Hamiltonian Approach to Conformal Coupling Scalar Field in the General Relativity

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Abstract.

The dynamic status of scalar fields is studied in the Hamiltonian approach to the General Relativity. We show that the conformal coupling of the scalar field violates the standard geometrical structure of the Einstein equations in GR and their solutions including the Schwarzschild one and the Newton static interaction.

In order to restore the standard geometrical structure of GR, the scalar field is mixed with the scale metric component by the Bekenstein type transformation. This "scalar-scale" mixing converts the conformal coupling scalar field with conformal weight (n= −1) into the minimal coupling scalar field with zero conformal weight (n=0) called a "scalar graviton".

Cosmological consequences of the "scalar-scale" mixing are considered in the finite space-time by extraction of the zero mode (homogeneous) harmonics of a "scalar graviton". The classical dynamics of "scalar graviton" testifies about a tremendous contribution of its kinetic energy into the Universe evolution at the beginning in the form of the rigid state.

PACS numbers: 11.10.Ef, 11.15.-q, 11.15.Ex, 11.25.Uv, 04.20.-q, 04.20.Fy

1. Introduction

The scalar fields play very important role in both particle physics as the Higgs field in Standard Model (SM) and standard cosmological model as one of elements of the inflation mechanism [1]. A scalar field (SF) can be introduced into General Relativity (GR) in two manners: without $R/6$ term (a minimal coupling) and with $R/6$ term (a conformal coupling) [2]. The difference between two couplings becomes essential at cosmological applications and the unification of SM with GR. Therefore, the research of difference of these two couplings from the dynamic point of view is the topical problem [3, 4].

The present paper is devoted to investigation of dynamical status of different couplings of scalar field by means of the Hamiltonian approach to GR [5, 6, 7, 8, 9] generalized for finite space in [10, 11] and the Lichnerowicz transformation to the unit determinant of the spatial metric [12]. We research self-consistences of initial conditions with equations of motion and boundary conditions, with the variation problem.

In Section 2, the status of a conformal coupling scalar field in GR is considered. In Section 3, the correspondence of the considered model with a relativistic brane is established. Sections 4, 5 are devoted to the Hamiltonian approach to the considered model in the finite space-time. In Section 6, cosmological consequences are studied.

2. Scalar field: action, interval, and symmetries

The sum of GR and SF actions is

$$S_{GR+SF,\xi} = \int d^4x\sqrt{-g}\left[-\frac{\varphi_0^2}{6}R(g) + \xi\frac{\Phi^2}{6}R(g) + g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right], \tag{1}$$

where

$$\varphi_0 = \sqrt{\frac{3}{8\pi}}\mathcal{M}_{Pl} \simeq 0.3\cdot 10^{18}\text{GeV} \tag{2}$$

is the mass scale factor, $R(g)$ is the curvature Ricci scalar and $g_{\mu\nu}$ is the metric tensor on the Riemann manifold with the *"geometric interval"*

$$ds^2 = g_{\mu\nu}dx^\mu x^\nu, \tag{3}$$

$\xi = 0, 1$ for minimal ($\xi = 0$) and conformal ($\xi = 1$) couplings. The *"geometric interval"* can be written in terms of linear differential forms as components of an orthogonal Fock's simplex of reference $\omega_{(\alpha)}$

$$ds^2 \equiv \omega_{(\alpha)}\omega_{(\alpha)} = \omega_{(0)}\omega_{(0)} - \omega_{(1)}\omega_{(1)} - \omega_{(2)}\omega_{(2)} - \omega_{(3)}\omega_{(3)}. \tag{4}$$

In terms of simplex the GR contains two principles of relativity: the *"geometric"* in the form of general coordinate transformations

$$x^\mu \quad\quad \to \tilde{x}^\mu = \tilde{x}^\mu(x^0, x^1, x^2, x^3) \tag{5}$$

$$\omega_{(\alpha)}(x^\mu) \to \omega_{(\alpha)}(\tilde{x}^\mu) = \omega_{(\alpha)}(x^\mu) \tag{6}$$

and the *"dynamic"* principle formulated as the Lorentz transformations of an orthogonal simplex of reference

$$\omega_{(\alpha)} \rightarrow \overline{\omega}_{(\alpha)} = L_{(\alpha)(\beta)}\omega_{(\beta)}. \tag{7}$$

The latter are considered as transformations of a frame of reference.

3. Conformal coupling scalar field in GR as a relativistic brane

The conformal coupling scalar field action

$$S_{SF,\xi=1} = \int d^4x \left[\sqrt{-g}\frac{\Phi^2}{6}R(g) - \Phi\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi)\right], \tag{8}$$

is invariant with respect to scale transformations of metric components and scalar field

$$g^{\Omega}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Phi^{\Omega} = \Omega^{-1}\Phi \tag{9}$$

with the conformal weights $n = 2, -1$ for the tensor and scalar fields, respectively. The conformal coupling of the scalar field with the weight $n = -1$ is required by unification of GR and SM. The latter is scale invariant except of the Higgs potential. However, the Hilbert – Einstein action S_{GR} in Eq. (1) is not invariant. After the scale transformation the total action (1) $S_{GR+SF,\xi=1}$ takes the form of the conformal relativistic brane

