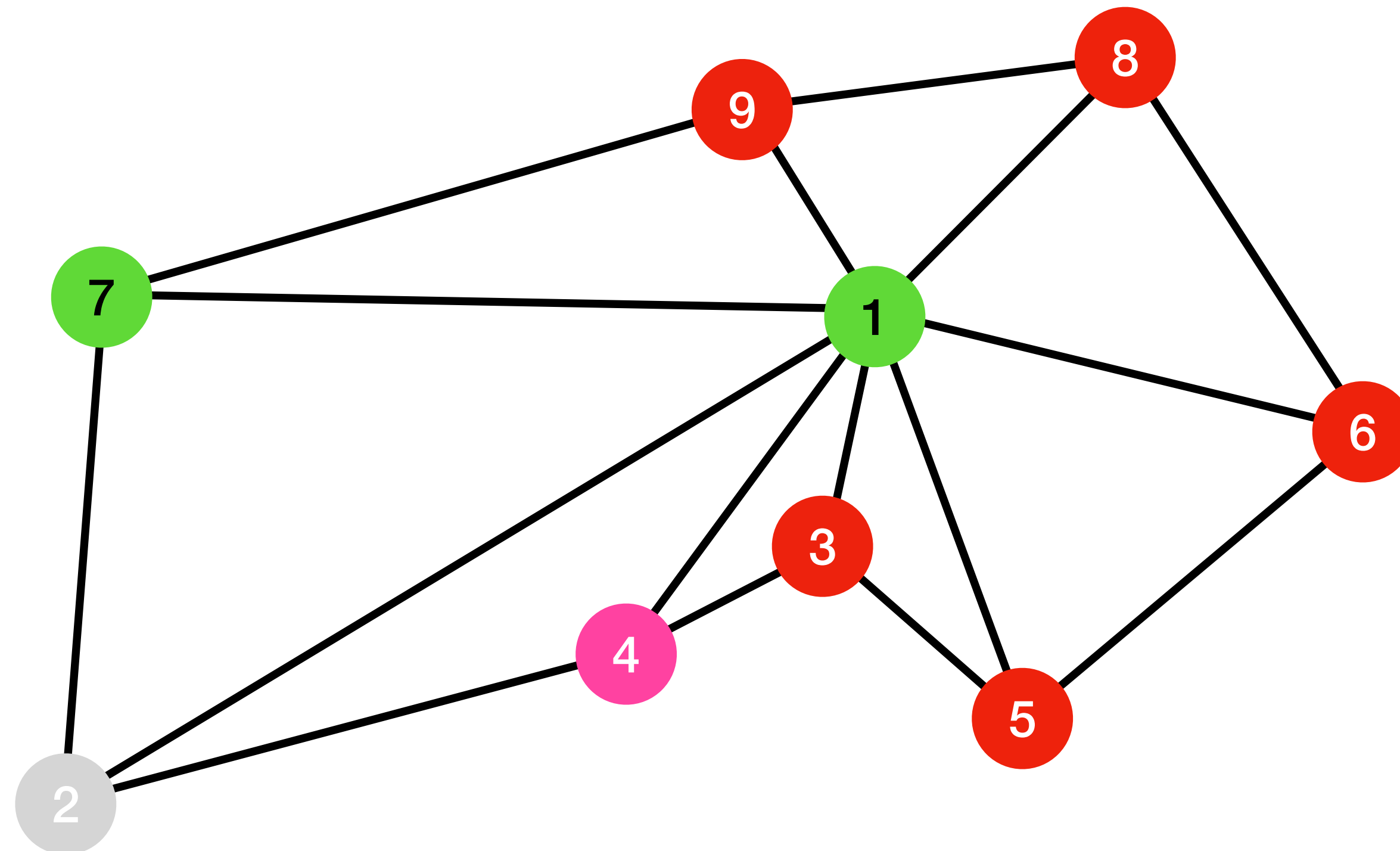
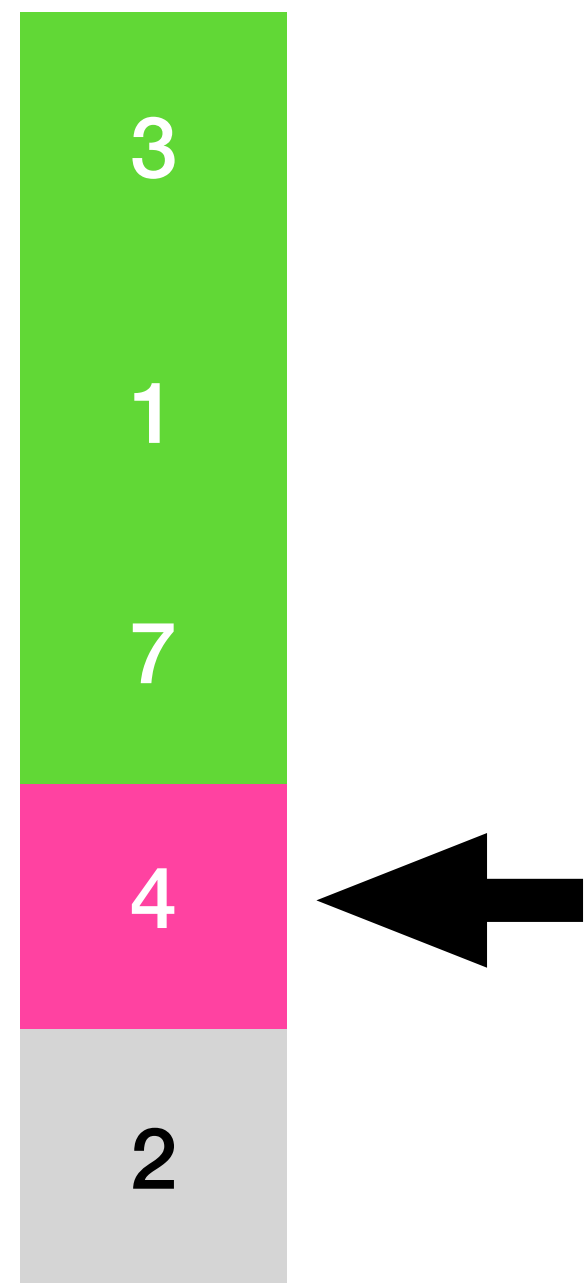


2

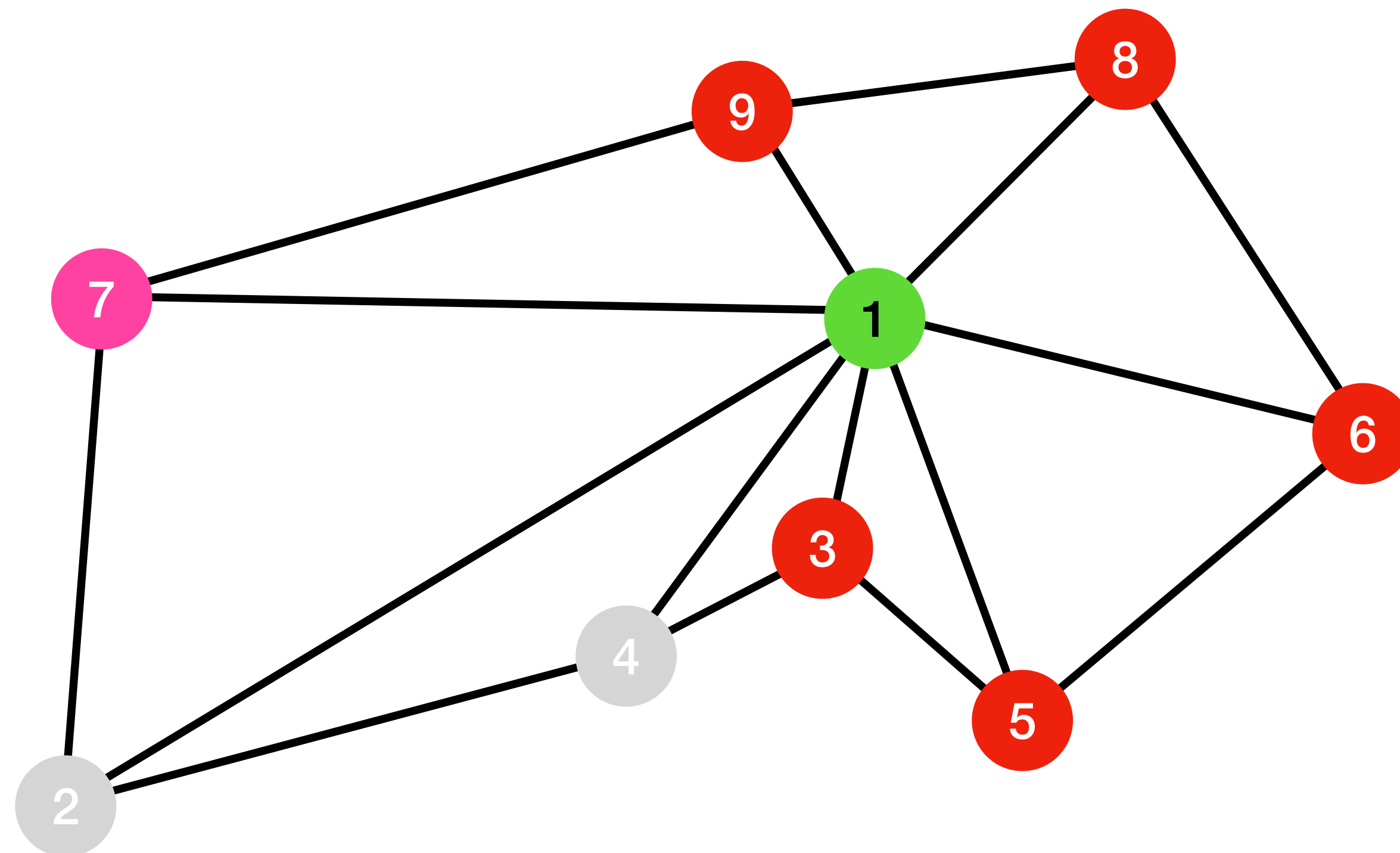
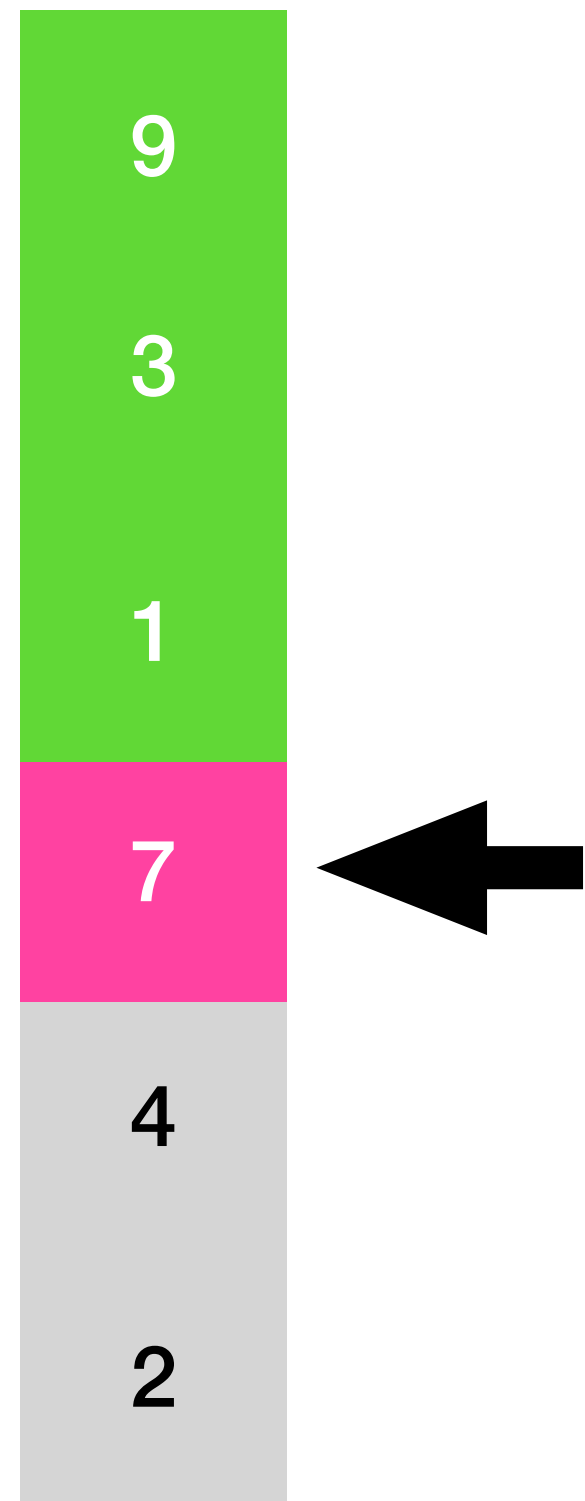
Brute force



