LQG Dynamics

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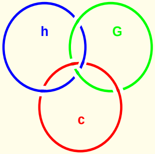
Zakopane 2010



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Status of Operator Constraint Approach to LQG. I.

- Hamiltonian Constraint Operator must be non – anomalous
- $\Rightarrow$ Operator necessarily modifies graph on which it acts
- All semiclassical tools developed so far insufficient to establish correctness of semiclassical limit
- Group averaging of Hamiltonian constraints too difficult due to ∞ no. of constraints
- $\Rightarrow$ physical HS not under sufficient control
- Commutant (Dirac observables) not under sufficient control
- $\Rightarrow$ Cannot do any physically interesting computations

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- Master Constraint Operator must be spat. diffeo. inv.
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Idea:

- Use suitable matter in order to gauge fix $\Leftrightarrow$ pass to the reduced phase space (Higgsing the diffeo group)
- Quantise directly the reduced phase space
- No constraints, no anomalies, no group averaging any more
- All phase space variables are Dirac observables
- The chosen HS rep. is the physical HS
- Automatically get physical, true Hamiltonian
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Words of Caution:

- In order to obtain managable expressions, scalar fields are preferred (using geometrical scalars leads to spatially non local expressions)
- While physical Higgs/SUSY/Dark Matter offer scalar fields, may not be realised in nature
- On the other hand, anyway Higgsed away, only influences the algebraic form of physical Hamiltonian, see next talk
- Have to make consistent restrictions on the phase space of the matter field in order that gauge fixing well defined
- In particular, scalar field must fill all spacetime (never and nowhere vanishing energy density)
- Despite these restrictions, this is a relatively small price to pay compared to the complications associated with operator constraint
- Strategy: Use mathematically convenient matter model to begin with to establish in – principle – proof, later refine physics of matter model

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Brown – Kuchař Scalars

- Imagine universe filled with test observers (non interacting point particles) in geodesic motion
- Specifying metric tensor (and observable matter fields) relative to these observers are Dirac observables
- Problem: test observers are mathematical idealisation, hence must couple point particles to gravity
- Solution: Brown – Kuchař Lagrangian

$$L_{BK} = \sqrt{|\det(g)|}\,\rho\,[g^{\mu\nu}\,U_\mu\,U_\nu + 1],\quad U_\mu = -\nabla_\mu T + W_j\,\nabla_\mu S^j$$

- Consistent (gauge invariant) restriction: $\det(\partial S/\partial x) \neq 0$

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Summary of a long analysis:

- Reduced phase space coordinatised by usual gravitational conjugate pair (A_a^j, E_j^a) (and standard matter) subject only to Gauß constraint but no longer to spatially diffeo constraint and Hamiltonian constraint!
- Physical Hamiltonian

$$H = \int d^3x \sqrt{|C^2 - q^{ab}\, C_a\, C_b|}$$

- Is invariant under active diffeos, no gauge diffeos
- Motivates to choose AIL representation as Physical Hilbert space
- H can be quantised using standard techniques

$$H\, T_\gamma = \sum_{v \in V(\gamma)} H_{\gamma,v}\, T_\gamma; \quad H_{\gamma,v} = \sqrt{|C^2_{\gamma,v} - [C^j_{\gamma,v}]^2|}$$

- Since $H\mathcal{H}_\gamma \subset \mathcal{H}_\gamma$, can establish semiclassical limit [Giesel's talk]
- Now can do scattering theory (eg Fermi's Golden Rule)

$$T_{fi} = < \psi_f,\ H\, \psi_i >$$

- Notice: $\psi_i,\ \psi_f,$ H are physical states/Dirac observables

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History

Recently, also SF researchers became interested in coherent states ...

- Perelomov coherent states for SU(2) [Livine, Speziale 07] and partly SU(1,1) [Conrady, Hnybida 10] enter FK model
- Hall coherent states for $SL(2,\mathbb{C})$ [Bianchi, Magliaro, Perrini 10] for twisted geometries [Freidel, Krasnov, Livine 09], [Freidel, Speziale 10]
- Hall coherent states were used 15y before already in LQG to provide Segal – Bargmann representation [Ashtekar, Lewandowski, Marolf, Mourão, TT 95]
- Inspired Complexifier Machine to Generate Coherent States with built – in semiclassical properties from a sinlge Input [Sahlmann, TT, Winkler 00], [Bahr, TT 08]

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The LQG coherent states were severely criticised by SF researchers eg