$$\begin{aligned} &S_{GR}[g^{\Omega}] + S_{SF,\xi=1}[g^{\Omega}, \Phi^{\Omega}] = S^{(D=4/N=2)}_{\text{brane}}[X_{(0)}, X_{(1)}] = \\ -&\int d^4x \Bigg[\sqrt{-g}\frac{X^2_{(0)} - X^2_{(1)}}{6}\,{}^{(4)}R(g) - X_{(0)}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu X_{(0)}) + \\ &X_{(1)}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu X_{(1)})\Bigg], \end{aligned} \tag{10}$$

where two external "coordinates" are defined as

$$X_{(0)} = \varphi_0\Omega, \qquad X_{(1)} = \Phi \tag{11}$$

in accord with the standard definition of the general action for brane in D/N dimensions given in [22] by

$$\begin{aligned} S^{(D/N)}_{\text{brane}} = -\int d^Dx \sum_{A,B=1}^{N} \eta^{AB}\Bigg[&\sqrt{-g}\frac{X_A X_B}{(D-2)(D-1)}\,{}^{(D)}R(g) - \\ &X_A\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu X_B)\Bigg] \end{aligned} \tag{12}$$

in the case for $D = 4$, $N = 2$ we have $\eta^{AB} = \text{diag}\{1, -1\}$. In this case, in order to keep conformal invariance of the theory (10), the Einstein definition of a measurable interval (3) in GR (1) should be replaced by its conformal invariant version as a Weyl-type ratio

$$ds^2_{(\text{L})} = \frac{ds^2}{ds^2_{\text{units}}}, \tag{13}$$

where ds^2_{units} is an interval of the units that is defined like the Einstein definition of a measurable interval (3) in GR.

From the relativistic brane viewpoint the Einstein GR (1) with the conformal coupling scalar field looks like the Ptolemaeus absolutizing of the present-day (PD) value of one of "coordinates" in field "superspace" of events $[X_{(0)}|X_{(1)}]$, in our case it is

$$X_{(0)}\Big|_{\rm PD} = \varphi_0. \tag{14}$$

This is equivalent to fixation of units of measurements [19]. Another choice of independent degrees of freedom in the brane theory (10) is the unit spatial metric determinant

$$|g^{(3)}_{(\rm L)}| = 1 \tag{15}$$

known as the Lichnerowicz variables [12]. In the last case, a relativistic system in each frame has its proper units, like a particle in each frame in classical mechanics has its proper initial position and velocity (Galilei), and a relativistic particle in each frame in special relativity has its proper time (Einstein). The "relative units" (13) supposes a new type of conformal cosmology (CC) [19, 20, 21] with another definition of the "measurable distance" (13), instead of the standard cosmology (SC) with absolute units (3) [18]. The "relative units" (13) in CC excludes the expansion of the "measurable" volume of the Universe in the process of its cosmological evolution, as this volume does not depend on any scale factor including the cosmological one, whereas all masses in CC including the Planck one are scaled by cosmological scale factor. The relative "measurable distance" (13) in CC explains the SN data on the luminosity-distance – redshift relation [23, 24, 25] by the rigid state without Λ – term [19, 21]. Thus, a conformal-invariant relativistic brane (10) is a more general theory than Einstein GR (1), and is reduced to GR for the absolute units (14) or to the scalar version of the Weyl conformal theory in terms of the Lichnerowicz variables (15). In the following, we call the theory (10) with condition (15) the Conformal Relativity (CR).

The problem is to determine the measurable Planck mass and cosmological scale factor in both the GR (1) and CR (10). Measurable quantities are determined by a frame of reference to initial data in both the "external superspace of events" $[X_{(0)}, X_{(1)}]$ and "internal" Riemannian space-time (x^0, x^k).

4. Reference frame in external "superspace of events"

4.1. Distortion of GR by the conformal coupling scalar field

One can see that the conformal coupling scalar field Φ distorts the Newton coupling constant in the Hilbert action (1) distinguished by (14)

$$S_{GR+SF,\xi=1} = \int d^4x\sqrt{-g}\left[-\left(1-\frac{|\Phi|^2}{\varphi_0^2}\right)\frac{\varphi_0^2}{6}R(g) + g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right], \tag{16}$$

This distortion changes the Einstein equations and their standard solutions of type of Schwarzschild one and other [13, 3, 4] due to the coefficient $[1-|\Phi|^2/(\varphi_0^2)]$. This coefficient restricts region of a scalar field motion by the condition $|\Phi|^2 \leq \varphi_0^2$, because in other region $|\Phi|^2 \geq \varphi_0^2$ the sign before the 4-dimensional curvature is changed in the

Hilbert action (1). The rough analogy of this restriction is the light cone in special relativity which defines the physically admissible region of a particle motion.

4.2. The Bekenstein's transformation of the Higgs field

In order to keep the Einstein theory (1), one needs to consider only the field configuration such that $|\Phi|^2 \le \varphi_0^2$. For this case one can introduce new variables [13]

$$g_{\mu\nu} = g^{(\mathrm{B})}_{\mu\nu} \cosh^2 Q, \tag{17}$$

$$|\Phi|^2 = \varphi_0^2 \sinh^2 Q \tag{18}$$

considered in [3, 4]. These variables restore the initial Einstein – Hilbert action

$$S_{GR+SF,\xi=1} = \int d^4x \sqrt{-g}\varphi_0^2 \left[-\frac{R(g_{(\mathrm{B})})}{6} + g^{\mu\nu}_{(\mathrm{B})}\partial_\mu Q \partial_\nu Q\right]. \tag{19}$$

One can see that *the Bekenstein transformation converts the "conformal coupling" scalar field with the weight* $n = -1$ *into the "minimal coupling"* angle Q of the scalar – scale mixing that looks like a *scalar graviton* with the conformal weight $n = 0$.