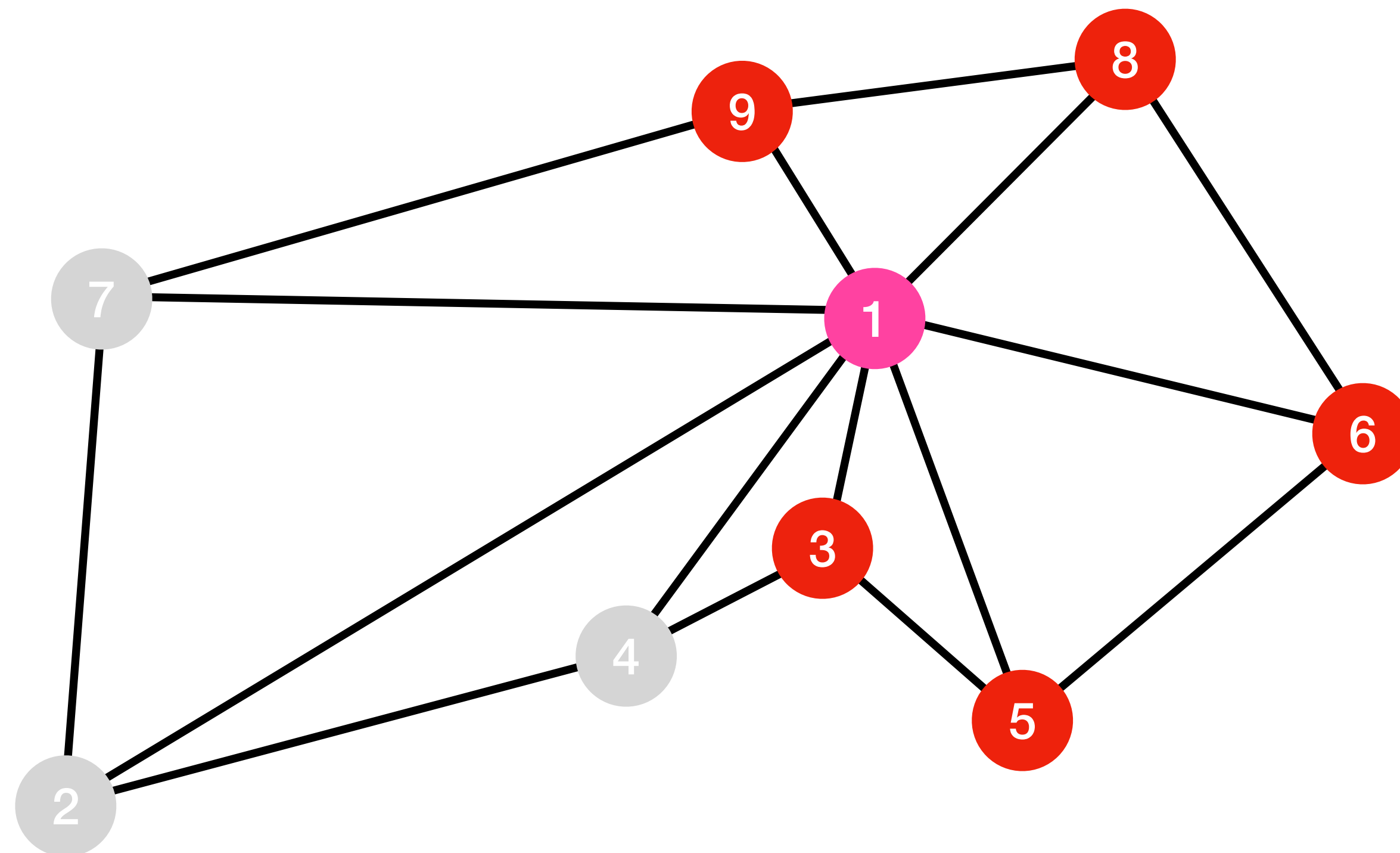
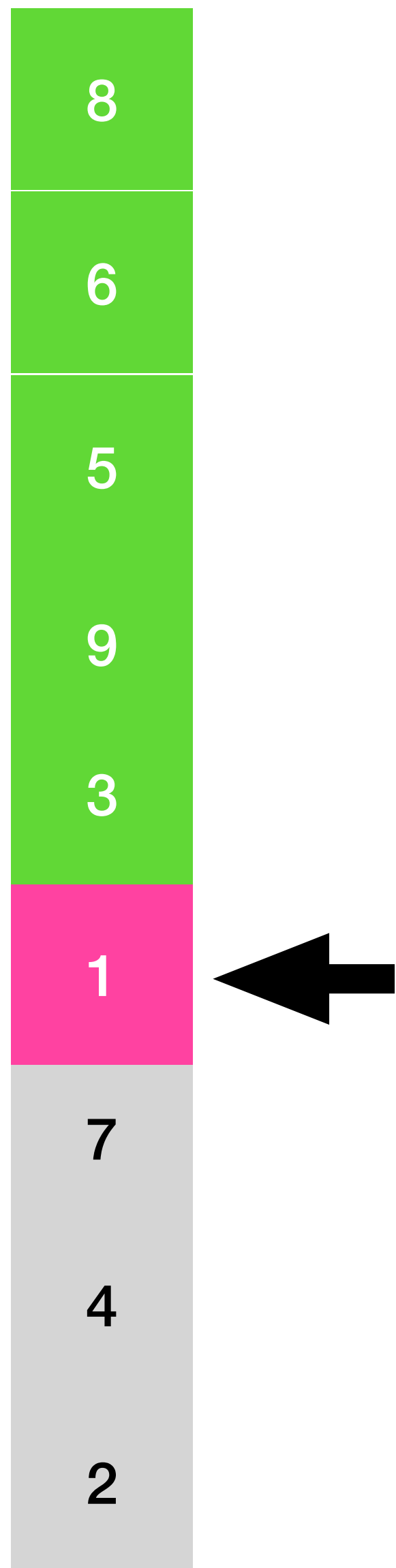
Brute force



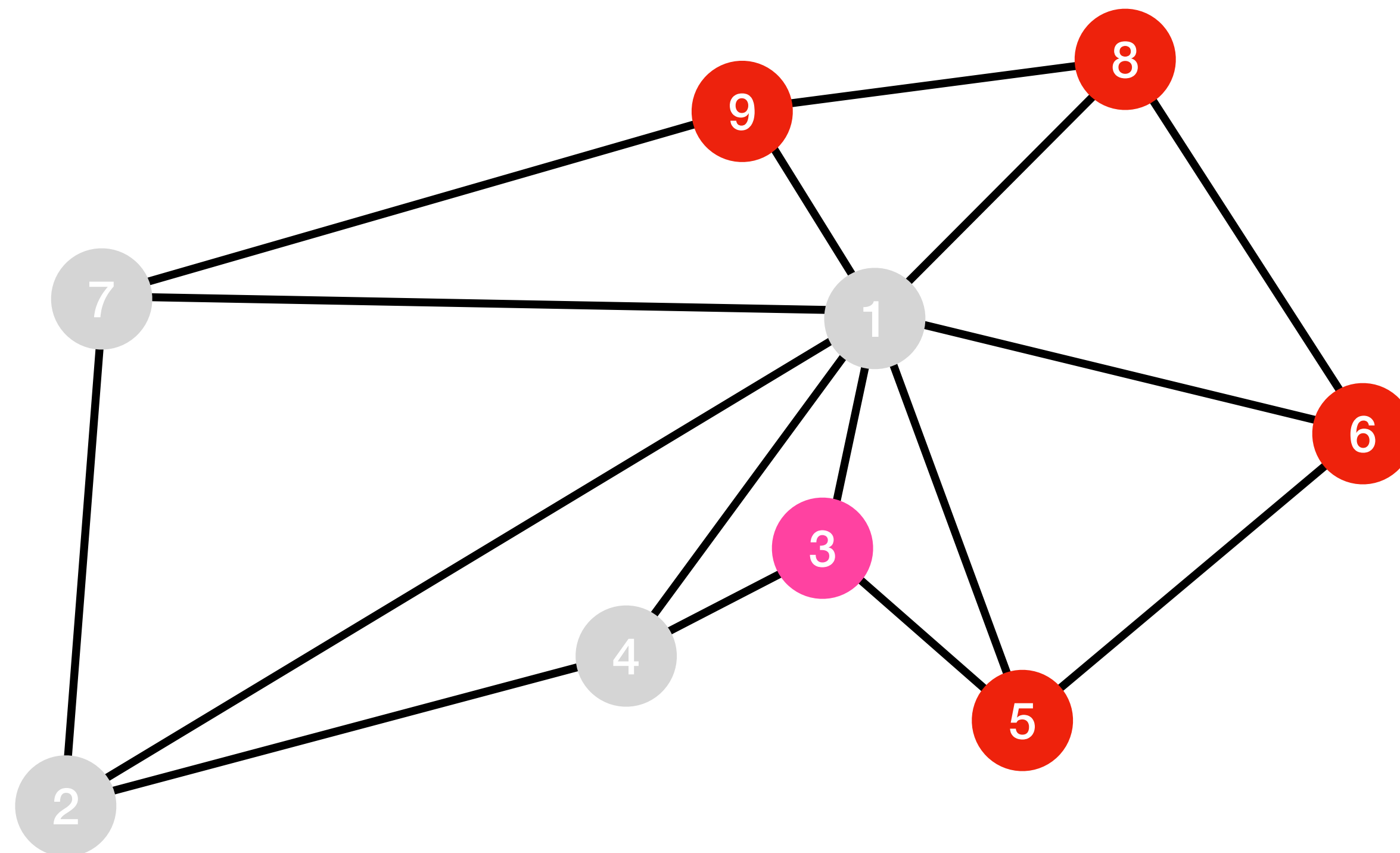
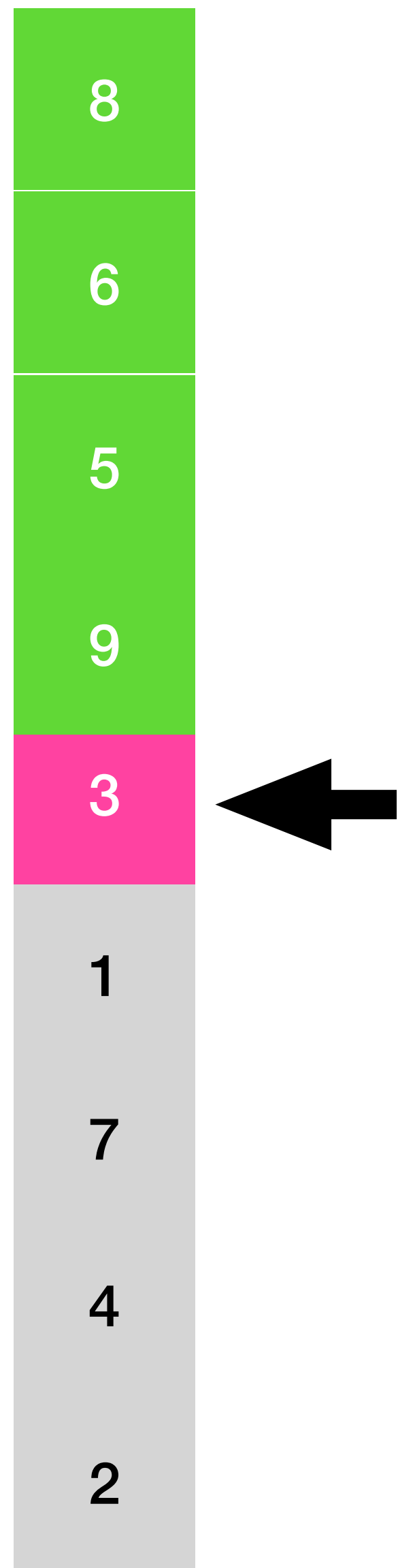
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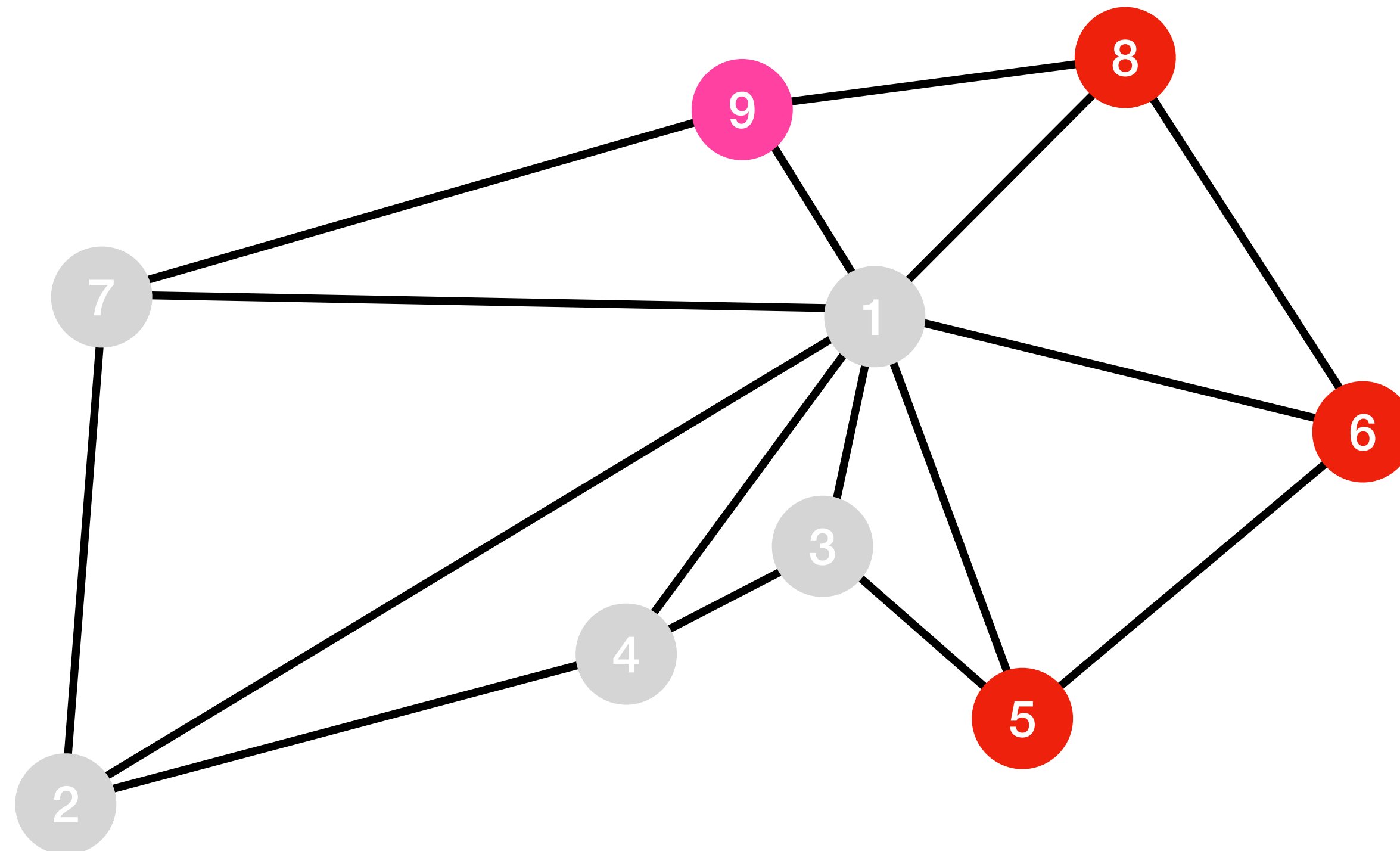
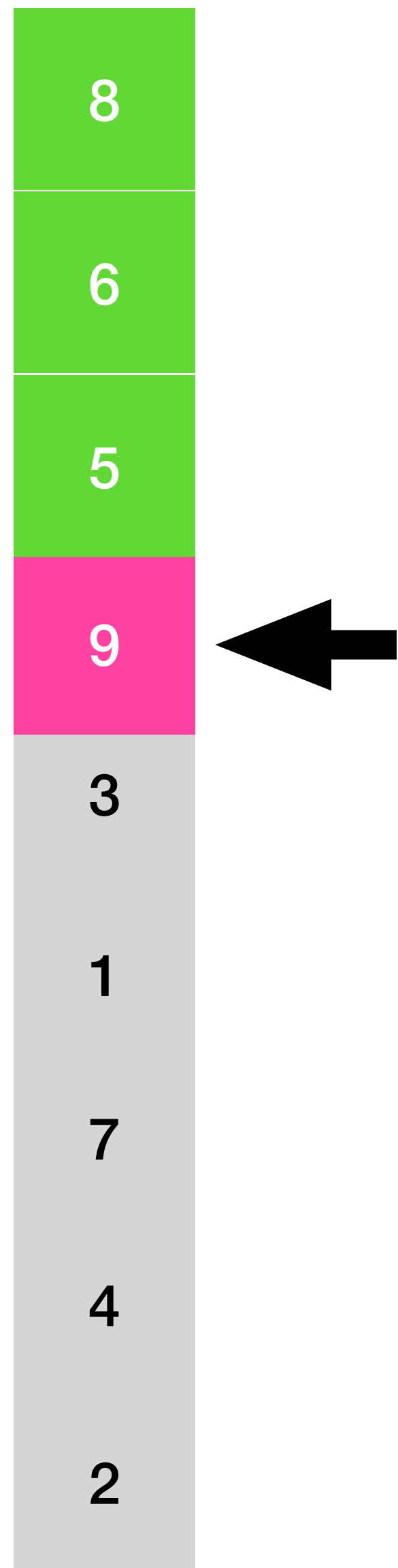
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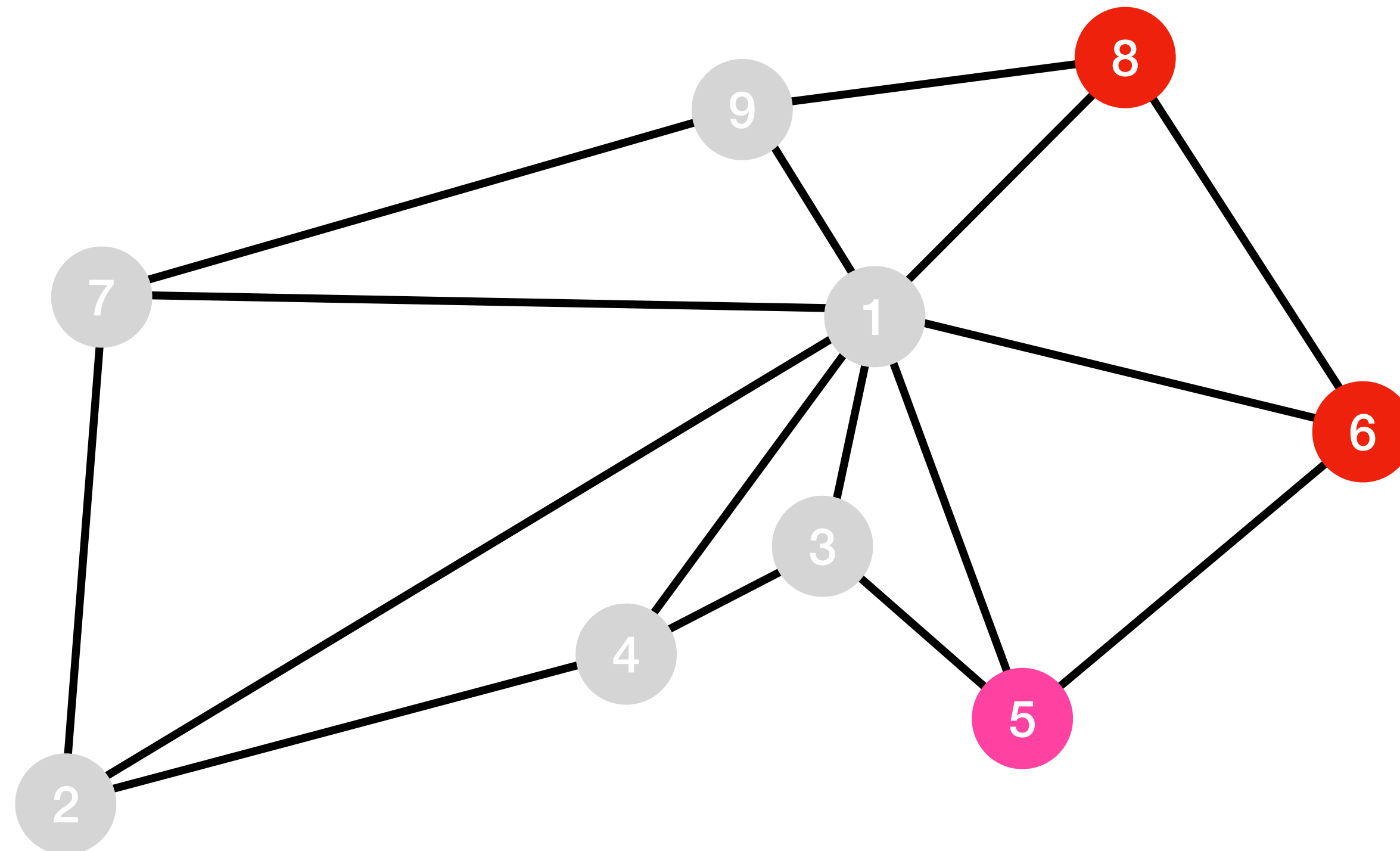
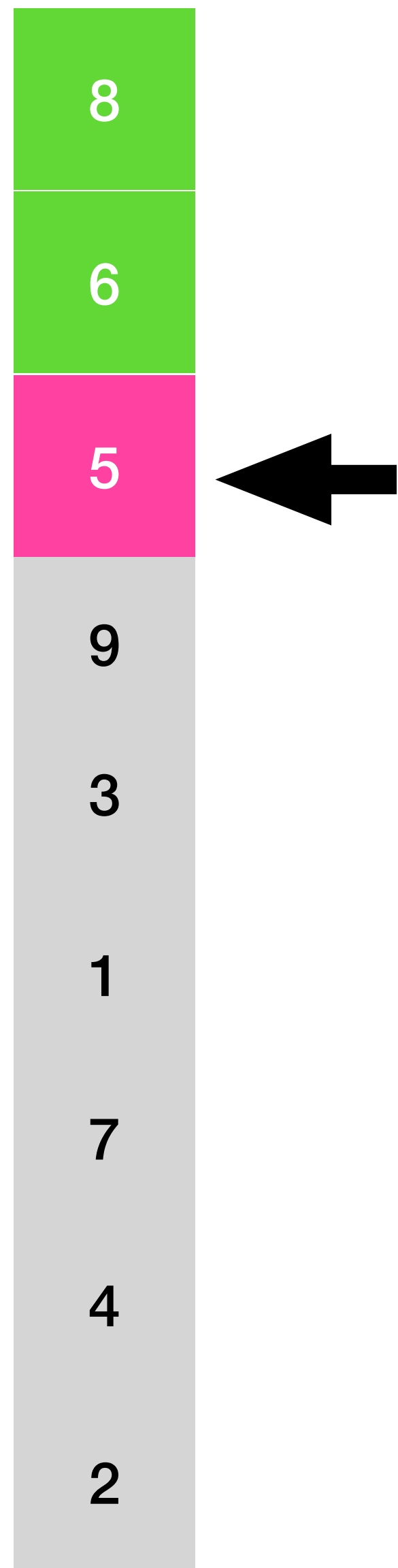
Brute force