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Complexifier Machine

Given a phase space $\mathcal{M} = \mathrm{T}^*(\mathcal{Q})$:

- Choose some function $\mathrm{C(q,p)}$ s.t. i. $\mathrm{C} \geq 0$ and $\mathrm{C} = 0 \quad \Leftrightarrow \quad ||\mathrm{p}|| = 0$ ii. $\lim_{\mathrm{p}\to\infty} \;\mathrm{C}/||\mathrm{p}|| = \infty$ iii. $[\mathrm{C}/\hbar]$ dimensionfree
- Define complex polarisation of phase space
$$\mathrm{z(q,p)} := \exp(-i\, \mathrm{X_C}) \cdot \mathrm{q}$$
- Choose rep. $\hat{\mathrm{q}}, \hat{\mathrm{p}}$ of q,p on HS $\mathcal{H} = \mathrm{L}_2(\mathcal{Q}, \mathrm{d}\mu)$, define Annihilation Operators
$$\hat{\mathrm{z}} := \exp(-\hat{\mathrm{C}}/\hbar)\; \hat{\mathrm{q}}\; \exp(\hat{\mathrm{C}}/\hbar)$$
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$$\psi_{\mathrm{q,p}} := [\mathrm{e}^{\mathrm{C}/\hbar} \cdot \delta_{\mathrm{x}}]_{\mathrm{x}\to \mathrm{z(q,p)}}$$
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Theorem

i. annihilation operator eigenstates $z\ \psi_{q,p} = z(q,p)\ \psi_{q,p}$
ii. Unquenched, minimal uncertainty states for
$x = [z + z^\dagger]/2,\ y = -i[z - z^\dagger]/2$
iii. Peaked at $x(p,q) = \Re(z(p,q)),\ y(p,q) = \Im(z(p,q))$
iv. Ehrenfest property $< [z, z^\dagger] >_{p,q} = i\hbar\{z, z^*\}(q,p)$
iv. Resolution of identity

$$\mathbf{1}_{\mathcal{H}} = \int_{\mathcal{M}} d\nu(z, z^*)\ |\psi_z > < \psi_z|$$

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  Harmonic oscillator: $C = \propto p^2/2$
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- Fix partition $\mathcal{P}$ of Σ into polyhedra p with faces S
- Define for certain L_S

$$C := \frac{1}{2\kappa} \sum_{S \in \partial\mathcal{P}} \frac{1}{L_S^2} [E^j(S)]^2$$

- Delta distribution

$$\delta_A = \sum_s T_s(A) \; < T_s, . >$$

- Complex polarisation (complex connection)

$$Z[A, E](x) = e^{-iX_C} \cdot A(x)$$

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$$\psi_{A,E} = [e^{-C/\hbar} \; \delta_{A'}]_{A' \to Z(A,E)}$$

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$$< \psi_{z'(q,p)}, q\psi_{z'(q,p)} >= \frac{q^3}{l'^2}, \; < \psi_{z'(q,p)}, p\psi_{z'(q,p)} >= \frac{l'^2}{l^2} p$$

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Compare with interpretation proposed in SF papers

- Only interpretation on a single graph dual to some polyhedronal (simplicial) partition (which one?)
- Only interpretation in terms of discretised phase space (no continuum)
- Proposal

$$g_e = n_{e,b(e)} \exp([j_e - i\xi_e]\sigma_3/2)\, n_{e,f(e)}^{-1}, \quad n_{e,p} = n^j_{e,p}\sigma_j, \quad [n^j_{e,p}]^2 = 1$$

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i. Let γ be dual to $\mathcal{P}$ then with $t_e = \ell_P^2/L_e^2$

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Recall

- Volume operator plays pivotal role to define quantum dynamics of LQG and LQC (singularity avoidance!), in particular to define co-triad – like operators
- Two volume operators have been proposed [Rovelli, Smolin 95], [Ashtekar, Lewandowski 95]
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- Want to consider graphs of arbitrary valence n
- Coherent states managable only when graph dual to partition
- Choose regular partition in terms of tetrahedra n=4, cubes n=6, octahedra n=8
- Compute expectation value to zeroth order in $\hbar$
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$$E_j(S) \approx [Y^* E]_j^I(v_P)\ F_I(S_0), \quad F_I(S_0) = \frac{1}{2}\int_{S_0} \epsilon_{IJK}\ dt^J \wedge dt^K$$