4.3. Choice of "coordinates" in brane "superspace of events"

The analogy of GR (1) with a relativistic brane (10) distinguished by the condition (15) allows us to formulate the choice of variables (17) and (18) as a choice of the "frame" in the brane "superspace of events"

$$\widetilde{X}_{(0)} = \sqrt{X^2_{(0)} - X^2_{(1)}}, \tag{20}$$

$$Q = \operatorname{arc\,coth} \frac{\mathrm{X}_{(0)}}{\mathrm{X}_{(1)}} \tag{21}$$

As we have seen above the argument in favor of the choice of these variables is the definition of the measurable value of the Newton constant

$$G = \frac{8\pi}{3} \widetilde{X}^{-2}_{(0)}\Big|_{\text{present-day}} = \frac{8\pi}{3}\varphi_0^{-2} \tag{22}$$

as the present-day value of the "coordinate" $\widetilde{X}_{(0)} = \varphi_0$.

In the case the action (10), (15) takes the form

$$\begin{aligned} &S_{GR}[g^\Omega] + S_{SF,\xi=1}[g^\Omega, \Phi^\Omega] = S^{(D=4/N=2)}_{\text{brane}}[X_{(0)}, X_{(1)}] = \\ &\int d^4x \Bigg[\sqrt{-g_{(L)}}\widetilde{X}^2_{(0)}\left(-\frac{{}^{(4)}R(g_{(L)})}{6} + g^{\mu\nu}_{(L)}\partial_\mu Q \partial_\nu Q\right) + \\ &\widetilde{X}_{(0)}\partial_\mu\left(\sqrt{-g_{(L)}} g^{\mu\nu}_{(L)} \partial_\nu \widetilde{X}_{(0)}\right)\Bigg]. \end{aligned} \tag{23}$$

This form is the brane generalization of the relativistic conformal mechanics

$$S^{(D=1/N=2)}_{\text{particle}}[X_0, Q_0] = \int ds \left[X_0^2\left(\frac{dQ_0}{ds}\right)^2 - \left(\frac{dX_0}{ds}\right)^2\right]; \tag{24}$$

$$ds = dx^0 e(x^0). \tag{25}$$

In the following this conformal mechanics will be considered as a simple example.

5. Reference frame in internal Riemannian space-time

5.1. The Dirac — ADM parametrization

Recall that the Hamiltonian approach to GR is formulated in a specific frame of reference in terms of the Dirac – ADM parametrization of metric [5, 6] defined as

$$ds^2 = g^{(\mathrm{B})}_{\mu\nu}dx^\mu dx^\nu \equiv \omega^2_{(0)} - \omega^2_{(b)} \tag{26}$$

$$\omega_{(0)} = \psi^6 N_{\mathrm{d}} dx^0 \equiv \psi^2\, \omega^{(L)}_{(0)} \tag{27}$$

$$\omega_{(b)} = \psi^2 \mathbf{e}_{(b)i}(dx^i + N^i dx^0) \equiv \psi^2\, \omega^{(L)}_{(b)} \tag{28}$$

here triads $\mathbf{e_{(a)i}}$ form the spatial metrics with $\det|\mathbf{e}| = 1$, N_{d} is the Dirac lapse function, N^i is shift vector and ψ is a determinant of the spatial metric and $\omega^{(L)}_{(\mu)}$ are the Lichnerowicz simplex distinguished by the condition of the unit determinant (15). In terms of these metric components the GR action takes the form

$$S[\varphi_0|F,Q] = \int dx^0 \varphi_0^2 \int d^3x \left[-\psi^{12} N_{\mathrm{d}} \frac{{}^{(4)}R(g)}{6} + \frac{(\partial_0 Q - N^k\partial_k Q)^2}{N_{\mathrm{d}}} - N_{\mathrm{d}}\psi^8 \partial_{(b)}Q\partial_{(b)}Q\right], \tag{29}$$

where $\partial_{(b)}Q = \mathbf{e}^k_{(b)}\partial_k Q$ and ${}^{(4)}R(g)$ is given in Appendix (see Eq. (68)). This action is invariant with respect to transformations [16]

$$x^0 \to \tilde{x}^0 = \tilde{x}^0(x^0) \tag{30}$$

$$x_i \to \tilde{x}_i = \tilde{x}_i(x^0, x_1, x_2, x_3), \tag{31}$$

$$\tilde{N}_d = N_d \frac{dx^0}{d\tilde{x}^0}, \tag{32}$$

$$\tilde{N}^k = N^i \frac{\partial \tilde{x}^k}{\partial x_i}\frac{dx^0}{d\tilde{x}^0} - \frac{\partial \tilde{x}^k}{\partial x_i}\frac{\partial x^i}{\partial \tilde{x}^0}\ . \tag{33}$$

This group of diffeomorphisms conserves a family of constant-time hypersurfaces, and is commonly known as the "kinemetric" subgroup of the group of general coordinate transformations $x^\mu \to \tilde{x}^\mu = \tilde{x}^\mu(x^0, x^1, x^2, x^3)$. The "kinemetric" subgroup contains reparametrizations of the coordinate evolution parameter x^0. This means that in finite space-time the coordinate evolution parameter x^0 is not measurable quantity, like the coordinate evolution parameter x^0 in the relativistic conformal mechanics (24) that is invariant with respect to diffeomorphisms $x^0 \to \tilde{x}^0 = \tilde{x}^0(x^0)$, because both parameters x^0 are not diffeo-invariant.