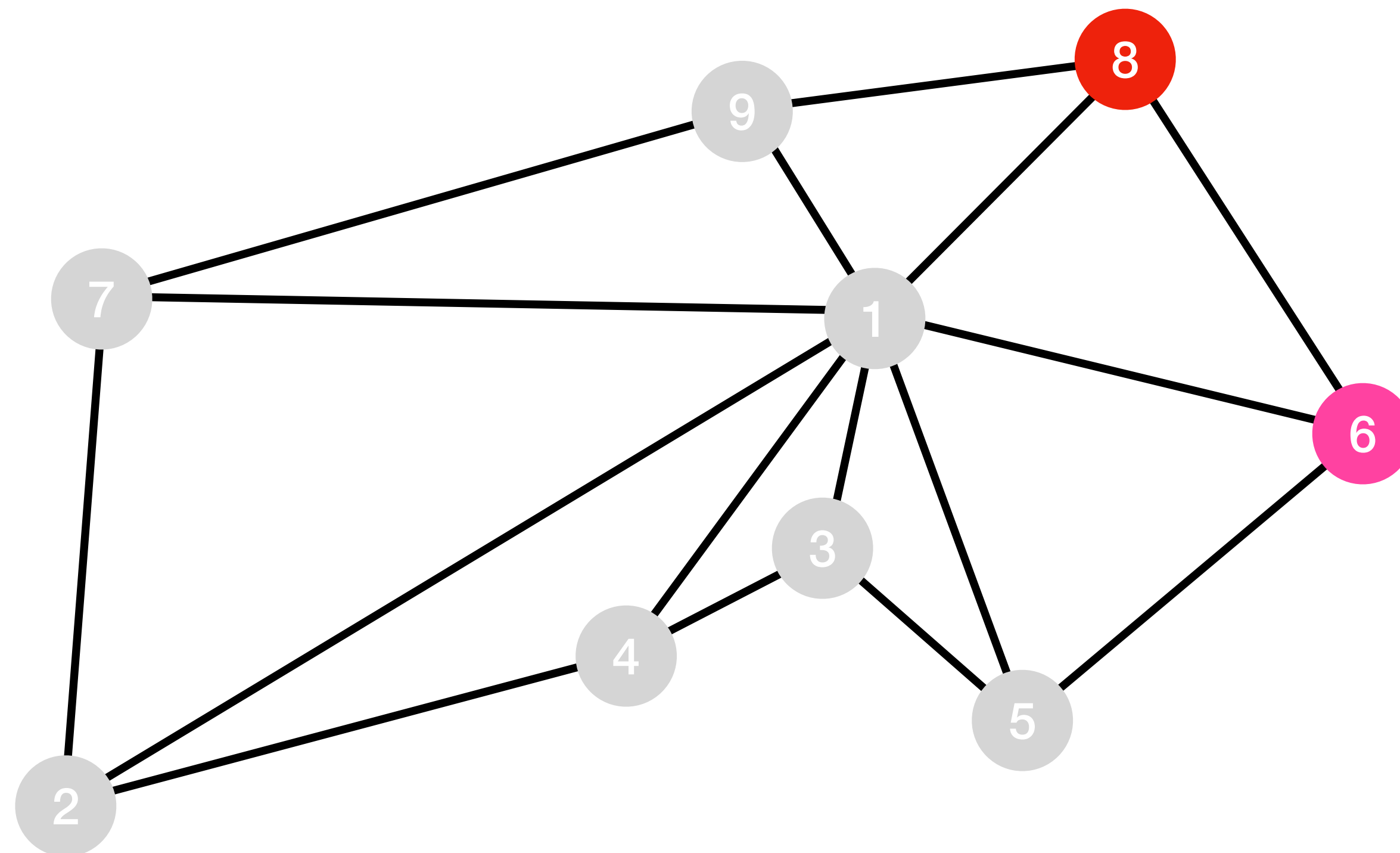
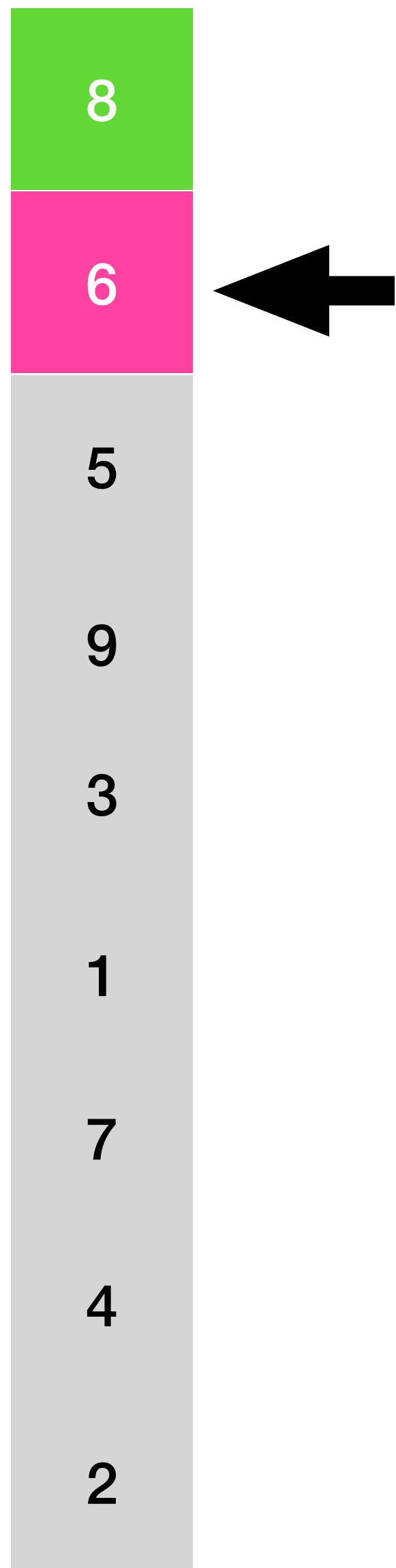
Brute force