- Accordingly

$$< V(P) > \approx \sqrt{|\det([Y^* E_j](v_P)|} \times$$

$$\times \sqrt{|\frac{1}{8} \sum_{1 \le A < B < C \le n} \sigma(e_A, e_B, e_C)\ \epsilon^{IJK}\ F_I(S_0^A)\ F_J(S_0^B)\ F_K(S_0^C)|}$$

- Quantum/classical ratio just depends on Euclidian computation

$$\frac{< V(P) >}{V(P)} \approx \kappa_n,$$

$$\kappa_n = \frac{\sqrt{\left|\frac{1}{8}\sum_{1\leq A<B<C\leq n} \sigma(e_A, e_B, e_C)\ \epsilon^{IJK}\ F_I(S_0^A)\ F_J(S_0^B)\ F_K(S_0^C)\right|}}{V_0(P_0)}$$

- End result for tetrahedron, cube and octahedron

$$\kappa_4 = 3\frac{\sqrt{2}}{4} > 1,\ \kappa_6 = 1,\ \kappa_8 = \frac{3}{2\sqrt{2}} > 1$$

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Discussion:

- One expects $\lim_{n \to \infty} \kappa_n = \infty$ too many triples contribute, overcounting
- Cubic graphs dynamically preferred in the semiclassical limit
- One cannot temper with $1/48$ (co-triad test)
- One could temper with t_e (rescale classical flux label in X^j_e)
- But then areas have incorrect expectation value
- Crucial that now SF also on arbitrary cell complexes are possible [Kaminski, Kiesilowski, Lewandowski 09]
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Motivation

Based on [Baratin, Flori, TT 08]

- Big issue in SFM: correct implementation of simplicity constraint
- In principle easy: Integrate over Plebanski Lagrange multiplier

$$Z := \int [DA\ DB\ D\lambda]\ e^{i\int [\mathrm{Tr}(B\wedge F)+\lambda^\alpha S_\alpha(B)]} = \int [DA\ DB]\ \delta[S(B)]\ e^{i\int [\mathrm{Tr}(B\wedge F)]}$$

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- Simplicity constraints still Abelian at this stage
- Now something funny happens: Discretisation $F \mapsto A(\alpha) - 1$ replaces $\frac{\delta}{\delta F} \mapsto X_\alpha$ and simplicity constraints become non – Abelian, anomalous
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Idea:

- If we honestly want to impose the simplicity constraint we must keep $\delta[S(B)]$
- Solving the δ distribution leads back to the Palatini – Holst action modulo mesure factors, see next topic
- Might as well start with Holst PI

$$Z = \int [DA\ De]\ e^{i \int \ \mathrm{Tr}(F_\gamma \wedge e \wedge e)}$$

- As pointed out in [Mikovic 05]: integral over e is oscillatory Gaussian, can be performed
- Care is due, since result depends critically on signature of 16 x 16 matrix F_γ
- One ends up with a final integral of the form

$$Z = \int [DA]\ [\frac{e^{i\pi \mathrm{ind}(F_\gamma)/4}}{\sqrt{|\det(F_\gamma)|}}$$

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Details

- As motivated by previous topic, discretise on cubulation, works for either signature
- Every manifold can be cubulated
- For sufficiently nice manifolds (admitting finite atlas) choose an atlas and consider its stratification
- Choose a regular cubulation (like in lattice gauge theory) in the interior of any chart (away from lower dimensional strata) and invent some gluing cubulation in the neighbourhood of the lower dimensional strata
- Non – regular “boundary” cubes contribute little as compared “bulk” cubes
- Use naive continuum limit as in lattice gauge theory rather than sum over cubulations
- Discretise both tetrads and connections on 1 – skeleton of cubulation $\Rightarrow$ preserves gauge invariance, will not use graph dual to cubulation

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- Regular cubulation: vertices $\mathsf{v} \in \mathbb{Z}^4$, directions $\mathsf{mu} = 0, 1, 2, 3$, links $\mathsf{l}_\mu(\mathsf{v})$, plaquettes

$$\partial \mathsf{f}_{\mu\nu}(\mathsf{v}) = \mathsf{l}_\mu(\mathsf{v}) \circ \mathsf{l}_\nu(\mathsf{v}+\mu) \circ \mathsf{l}_\mu(\mathsf{v}+\nu)^{-1} \circ \mathsf{l}_\nu(\mathsf{v})^{-1}$$