The relativistic mechanics (24) has two diffeo-invariant measurable times. They are the geometrical interval (25) and the time-like variable X_0 in the external "superspace of events". The relation between these two "times" $X_0(s)$ are conventionally treated as a relativistic transformation. The main problem is to point out similar two measurable time-like diffeo-invariant quantities in both GR (29) and a brane (23).

5.2. *External diffeo-invariant evolution parameter as zero mode in finite volume*

The brane/GR correspondence (10) and special relativity (24) allows us to treat an external time as homogeneous component of the time-like external "coordinate" $\widetilde{X}_{(0)}(x^0, x^k)$ identifying this homogeneous component with the cosmological scale factor a

$$\widetilde{X}_{(0)}(x^0, x^k) \to \varphi_0 a(x^0) = \varphi(x^0) \tag{34}$$

because this factor is introduced in the cosmological perturbation theory [26] by the scale transformation of the metrics (11) too

$$g_{\mu\nu} = a^2(x^0)\widetilde{g}_{\mu\nu}. \tag{35}$$

Recall that, in this case, any field $F^{(n)}$ with the conformal weight (n) takes the form

$$F^{(n)} = a^n(x_0)\widetilde{F}^{(n)}. \tag{36}$$

In particular, the curvature $\sqrt{-g}\ {}^{(4)}R(g) = a^2\sqrt{-\widetilde{g}}\ {}^{(4)}R(\widetilde{g}) - 6a\partial_0\left[\partial_0 a\sqrt{-\widetilde{g}}\ \widetilde{g}^{00}\right]$ can be expressed in terms of the new lapse function $\widetilde{N}_d$ and spatial determinant $\widetilde{\psi}$ in Eq. (26)

$$\widetilde{N}_d = [\sqrt{-\widetilde{g}}\ \widetilde{g}^{00}]^{-1} = a^2 N_d, \qquad \widetilde{\psi} = (\sqrt{a})^{-1}\psi. \tag{37}$$

In order to keep the number of variables in GR, in contrast to [26], we identify $\log\sqrt{a}$ with the spatial volume "averaging" of $\log\psi$, and $\log\widetilde{\psi}$, with the nonzero Fourier harmonics [10, 11]

$$\log\sqrt{a} = \langle\log\psi\rangle \equiv \frac{1}{V_0}\int d^3x \log\psi, \qquad \langle\log\widetilde{\psi}\rangle \equiv 0 \tag{38}$$

here the Lichnerowicz diffeo-invariant volume $V_0 = \int d^3x$ is introduced. One should emphasize that modern cosmological models [26] are considered in the finite space and "internal finite time" in a reference frame identified with the frame of the Cosmic Background Microwave Radiation.

A scalar field can be also presented as a sum of a zero Fourier harmonics and nonzero ones like (38)

$$Q = \langle Q\rangle + \overline{Q}; \qquad \langle\overline{Q}\rangle = 0 \tag{39}$$

After the separation of all zero modes the action (29) takes the form

$$S[\varphi_0|F,Q] = S[\varphi|\widetilde{F},\overline{Q}] + \underbrace{V_0\int dx^0\frac{1}{N_0}\left[\varphi^2\left(\frac{d\langle Q\rangle}{dx^0}\right)^2 - \left(\frac{d\varphi}{dx^0}\right)^2\right]}_{zero-mode\ contribution}; \tag{40}$$

here

$$S[\varphi|\widetilde{F},\overline{Q}] = \int dx^0\varphi^2\int d^3x\left[-\widetilde{\psi}^{12}\widetilde{N}_{\rm d}\frac{{}^{(4)}R(\widetilde{g})}{6} + \frac{(\partial_0\overline{Q} - N^k\partial_k\overline{Q})^2}{\widetilde{N}_{\rm d}} - \widetilde{N}_{\rm d}\widetilde{\psi}^8\partial_{(b)}\overline{Q}\partial_{(b)}\overline{Q}\right] \tag{41}$$

repeats action $S[\varphi_0|F,Q]$ (29), where $[\varphi_0|F,Q]$ are replaced by $[\varphi|\widetilde{F},\overline{Q}]$, and

$$\frac{1}{N_0}=\frac{1}{V_0}\int\frac{d^3x}{\widetilde{N}_d}\equiv\left\langle\frac{1}{\widetilde{N}_d}\right\rangle \tag{42}$$

is the homogeneous component of the lapse function. The action of the local variables (41) determines the correspondent local energy density for the local variables

$$-\,\widetilde{T}_{\rm d}=\frac{\delta S[\varphi_0|\widetilde{F},\overline{Q}]}{\delta\widetilde{N}_{\rm d}}. \tag{43}$$

5.3. *"Internal diffeo-invariant homogeneous time"*

The homogeneous component of the lapse function (42) N_0 determines difeo-invariant local lapse function

$$\mathcal{N}=\widetilde{N}_{\rm d}\langle\widetilde{N}_{\rm d}^{-1}\rangle,\qquad\langle\mathcal{N}^{-1}\rangle=1, \tag{44}$$

and the "internal diffeo-invariant homogeneous time" with its derivative

$$\int dx^0N_0=\zeta,\qquad f'=\frac{df}{d\zeta}. \tag{45}$$

5.4. *Resolution of the energy constraints*

The action principle for the $S[\varphi_0|F,Q]$ with respect to the lapse function $\widetilde{N}_{\rm d}$ gives us the energy constraints equation

$$\frac{1}{\mathcal{N}^2}\left(\varphi'^2-\varphi^2\langle Q\rangle'^2\right)-\widetilde{T}_{\rm d}=0. \tag{46}$$