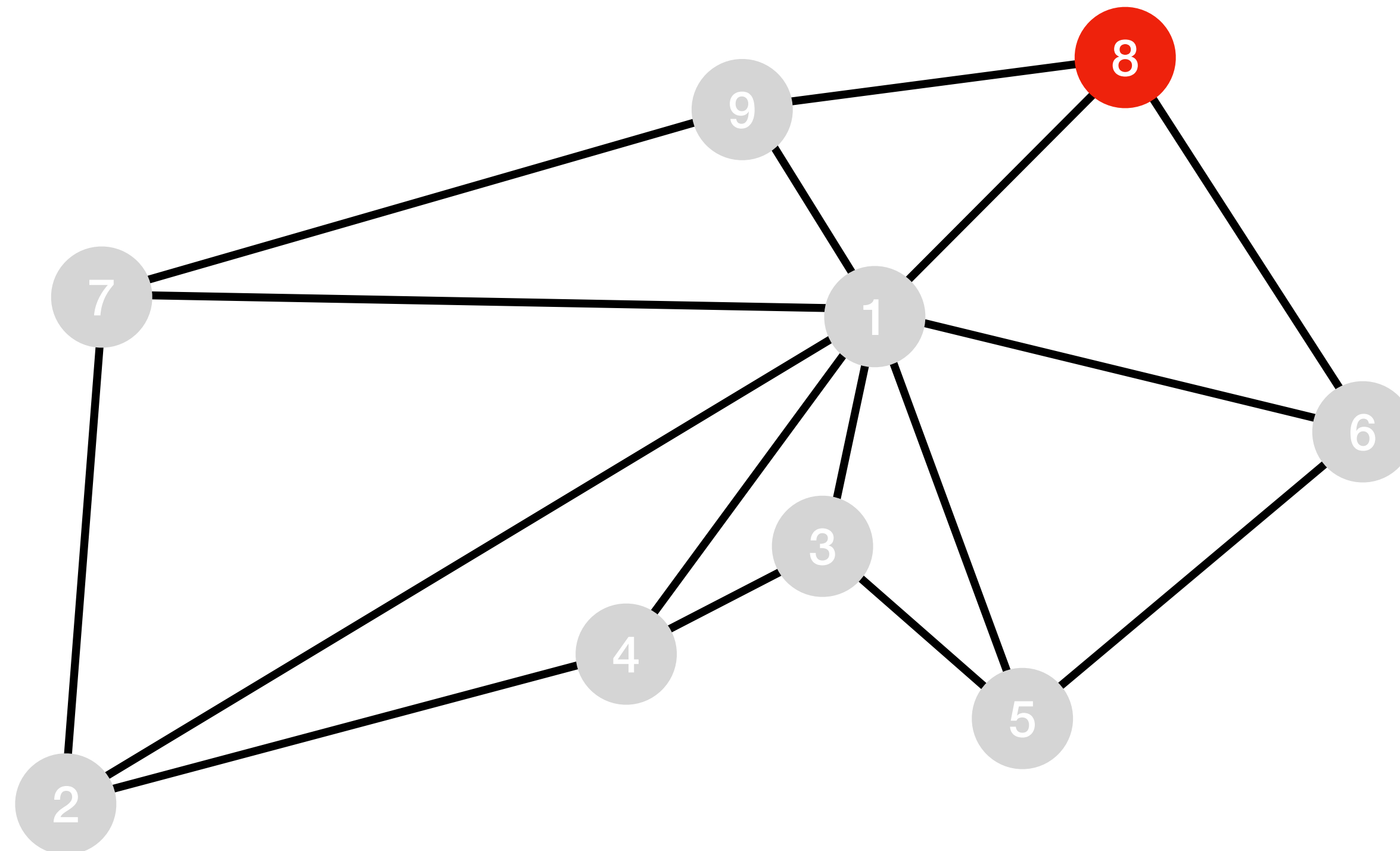
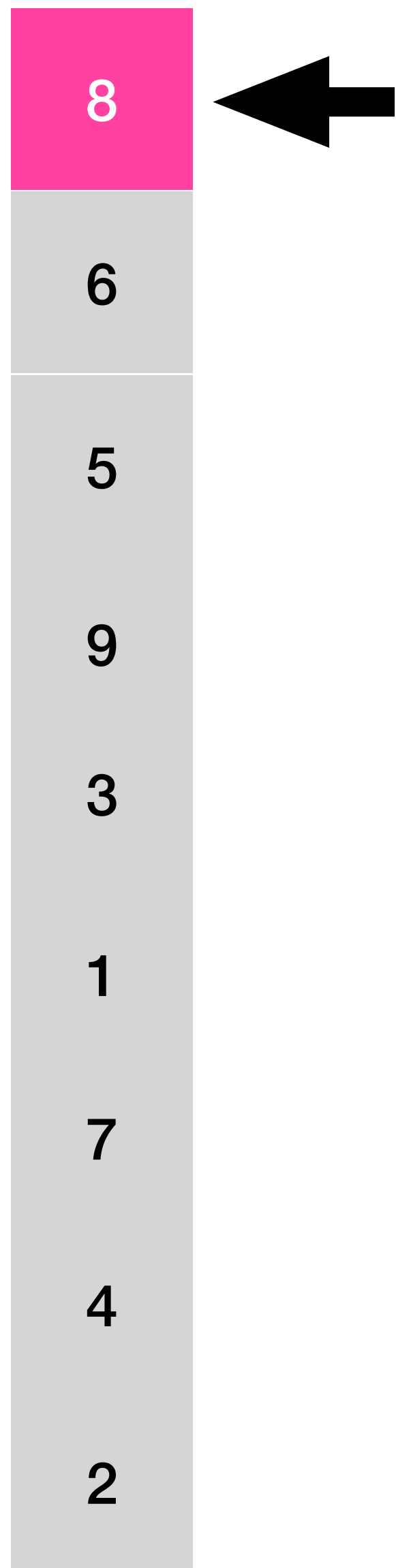
Brute force



Brute force



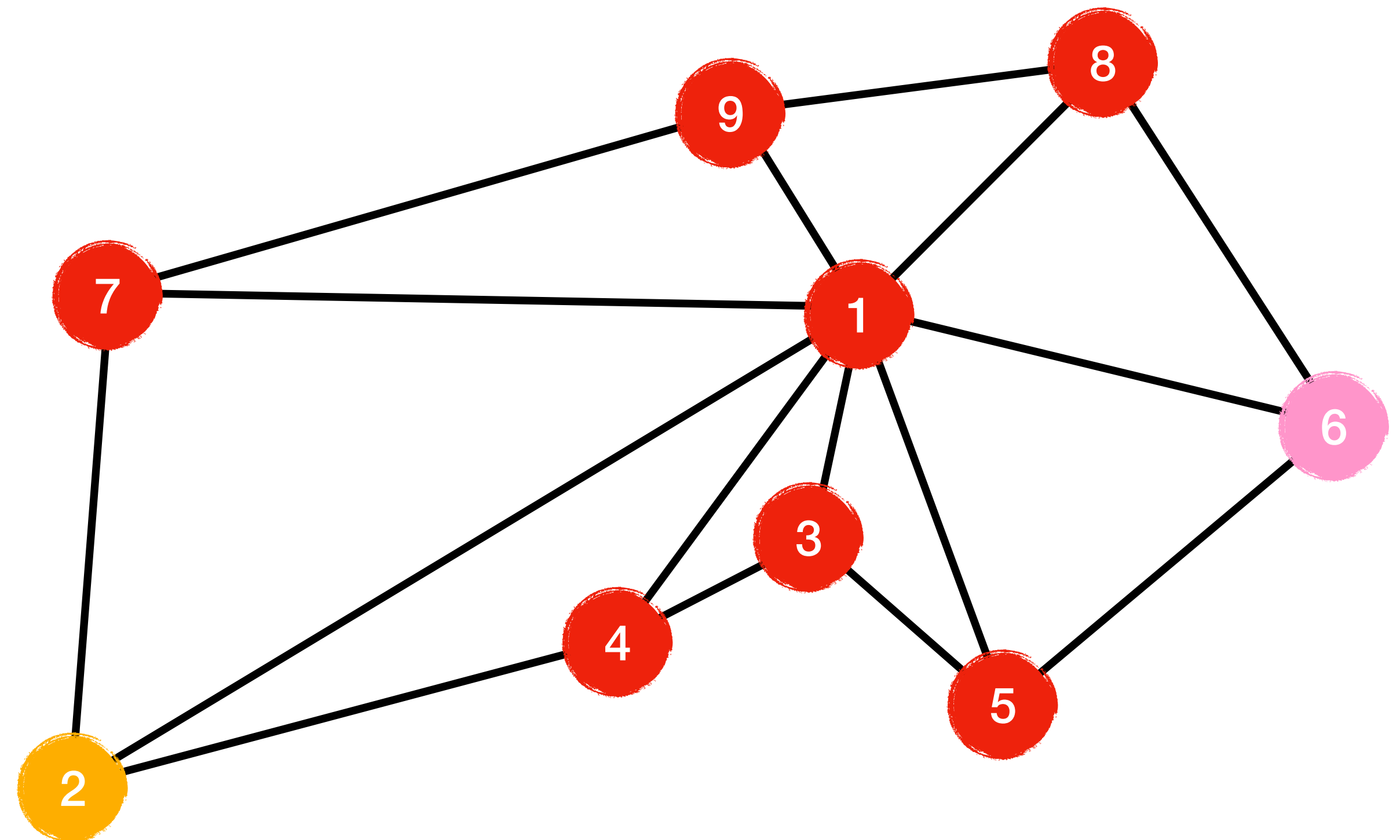
Brute force



Brute force

Adjacency matrix

	1	2	3	4	5	6	7	8	9
1	0	1	1	1	1	1	1	1	1
2	1	0	-1	1	-1	-1	1	-1	-1
3	1	-1	0	1	1	-1	-1	-1	-1
4	1	1	1	0	-1	-1	-1	-1	-1
5	1	-1	1	-1	0	1	-1	-1	-1
6	1	-1	-1	-1	1	0	-1	1	-1
7	1	1	-1	-1	-1	-1	0	-1	1
8	1	-1	-1	-1	-1	1	-1	0	1
9	1	-1	-1	-1	-1	-1	1	1	0