- Discrete variables

$$\mathsf{e}^{\mathsf{I}}_\mu(\mathsf{v}) := \int_{\mathsf{l}_\mu(\mathsf{v})} [\mathsf{A}(\mathsf{l}_\mu(\mathsf{v},\mathsf{x}))]^{\mathsf{I}}{}_{\mathsf{J}}\, \mathsf{e}^{\mathsf{J}}(\mathsf{x}), \quad \mathsf{G}^{\mu\nu}{}_{\mathsf{IJ}}(\mathsf{v}) = \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}([*\mathsf{T}_{\mathsf{IJ}}]\, \mathsf{A}(\partial \mathsf{f}_{\rho\sigma}(\mathsf{v}))$$

- Discrete action

$$\mathsf{S} = \frac{1}{2\kappa} \sum_{\mathsf{v} \in \mathbb{Z}^4} \mathsf{e}^{\mathsf{I}}_\mu(\mathsf{v})\, \mathsf{e}^{\mathsf{J}}_\nu(\mathsf{v})\, \mathsf{G}^{\mu\nu}{}_{\mathsf{IJ}}(\mathsf{v})$$

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$$\frac{\mathsf{S}}{\hbar} = \frac{1}{2\ell_{\mathsf{P}}^2} \sum_{\mathsf{v}} \mathsf{e}^{\mathsf{T}}(\mathsf{v})\, \mathsf{G}(\mathsf{v})\, \mathsf{e}(\mathsf{v})$$

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$$\frac{S}{\hbar} = \frac{1}{2\ell_P^2} \sum_v e^T(v) \; G(v) \; e(v)$$

- Generating functional of n – point functions

$$Z(j) = \int [\prod_{v,\mu} [d\mu_H(A(l_\mu(v))] \, [d^4 e_\mu(v)]] \;\; e^{\frac{i}{2\ell_P^2} \sum_v \; e^T(v) \; G(v) \; e(v)} \;\; e^{i \sum_v \; j^T(v) \; e(v)}$$

- Integrating out tetrad yields

$$Z(j) = \int [\prod_{v,\mu} [d\mu_H(A(l_\mu(v))]][\prod_v \frac{e^{i\pi \mathrm{ind}(G(v))/4}}{\sqrt{|\det(G)(v)|}}] \; e^{-i\frac{\ell_P^2}{2} \sum_v \; j^T(v) \; G^{-1}(v) \; j(v)}$$

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Remarks:

- Looks like generating functional for free field theory
- But not a quasi – free "state" because of averaging over G
- Wick identities hold in averaged sense, all odd n – point functions vanish
- "Graviton propagator" from 4 – point function

$$< g_{\mu_1\nu_1}(v_1)\ g_{\mu_2\nu_2}(v_2) >= \ell_P^4\ < \mathrm{Tr}([G(v_1)^{-1}]_{\mu_1\nu_1}([G(v_2)^{-1}]_{\mu_2\nu_2}) >'$$

- If wanted, can formally expand in terms of irreps
  $\Rightarrow$ octagon diagramme (involves 48 irreps of Spin(4) or Spin(1,3))
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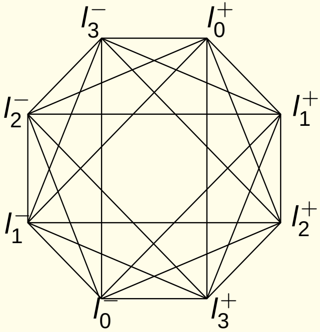
Octagon diagramme for vertex amplitude:

8 corners = adjacent edges, 24 lines between corners = plaquettes using those edges



Path Integrals from Canonical Quantisation

- The only safe route to a path integral formulation with manifest relation to canonical theory starts from the reduced phase space
- Studied by Field theorists in great detail, e.g. [Henneaux, Teitelboim 95]
- Special care for gauge theories with 2nd class constraints
- It is possible to unfold the path integral to the kinematical phase space, but generically this leads to corrections to the naive measure, formally (q', p' kinematical phase space coordinates not including Lagrange multiplicators)

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- Gauge fixing conditions G (clocks and rods) select preferred true degrees of freedom Q (Dirac observables)
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- One must get rid of the secondary second class constraints in order to produce the wanted exp(iS). General technique developed by [Henneaux, Slavnov 94]
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Results for Holst action

- Explicit computation of ρ performed for Holst action [Engle, Han, TT 09] to make contact with new spin foam models
- Result (full phase space variables: tetrad and connection)

$$\rho = \sqrt{|\det(g)|}^3 \sqrt{\det(q)}$$

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