This equation is the algebraic one with respect to the diffeo-invariant lapse function $\mathcal{N}$ and has solution satisfying the constraint (44)

$$\mathcal{N}=\frac{\langle\widetilde{T}_{\rm d}^{1/2}\rangle}{\widetilde{T}_{\rm d}^{1/2}}. \tag{47}$$

The substitution of this solution into the energy constraint (46) leads to the cosmological type equation

$$\varphi'^2=\varphi^2\langle Q\rangle'^2+\langle(\widetilde{T}_{\rm d})^{1/2}\rangle^2\equiv\rho_{\rm tot}(\varphi)=\frac{P^2_{\langle Q\rangle}}{4V_0^2\varphi^2}+\langle(\widetilde{T}_{\rm d})^{1/2}\rangle^2 \tag{48}$$

here the total energy density $\rho_{\rm tot}(\varphi)$ is split on the sum of the energy density of local fields $\langle(\widetilde{T}_{\rm d})^{1/2}\rangle^2$ and the zero mode one, where

$$P_{\langle Q\rangle}=2V^0\varphi^2\langle Q\rangle'\equiv 2V^0p_0 \tag{49}$$

is the scalar field zero mode momentum that is an integral of motion of the considered model because the action does not depend on $\langle Q\rangle$. The value of the local energy density onto solutions of motion equations depends on only φ too, because momentum of the external time φ

$$P_\varphi=2V_0\varphi'=\pm 2V_0\sqrt{\frac{p_0^2}{\varphi^2}+\langle(\widetilde{T}_{\rm d})^{1/2}\rangle^2}\equiv\mp E_\varphi \tag{50}$$

can be considered as the Hamiltonian of evolution in the "superspace of events". The value of the momentum $P_\varphi = \pm E_\varphi$ onto solutions of motion equation is defined as an energy of the universe, in accord with the second Nöther theorem removing momenta by constraints following from diffeomorphisms. We can see that the dimension of the group of diffeomorphisms (30) and (31) coincides with the dimension of the first class constraint momenta, because the local part of the energy constraint (46) determines the diffeo-invariant local lapse function $\mathcal{N}$ (47).

The solution (50) gives the diffeo-invariant constraint-shell action (90) obtained in Appendix

$$S_{\mathcal{H}=0} = \int\limits_{\varphi_I}^{\varphi_0} d\widetilde{\varphi} \left\{ \int\limits_{V_0} d^3x \sum_{\widetilde{F}} P_{\widetilde{F}} \partial_\varphi \widetilde{F} \mp 2E_\varphi \right\}. \qquad (51)$$

The GR version of the Friedmann equation (48) leads to the diffeo-invariant Hubble law as the relation between the geometric time (45) and the cosmological scale factor $\varphi = \varphi_0 a$ [10, 11]

$$\zeta_{(\pm)} = \int dx^0 N_0 = \pm \int_{\varphi_I}^{\varphi_0} d\varphi \ \left[p_0^2/\varphi^2 + \langle (\widetilde{T}_{\rm d})^{1/2} \rangle^2 \right]^{-1/2} \geq 0,$$

where

$$\widetilde{T}_{\rm d} = \frac{4\varphi^2}{3} \widetilde{\psi}^7 \triangle \widetilde{\psi} + \sum_I \varphi^{I/2-2} \widetilde{\psi}^I \overline{\mathcal{T}}_I; \qquad (52)$$

here $\overline{\mathcal{T}}_I$ is partial energy density marked by the index I running, in general case, a set of values I=0 (stiff), 4 (radiation), 6 (mass), 8 (curvature)‡ in correspondence with a type of matter field contributions (see Appendix, Eqs. (87) and (88)).

The second class Dirac condition of the minimal 3-dimensional hyper-surface [5]

$$p_{\widetilde{\psi}} = 0 \to (\partial_\zeta - N_{(b)} \partial_{(b)}) \log \widetilde{\psi} = \frac{1}{6} \partial_{(b)} N_{(b)}, \qquad (53)$$

is included in order to give a positive value of the Hamiltonian density $\widetilde{T}_{\rm d}$ given by Eq. (52) and Eq. (87) in the Appendix. The equations (47) and $\widetilde{T}_\psi - \langle \widetilde{T}_\psi \rangle = 0$ (where $\widetilde{T}_\psi = T_\psi[\varphi|\widetilde{\psi}]$ and T_ψ is given by Eq. (85)) determine the lapse function $\mathcal{N}$ and the scalar component $\widetilde{\psi}$.

Thus, we give the diffeo-invariant formulation of the GR with the conformal scalar field in the comoving CMB frame compatible with the Einstein equations $T_{\rm d} = T_\psi = 0$ and their Schwarzschild-type solutions $\triangle\psi = 0$, $\triangle(\psi^7 \mathcal{N}) = 0$ in the infinite volume limit [10], in contrast to all other approach to a scalar field in GR [1, 27].