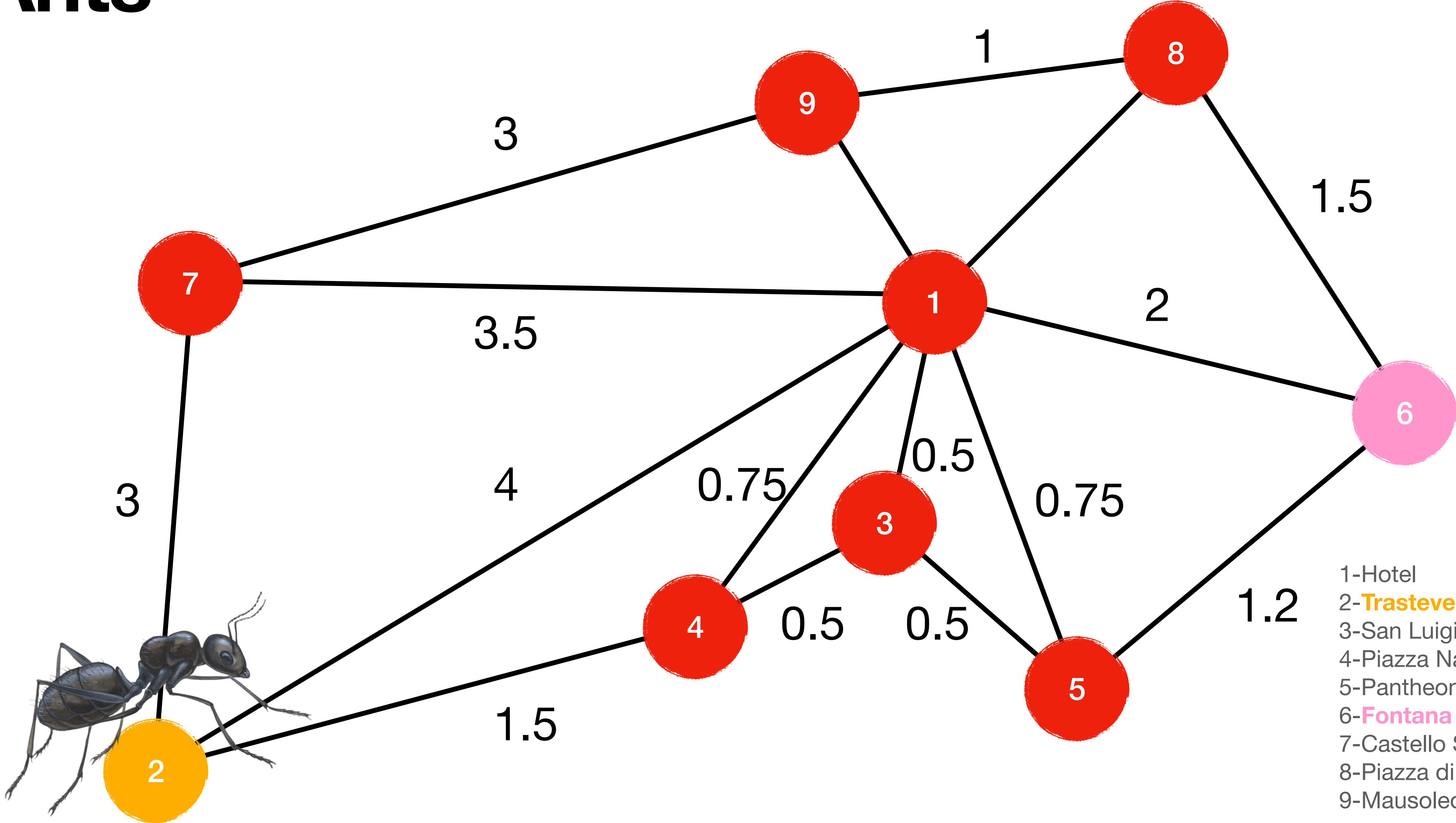




Ants

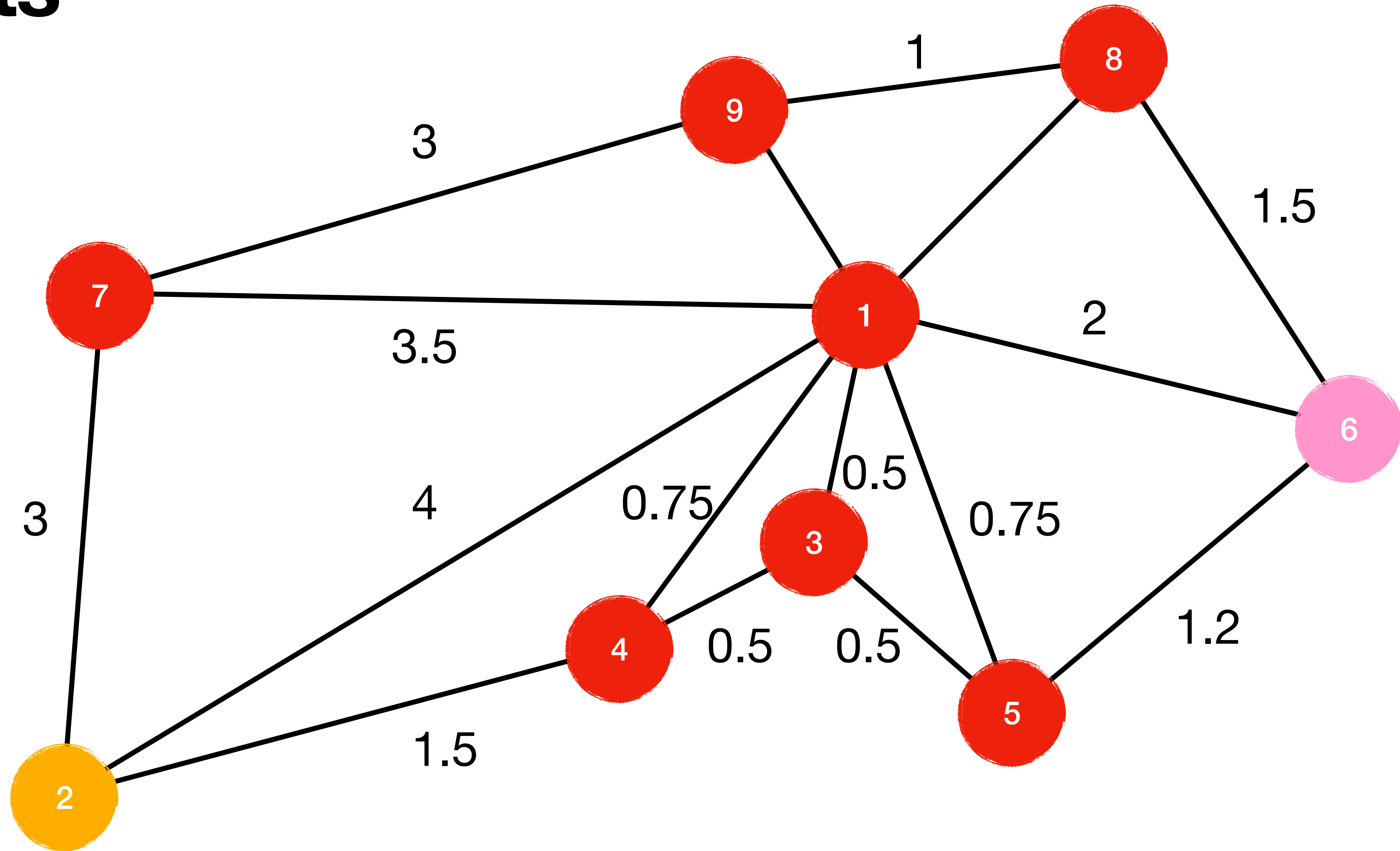
Swarm Intelligence

Ants

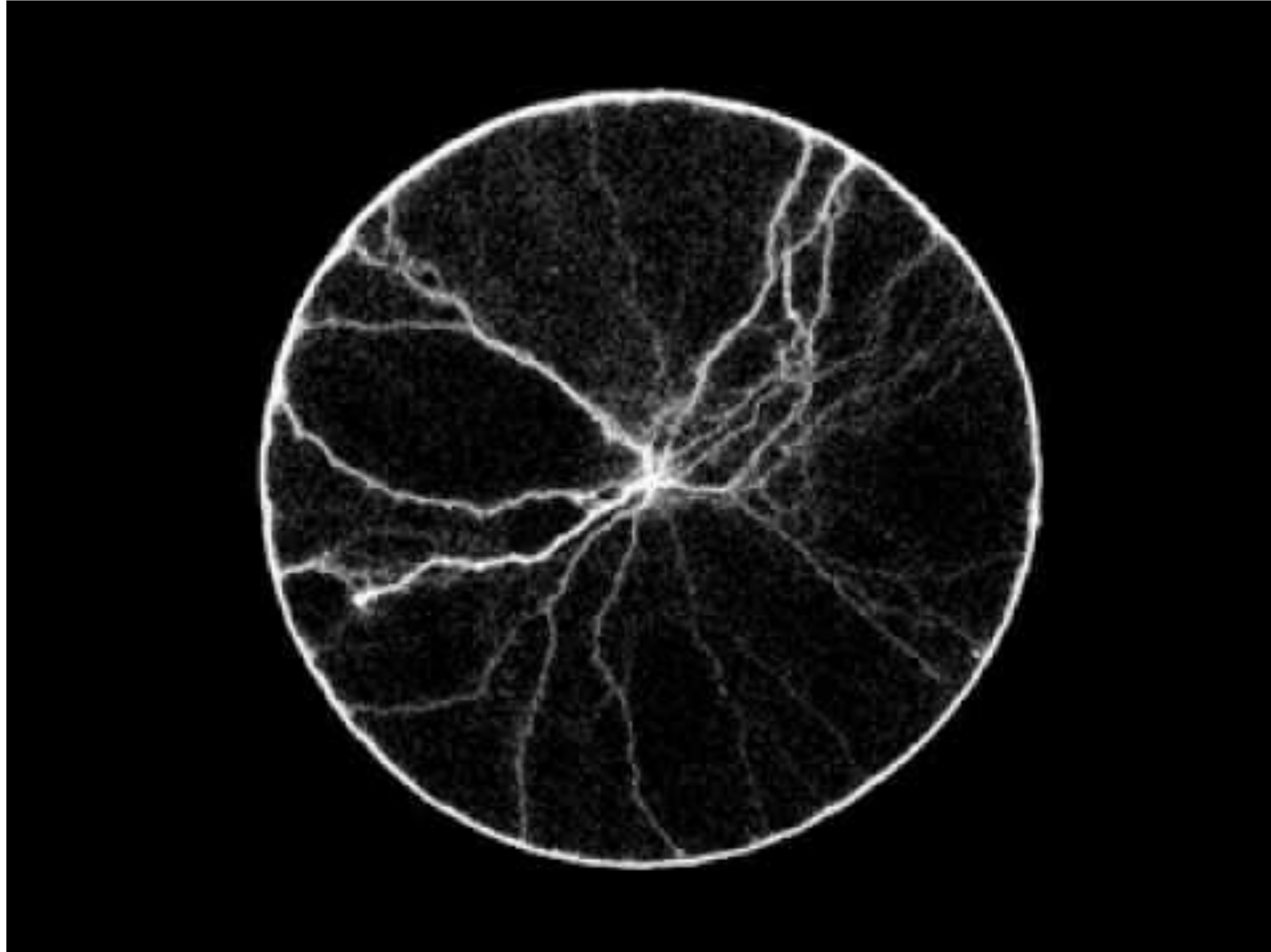
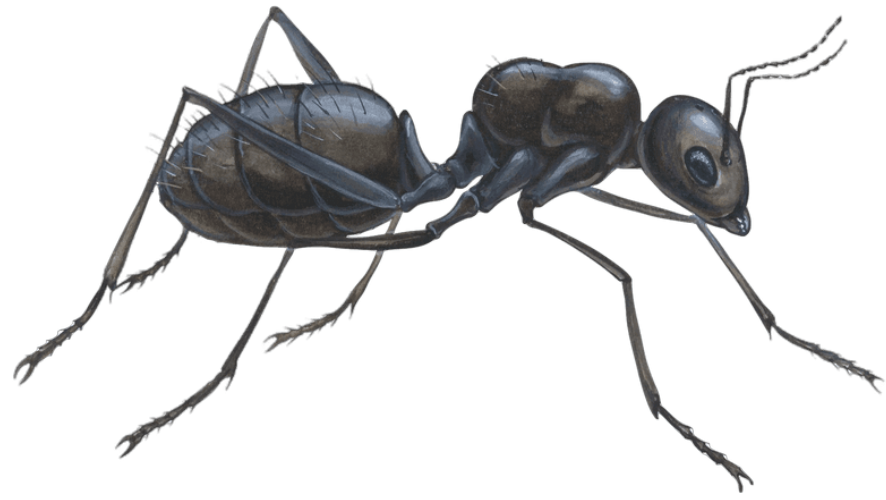