The special relativity identification of the brane external time-like "coordinate" with the diffeo-invariant evolution parameter (34)

$$\widetilde{X}_{(0)}(x^0, x^k) = \sqrt{X_{(0)}^2 - X_{(1)}^2} = \varphi(\zeta) \widetilde{\psi}^2$$

‡ Λ-term corresponds to $I = 12$

arising after resolving energy constraint with respect its momentum P_φ is in agreement with the Hamiltonian version [10, 11, 20] of cosmological perturbation theory [26] identifying this external time-like "coordinate" with the cosmological scale factor $a(\eta) = \varphi(\eta)/\varphi_0$ provided that

(i) the cosmological scale factor is a zero mode of the scalar component of metric (but it is not additional variable as it is supposed in the accepted cosmological perturbation theory [26])

(ii) the conformal time in the redshift – luminosity distance relation is gauge-invariant measurable quantity, in accord with the Dirac definition of observable quantities as diffeo-invariants (but it is not diffeo-variant quantity as an object of Bardeen's gauge transformations in the accepted perturbation theory [26]),

(iii) the initial datum $a(\eta = 0) = a_I$ is free from the current data $a'(\eta_0)$ and fundamental parameter $M_{\rm Planck}$ of the motion equations (because the Planck epoch data $a(\eta = 0) = a'(\eta_0)/M_{\rm Planck}$ violate the causality principle in the constraint action (51) and they are the origin of numerous problems in the Inflationary Model [1]),

(iv) there is the vacuum as a state with the minimal energy as explicit resolving the energy constraint. The vacuum postulate is provided by the second class Dirac condition of the minimal 3-dimensional hyper-surface [5] (53) that removes the kinetic perturbations of the accepted cosmological perturbation theory explaining the power CMB spectrum [27].

However, the accepted cosmological perturbation theory [27] omitted the potential perturbations going from the scalar metric component $\widetilde{\psi} = 1 - \Psi/2$ in partial energy density (52) that leads to additional fluctuations of the CMB temperature [10].

6. Cosmology and the Cauchy problem of the zero mode dynamics

The conformal-invariant unified theory alternative in the homogeneous approximation

$$w(x^0, x^k) = \varphi(x^0) \equiv \varphi_0 a(x^0), \tag{54}$$

$$Q(x^0, x^k) = \langle Q\rangle(x^0) \equiv \frac{1}{V_0}\int d^3x Q(x^0, x^k), \tag{55}$$

$$N_d(x^0, x^k) = N_0(x^0), \tag{56}$$

$$N_0(x^0)dx^0 = d\eta \tag{57}$$

leads to the cosmological model given by the action

$$S = V_0 \int dx^0 \left[\frac{-(\partial_0\varphi)^2 + \varphi^2(\partial_0\langle Q\rangle)^2}{N_0}\right] = \tag{58}$$

$$= \int dx^0 \left\{P_Q \frac{d}{dx^0}\langle Q\rangle - P_\varphi \frac{d}{dx^0}\varphi + \frac{N_0}{4V_0}\left[P_\varphi^2 - \frac{P_Q^2}{\varphi^2}\right]\right\} \tag{59}$$

where $V_0 = \int d^3x$ is finite coordinate volume,

$$P_\varphi = 2V_0\varphi' \equiv 2V_0\frac{d\varphi}{d\eta}, \tag{60}$$

$$P_Q = 2V_0\,\varphi^2\,\langle Q\rangle' \tag{61}$$

are canonical conjugated momenta, $\eta = \int dx^0 N_0(x^0)$ is the conformal time.

The energy constraint in the model

$$P_\varphi^2 - E_\varphi^2 = 0; \qquad\qquad E_\varphi = \frac{|P_Q|}{\varphi} \tag{62}$$

repeat completely the cosmological equations of the GR in the case of a rigid equation of state $\Omega_{\rm rigid} = 1$

$$\varphi_0^2 a'^2 = \frac{P_Q^2}{4V_0^2\varphi^2} \equiv \frac{\rho_0}{a^2} = H_0^2\frac{\Omega_{\rm rigid}}{a^2}, \tag{63}$$

where P_Q is the constant of the motion, because

$$P_Q' = 0. \tag{64}$$

The solution of these equations take the form

$$\varphi(\eta) = \varphi_I\sqrt{1+2\mathcal{H}_I\eta}, \qquad Q(\eta) = Q_I + \log\sqrt{1+2\mathcal{H}_I\eta}, \tag{65}$$

where

$$\varphi_I = \varphi(\eta=0), \tag{66}$$

$$Q_I = Q(\eta=0), \qquad P_Q = \text{const} \tag{67}$$

are the ordinary initial data. These data do not depend on the current values of variables $\varphi_0 = \varphi_0 a(\eta=\eta_0)$ in contrast to the Planck epoch one, where the initial data of the scale variable $a_I = a(\eta=0) = a'(\eta_0)/M_{\rm Planck}$ are determined by its velocity at present-day epoch. We have seen above that this determination violates the causality principle in the constraint-shell action (51).

7. Conclusion

We convince that the conformal symmetry is the way for classification of scalar field dynamics in GR. Conformal transformation allows us to convert the conformal coupling scalar field into a conformal relativistic brane without any dimensional parameter. Spontaneous conformal symmetry breaking in this case can be provided by initial data.