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Ants

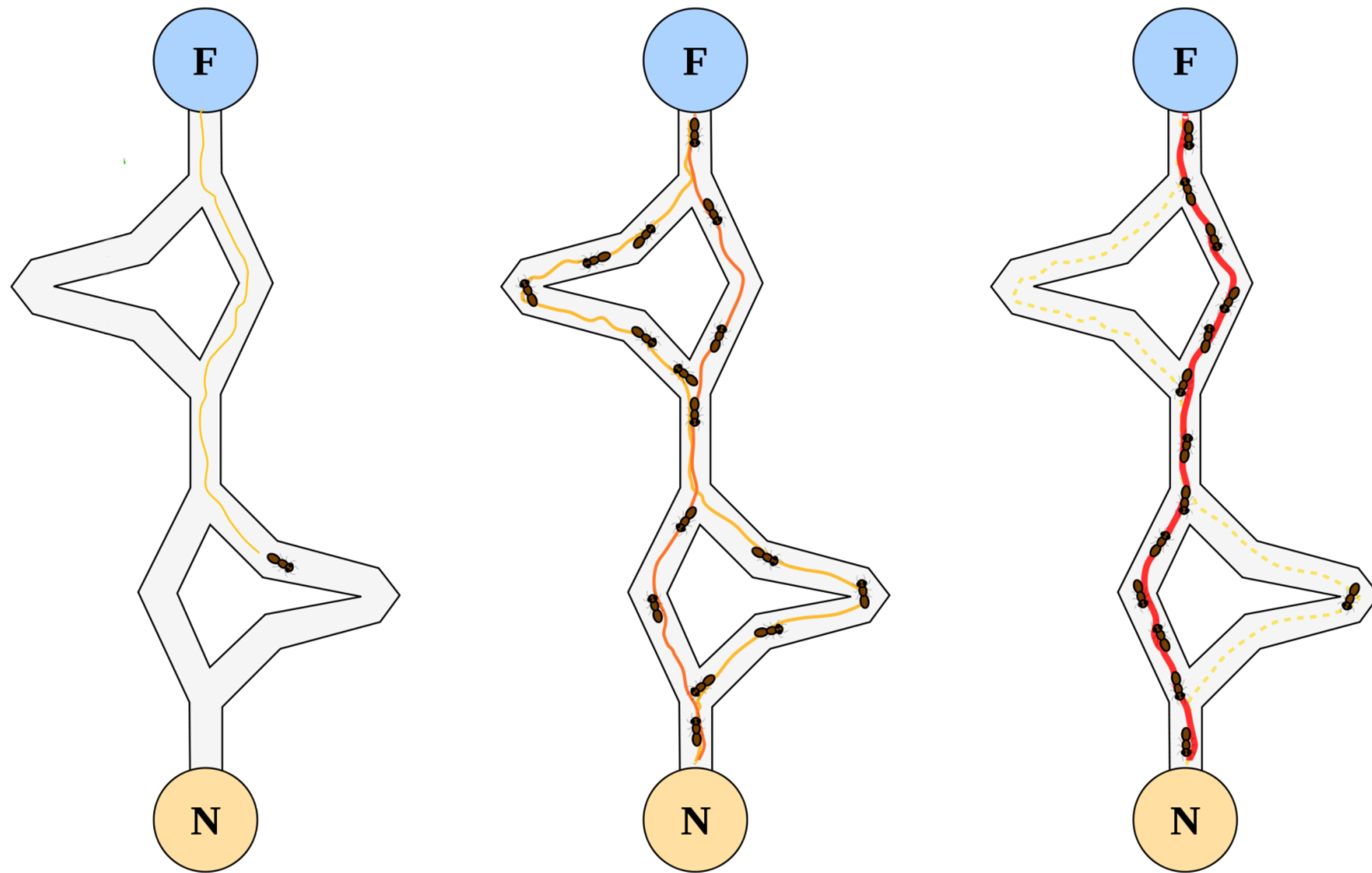


Pheromone trails



Modelling ant “social” behaviour

... with artificial ants

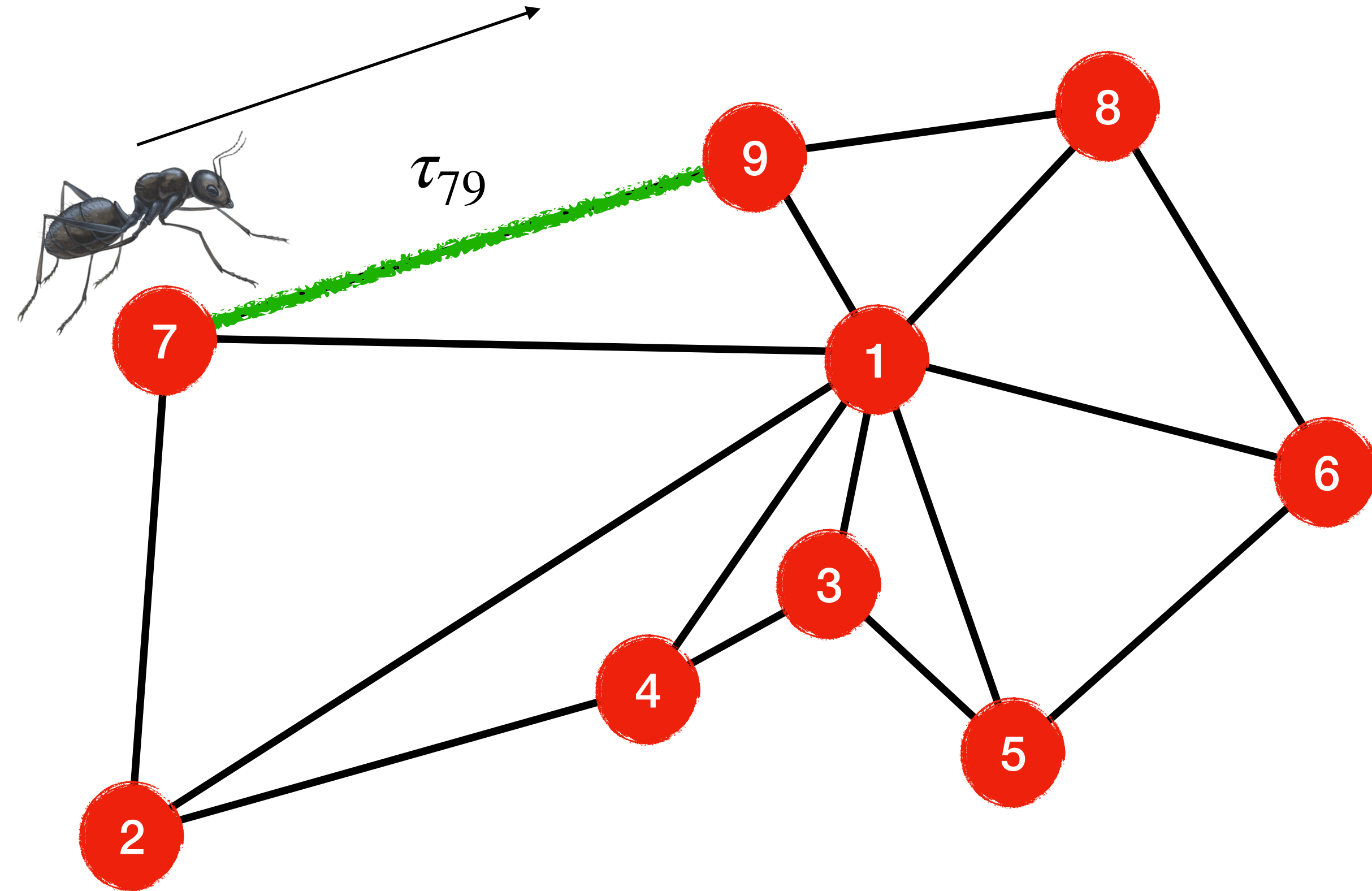


“The shortest path contains more pheromones due high traffic”

Modelling ant “social” behaviour

... with artificial ants

$$p_{ij} = \frac{w_{ij}^{\alpha} \tau_{ij}^{\beta}}{\sum_{\langle i,k \rangle} w_{ik}^{\alpha} \tau_{ik}^{\beta}} \text{ with } 0 < \alpha < 1 \text{ and } \beta \geq 1$$



τ_{ij} pheromones laid at ij edge

w_{ij} weight between ij edge

Modelling ant “social” behaviour

... with artificial ants

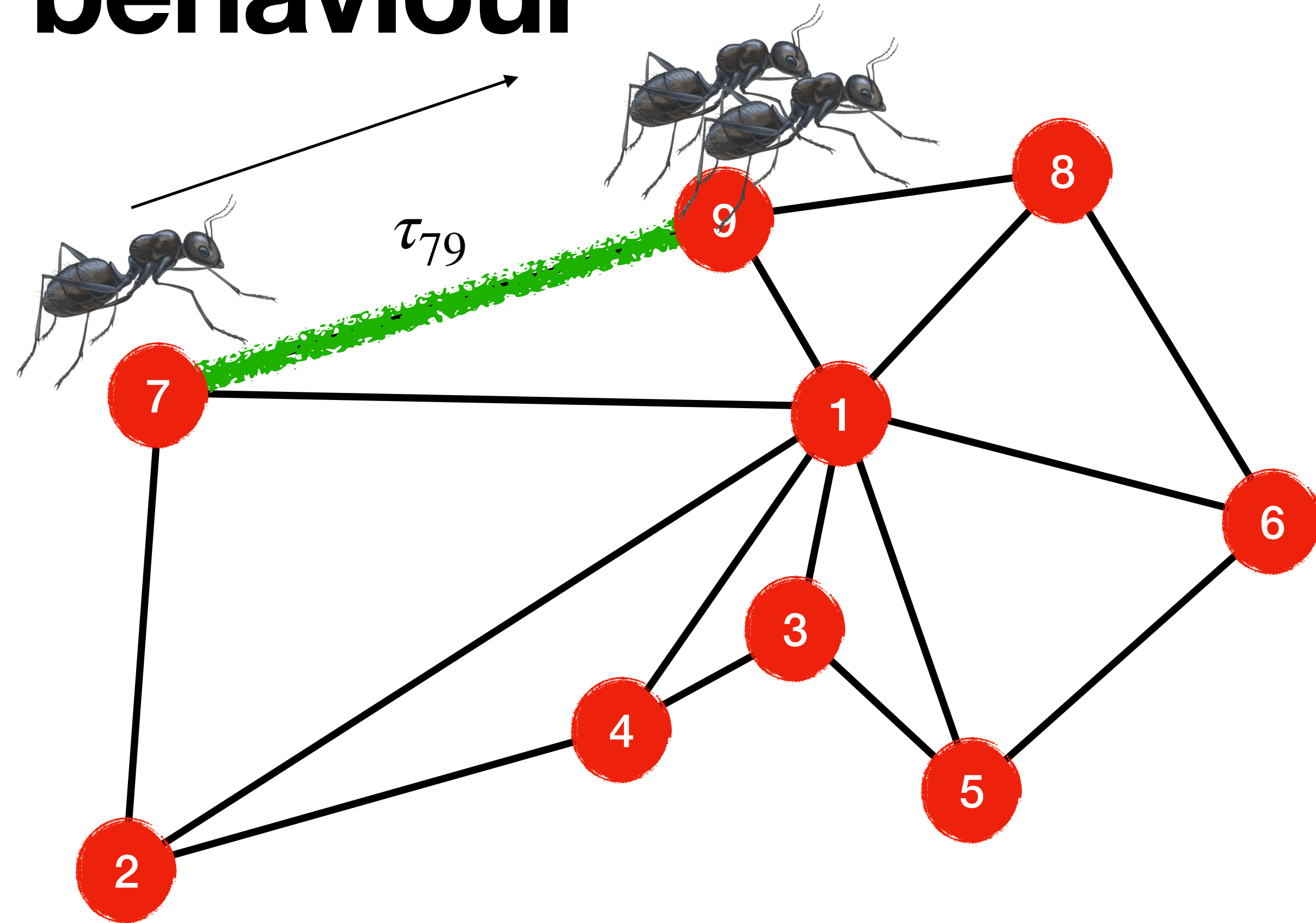
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Pheromones updates

$$\Delta \tau_{ij,n} = \begin{cases} \frac{Q}{L(P_n)} & \text{if an ant } n \text{ crossed } ij \\ 0 & \text{otherwise} \end{cases}$$