Consideration of the diffeo-invariant initial data in a specific frame differs our approach to scalar field in this paper from other approaches to this problem. A definition of initial data as diffeo-invariant measurable quantities supposes two distinguished reference frames - the observer rest frame and the observable comoving frame. In particular, comoving frame of the Universe is identified with the CMB frame that differs from the rest frame by the non zero dipole component of the temperature fluctuations. *Differences* between these two frames lie in essence of all principles of relativity including the Galilei's relativity as a *difference* of initial positions and velocities, the Einstein's relativity as a *difference* of proper times, and the Weyl's relativity of a *difference* of units. A definition of reference frame, in our paper, is based on the Fock simplex (in order to separate diffeomorphism from frame transformation), the Dirac–ADM parametrization of metric (in order to classify of the metric components), and the Zel'manov "kinemetric" diffeomorphisms as parametrizations of the internal coordinate (in order to identify the diffeo-invariant evolution parameter with the cosmological scale factor as the zero mode of metric determinant and to define the energy as the constraint-shell value of the scale momentum). Finally, the Hamiltonian action in GR coincides with the relativistic brane one, where time-like external coordinate plays the role of the diffeo-invariant evolution parameter in the field "superspace of events", and its momentum plays the role of the energy in accord with special relativity given in the Minkowskian space of events. Therefore, the generalization of the Dirac Hamiltonian approach to the conformal coupling scalar field gives us the possibility to restore the universal Hamiltonian description of relativistic brane-like systems with the action $S^{D/N}$ with any number of external and internal coordinates. Thus, we show that the Dirac Hamiltonian approach to the conformal coupling scalar field in GR coincides with the similar consideration of the conformal brane $S^{D=4/N=2}$. Both these theories (the conformal coupling scalar field and the brane) lead to the rigid state in agreement with the SN data on the luminosity-distance – redshift relation [23, 24, 25] in framework of the conformal cosmology [19, 20, 21], where the Weyl relativity of units (13) is supposed.

Acknowledgements

All authors are grateful to Dmitry Kazakov and Igor Tkachev for discussions of statement of the problem of the rigid state in the modern cosmology. The authors are grateful to B.M. Barbashov, K.A. Bronnikov, V.V. Kassandrov, E.A. Kuraev, D.G. Pavlov, Yu.P. Rybakov and A.F. Zakharov for interesting and critical discussions. Ł.A. Glinka is thankful to the Bogoliubov-Infeld program of grants for partial financial support. R.P. Kostecki is thankful to Ł.A.G. and V.N.P. for hospitability.

The Appendix. The Dirac–ADM approach to GR

The Hilbert action $S = S_{\rm GR} + S_Q$ in terms of the Dirac – ADM variables (27) and (28) is as follows

$$S_{\rm GR} = -\int d^4x\sqrt{-g}\frac{\varphi_0^2}{6}\,{}^{(4)}R(g) = \int d^4x(\mathcal{K}[\varphi_0|g] - \mathcal{P}[\varphi_0|g] + \mathcal{S}[\varphi_0|g])$$
$$S_{\rm Q} = \varphi_0^2\int dx^0d^3x\left[\frac{(\partial_0Q-N^k\partial_kQ)^2}{N_d} - N_d\psi^8(\partial_{(b)}Q)^2\right], \qquad (68)$$

where

$$\mathcal{K}[\varphi_0|e] = N_d\varphi_0^2\left(-4v^2 + \frac{v_{(ab)}^2}{6}\right), \qquad (69)$$

$$\mathcal{P}[\varphi_0|e] = \frac{N_d\varphi_0^2\psi^7}{6}\left({}^{(3)}R(\mathbf{e})\psi + 8\triangle\psi\right), \tag{70}$$

$$\mathcal{S}[\varphi_0|e] = 2\varphi_0^2\left[\partial_0 v_\psi - \partial_l(N^l v_\psi)\right] - \frac{\varphi_0^2}{3}\partial_j[\psi^2\partial^j(\psi^6 N_d)]; \tag{71}$$

are the kinetic and potential terms, respectively,

$$v = \frac{1}{N_d}\left[(\partial_0 - N^l\partial_l)\log\psi - \frac{1}{6}\partial_l N^l\right], \tag{72}$$

$$v_{(ab)} = \frac{1}{2}\left(\mathbf{e}_{(a)i}v^i_{(b)} + \mathbf{e}_{(b)i}v^i_{(a)}\right), \tag{73}$$

$$v_{(a)i} = \frac{1}{N_d}\left[(\partial_0 - N^l\partial_l)\mathbf{e}_{(a)i} + \frac{1}{3}\mathbf{e}_{(a)i}\partial_l N^l - \mathbf{e}_{(a)l}\partial_i N^l\right], \tag{74}$$

$$v_Q = \frac{\partial_0 Q - N^k\partial_k Q}{N_d} \tag{75}$$

are velocities of the metric components, $\triangle\psi = \partial_i(\mathbf{e}^i_{(a)}\mathbf{e}^j_{(a)}\partial_j\psi)$ is the covariant Beltrami–Laplace operator, ${}^{(3)}R(\mathbf{e})$ is a three-dimensional curvature expressed in terms of triads $\mathbf{e}_{(a)i}$:

$${}^{(3)}R(\mathbf{e}) = -2\partial_i\left[\mathbf{e}^i_{(b)}\sigma_{(c)|(b)(c)}\right] - \sigma_{(c)|(b)(c)}\sigma_{(a)|(b)(a)} + \sigma_{(c)|(d)(f)}\sigma_{(f)|(d)(c)}. \tag{76}$$