$P_n = \{s, i_1, i_2, \dots, i_k, f\}$ vertex sequence

$$L(P_n) = \sum_{j \in P_n} d_{j,j+1} \text{ path distance}$$



τ_{ij} pheromones laid at ij edge

w_{ij} weight between ij edge

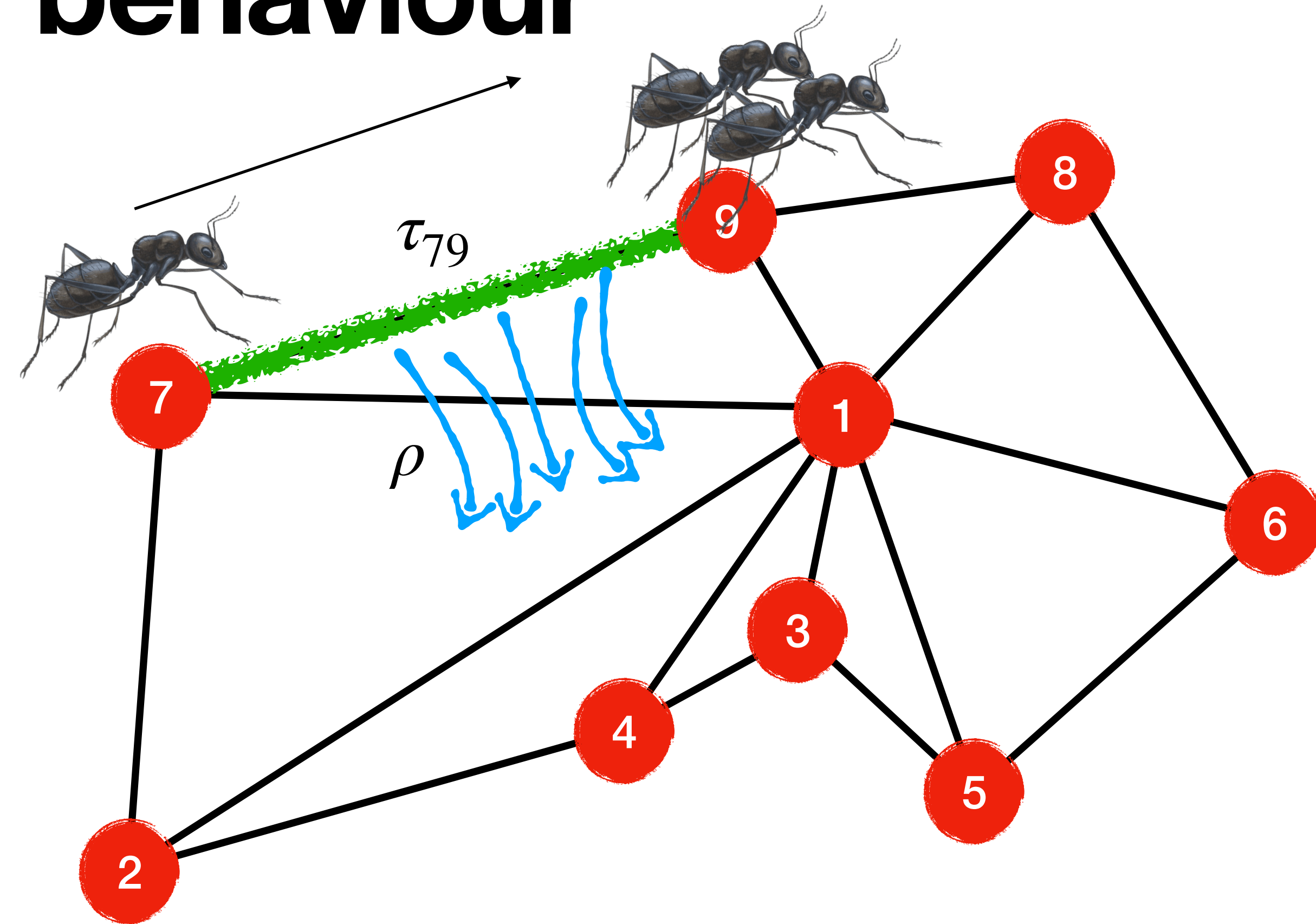
Modelling ant “social” behaviour

... with artificial ants

Pheromones updates

$$\tau_{ij} = (1 - \rho)\tau_{ij} + \sum_{n \in \bar{ij}} \Delta\tau_{ij,n}$$

$\sum_{\bar{ij}}$ sum over all the ants that reached f



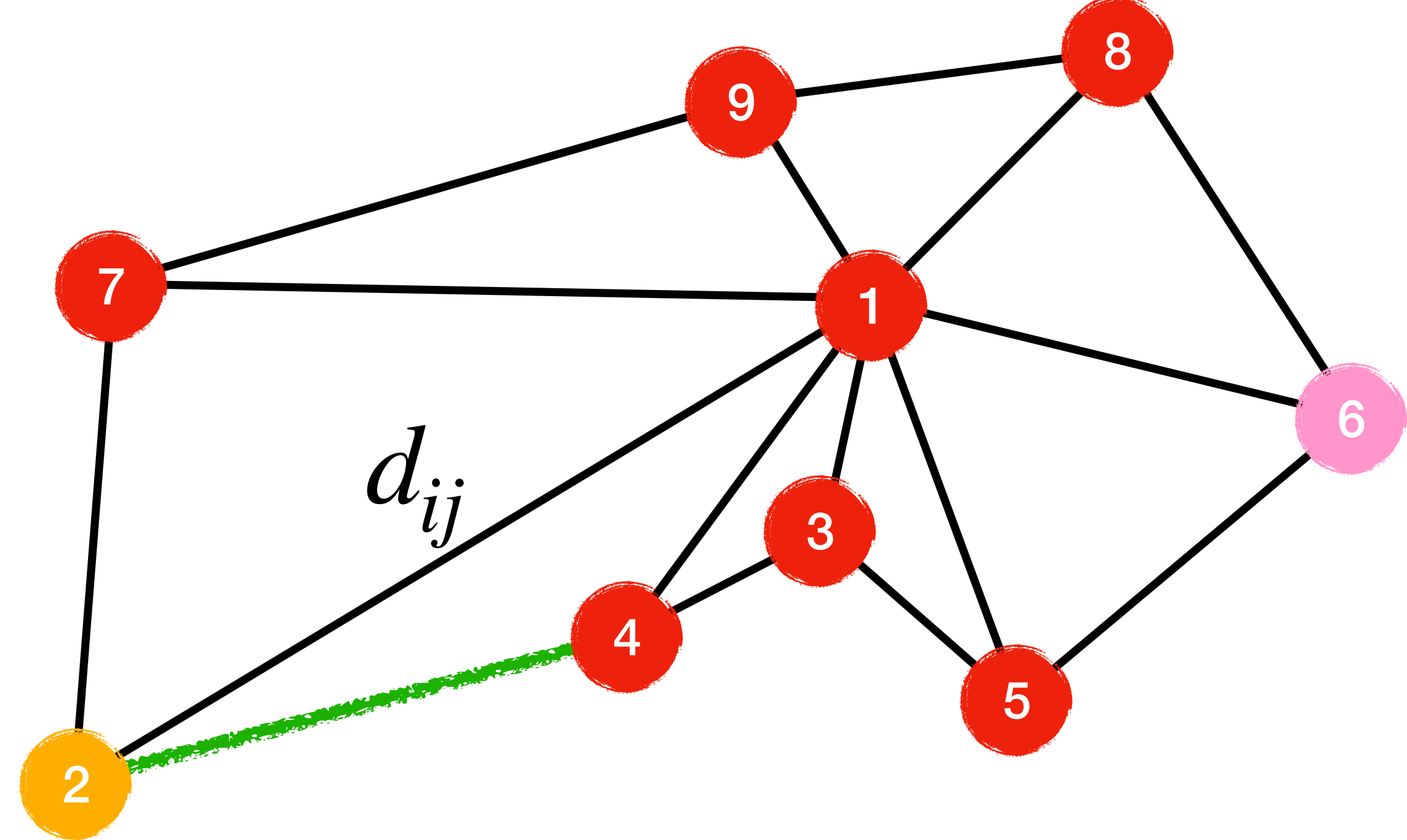
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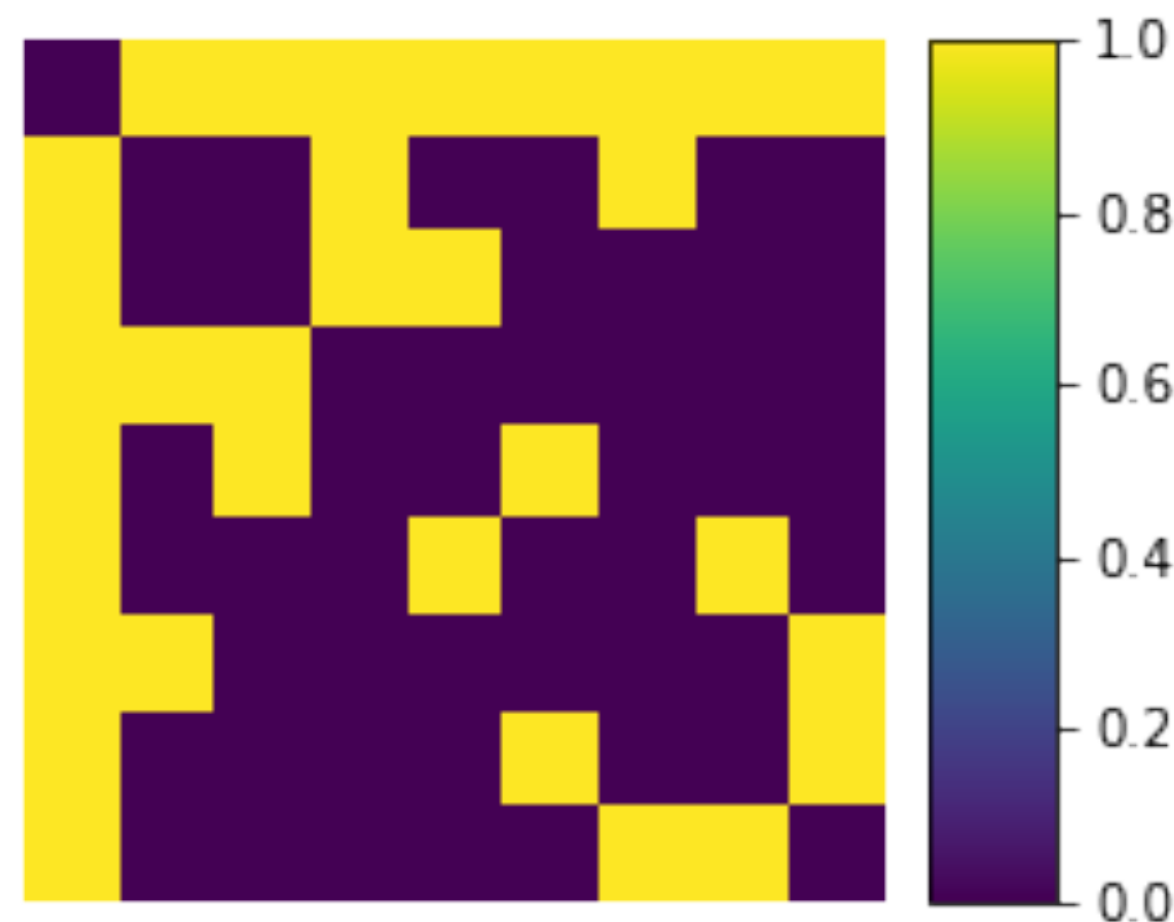
ρ evaporation rate

Ants in action

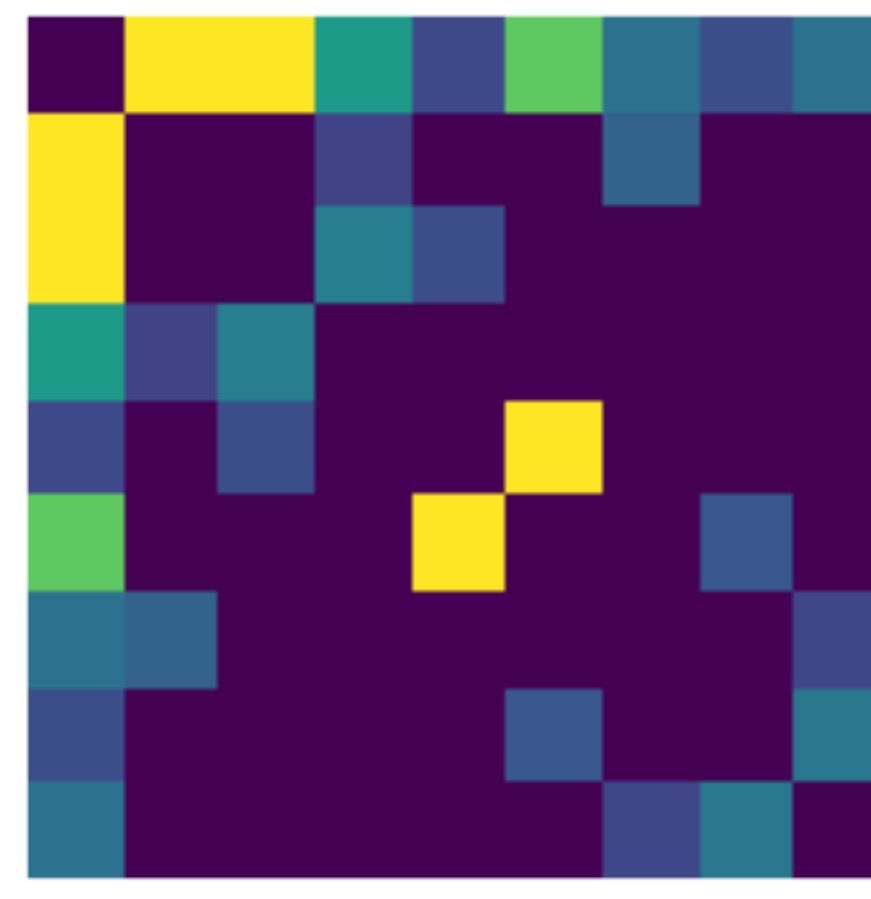
- Adjacency Matrix
- Weight Matrix
- Pheromone Matrix



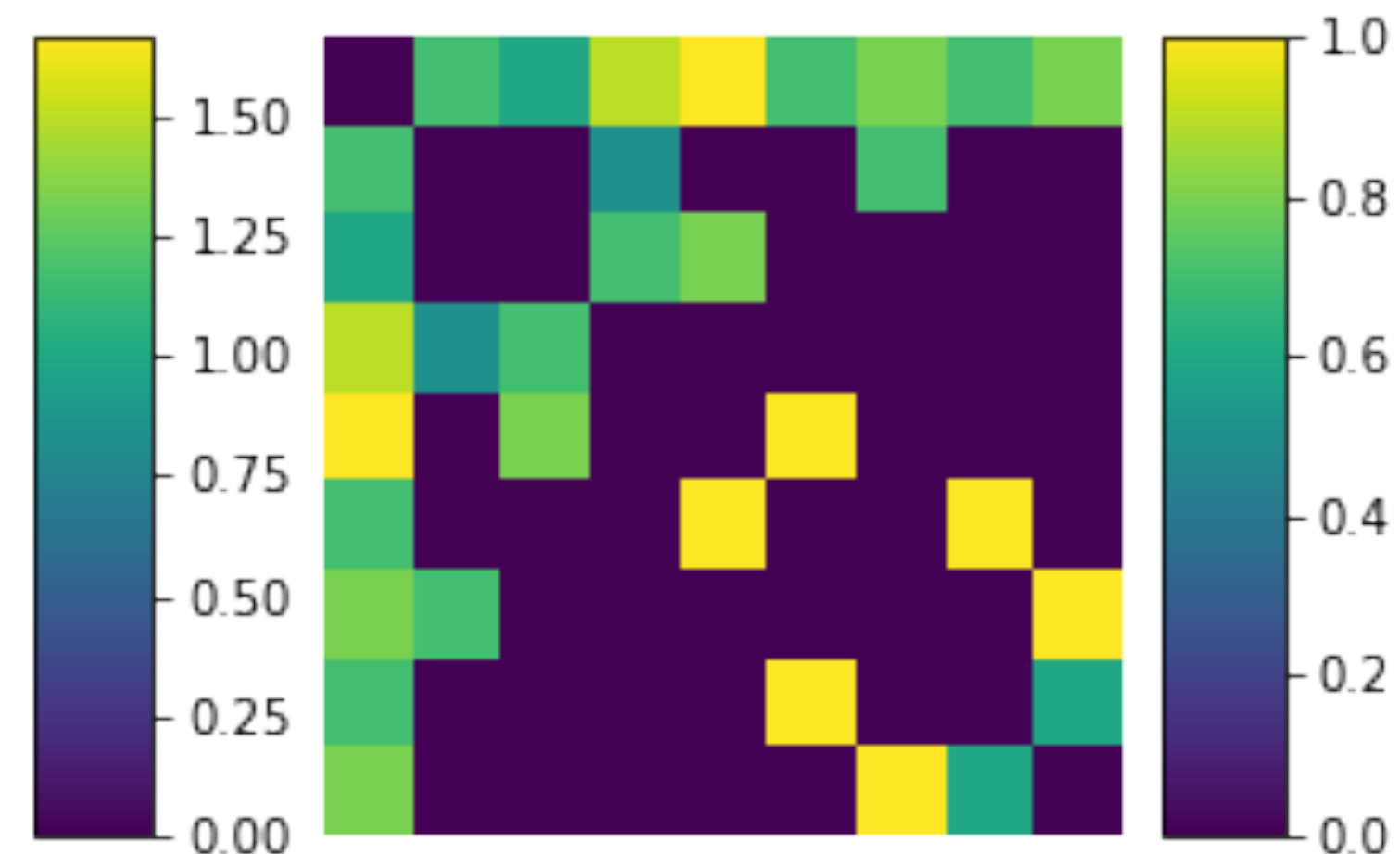
Adjacency Matrix



Weight Matrix := d_{ij}^{-1}



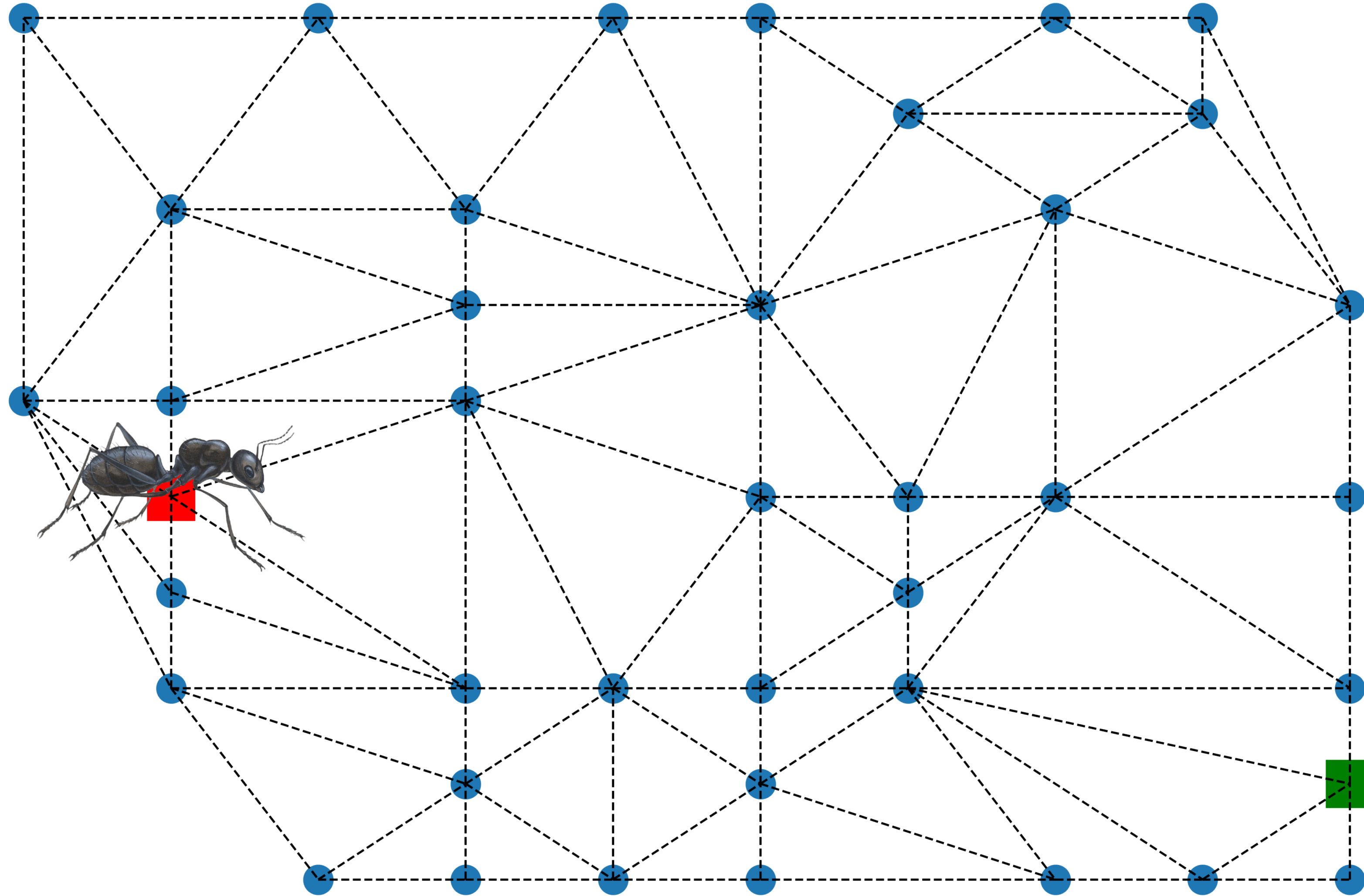
Pheromone Matrix



Ants in action

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$$\tau_{ij} |_{t=0} = 1$$

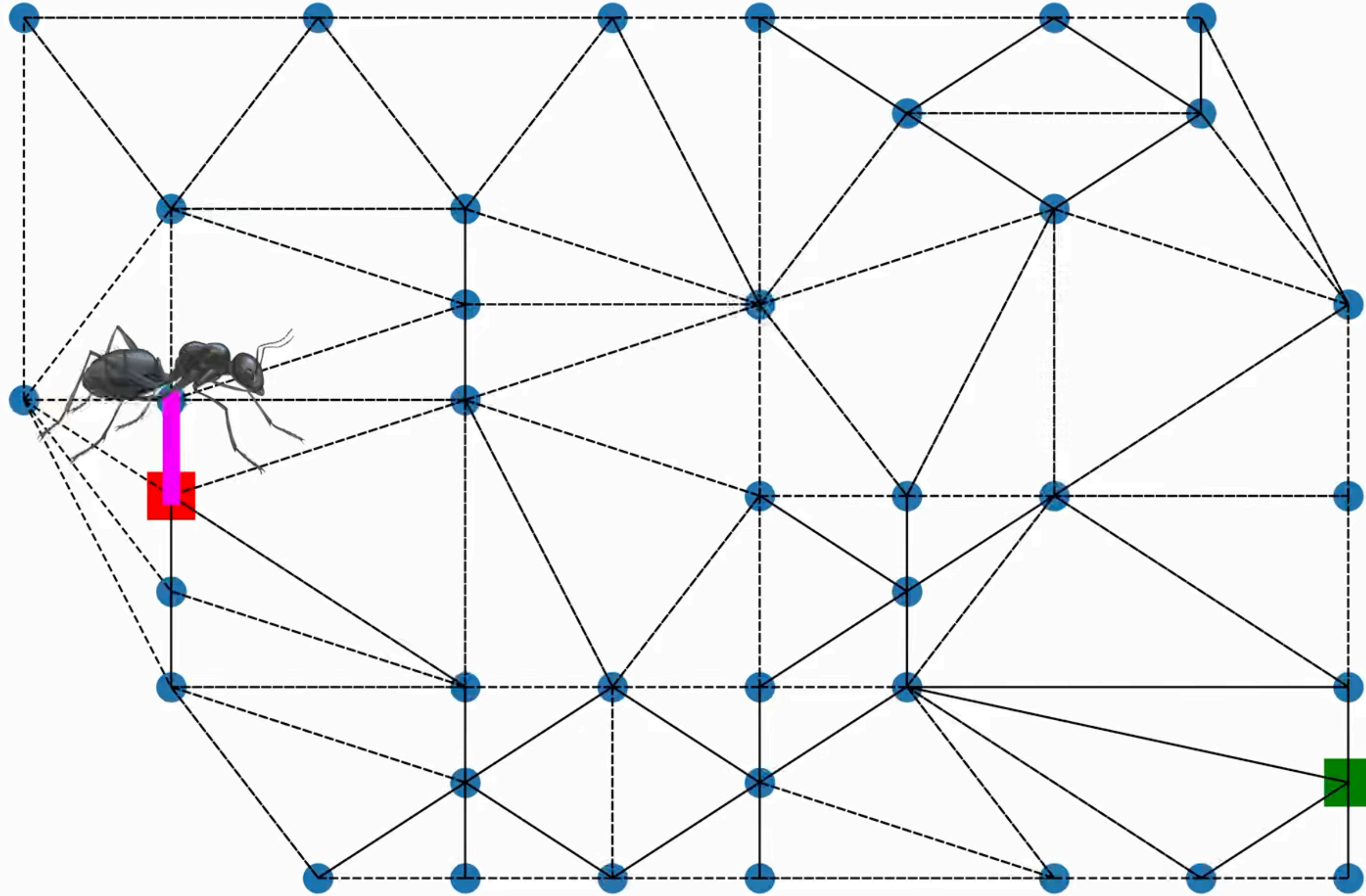


Iteration = 0

Ants in action

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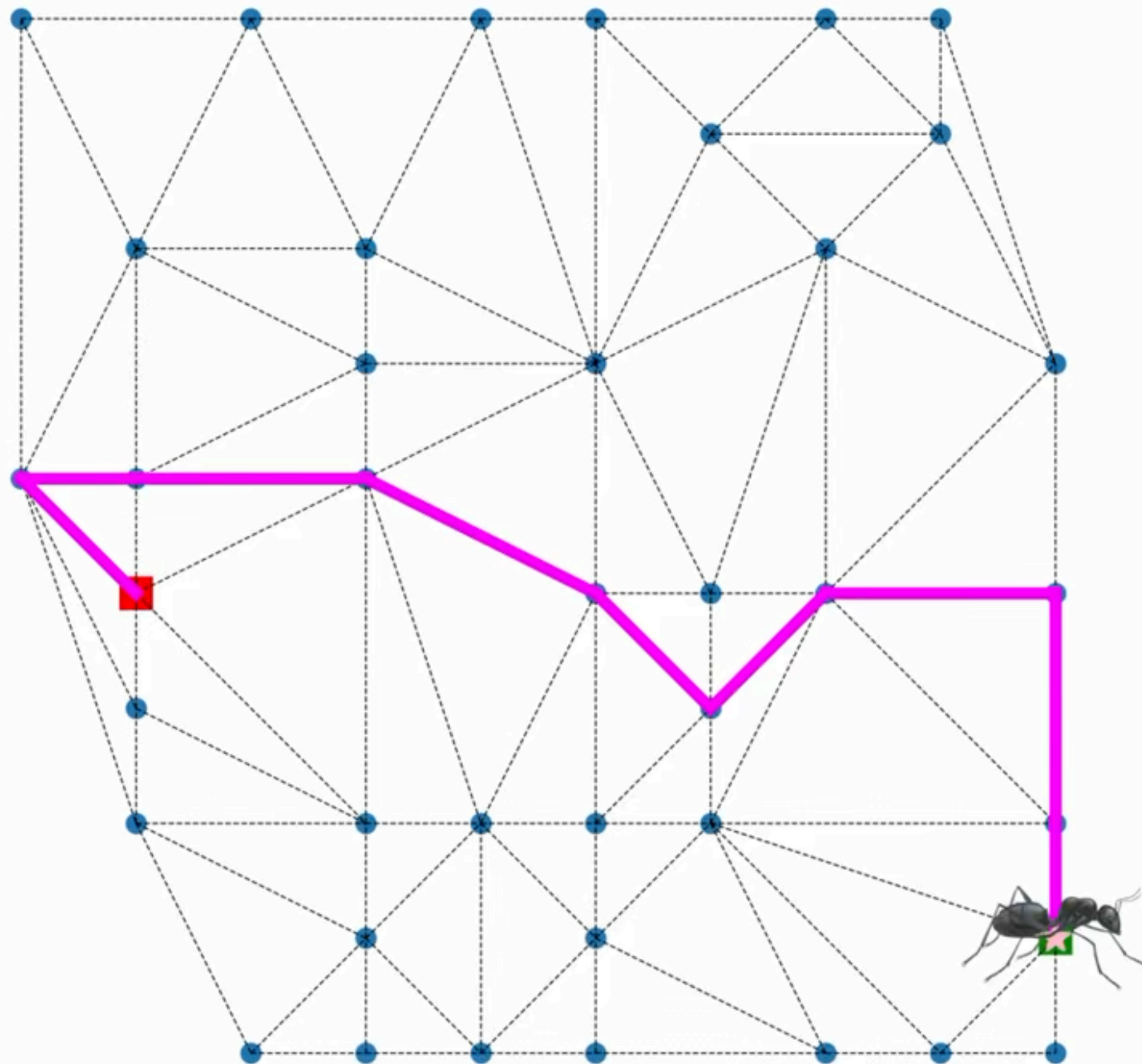
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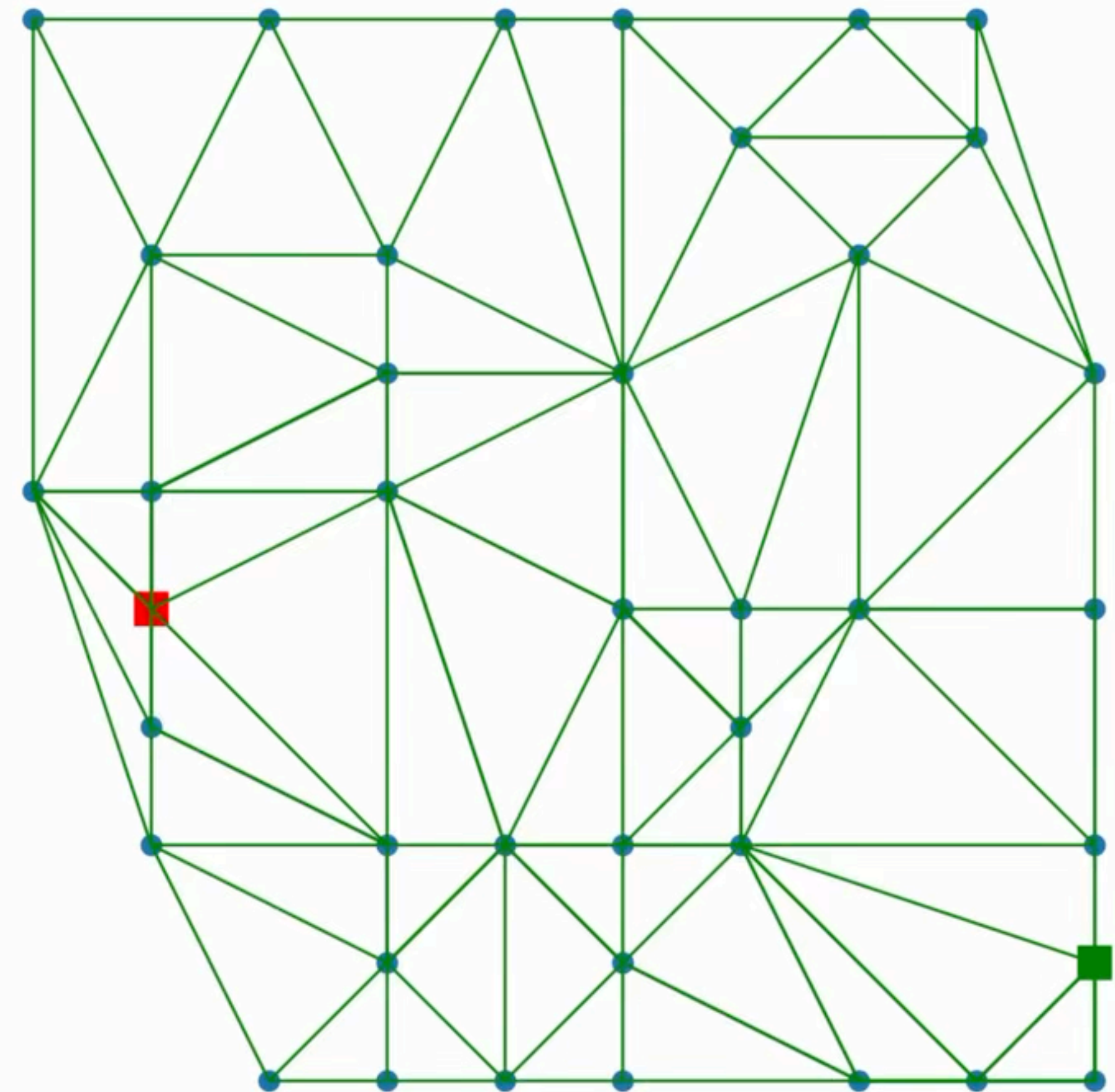
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Pheromones

Iteration 0



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