Here

$$\sigma_{(a)|(b)(c)} = \mathbf{e}^j_{(c)}\nabla_i\mathbf{e}_{(a)k}\mathbf{e}^k_{(b)} = \frac{1}{2}\mathbf{e}_{(a)j}\left[\partial_{(b)}\mathbf{e}^j_{(c)} - \partial_{(c)}\mathbf{e}^j_{(b)}\right] \tag{77}$$

are the coefficients of the spin-connection (see [11]),

$$\nabla_i\mathbf{e}_{(a)j} = \partial_i\mathbf{e}_{(a)j} - \Gamma^k_{ij}\mathbf{e}_{(a)k} \tag{78}$$

are covariant derivatives, and $\Gamma^k_{ij} = \frac{1}{2}\mathbf{e}^k_{(b)}(\partial_i\mathbf{e}_{(b)j} + \partial_j\mathbf{e}_{(b)i})$. The canonical conjugated momenta are

$$p_\psi = \frac{\partial\mathcal{K}[\varphi_0|e]}{\partial(\partial_0\ln\psi)} = -8\varphi_0^2 v, \tag{79}$$

$$p^i_{(b)} = \frac{\partial\mathcal{K}[\varphi_0|e]}{\partial(\partial_0\mathbf{e}_{(a)i})} = \frac{\varphi^2}{3}\mathbf{e}^i_{(a)}v_{(ab)}, \tag{80}$$

$$P_Q = 2\varphi_0^2\frac{\partial_0 Q - N^k\partial_k Q}{N_d}. \tag{81}$$

The Hamiltonian action takes the form [10, 11]

$$S = \int d^4x\left[\sum_{F=e,\log\psi,Q} P_F\partial_0 F - \mathcal{H}\right] \tag{82}$$

where

$$\mathcal{H} = N_d T_{\rm d} + N_{(b)}T^0_{(b)} + \lambda_0 p_\psi + \lambda_{(a)}\partial_k\mathbf{e}^k_{(a)} \tag{83}$$

is the sum of constraints with the Lagrangian multipliers N_d, $N_{(b)} = \mathbf{e}_{k(b)}N^k$, λ_0 , $\lambda_{(a)}$, and , $T^0_{\rm (a)} = -p_\psi\partial_{(a)}\psi + \frac{1}{6}\partial_{(a)}(p_\psi\psi) + 2p_{(b)(c)}\gamma_{(b)|(a)(c)} - \partial_{(b)}p_{(b)(a)} + P_Q\partial_{(a)}Q$ are the components of the total energy-momentum tensor $T^0_{(a)} = -\frac{\delta S}{\delta N_k}\mathbf{e}_{k(a)}$, and

$$T_{\rm d}[\varphi_0|\psi] = -\frac{\delta S}{\delta N_{\rm d}} = \frac{4\varphi_0^2}{3}\psi^7\triangle\psi + \sum_I \psi^I \mathcal{T}_I, \tag{84}$$

$$T_\psi[\varphi_0|\psi] = -\psi\frac{\delta S}{\delta\psi} \equiv \frac{4\varphi_0^2}{3}\left[7N_d\psi^7\triangle\psi + \psi\triangle[N_d\psi^7]\right] + N_d\sum_I I\psi^I\mathcal{T}_I = 0; \tag{85}$$

here $\mathcal{T_I}$ is partial energy density marked by the index I running, in general case, a set of values I=0 (stiff), 4 (radiation), 6 (mass), 8 (curvature) in correspondence with a type of matter field contributions

$$\psi^7\triangle\psi \equiv \psi^7\partial_{(b)}\partial_{(b)}\psi \tag{86}$$

$$\mathcal{T}_{I=0} = \frac{6p_{(ab)}p_{(ab)}}{\varphi_0^2} - \frac{16}{\varphi_0^2}{p_\psi}^2 + \frac{P_Q^2}{4\varphi_0^2} \tag{87}$$

$$\mathcal{T}_{I=8} = \varphi_0^2\left[\frac{1}{6}R^{(3)}(\mathbf{e}) + \partial_{(b)}Q\partial_{(b)}Q\right], \tag{88}$$

here $p_{(ab)} = \frac{1}{2}(\mathbf{e}^i_{(a)}\widetilde{p}_{(b)i} + \mathbf{e}^i_{(b)}p_{(a)i})$, we include the Dirac local condition of the minimal 3-dimensional hyper-surface [5] too

$$p_{\widetilde{\psi}} = 0 \to (\partial_0 - N^l\partial_l)\log\widetilde{\psi} = \frac{1}{6}\partial_l N^l, \tag{89}$$

in order to obtain a positive value of the Hamiltonian density (87) after the separation of the cosmological scale factor (38).

The constraint-shell action (82) after the separation of the zero modes (38) and (39) takes the form

$$\begin{aligned} S|_{\mathcal{H}=0} &= \int dx^0 \int d^3x \sum_{F=\psi,e,Q} P_F\partial_0 F|_{\varphi_0 a=\varphi} = \\ &= \int dx^0\left\{\int\limits_{V_0} d^3x \sum_{\widetilde{F}=\widetilde{\psi},e,\overline{Q}} P_{\widetilde{F}}\partial_0\widetilde{F} - P_\varphi\frac{d\varphi}{dx^0} + P_{\langle Q\rangle}\frac{d\langle Q\rangle}{dx^0}\right\} = \\ &= \int\limits_{\varphi_I}^{\varphi_0} d\varphi\left\{\int\limits_{V_0} d^3x \sum_{\widetilde{F}=\widetilde{\psi},e,\overline{Q}} P_{\widetilde{F}}\partial_\varphi\widetilde{F} + P_{\langle Q\rangle}\frac{d\langle Q\rangle}{d\varphi} - P_\varphi\right\}. \end{aligned} \tag{90}$$

where $P_\varphi = \pm E_\varphi$ is the constraint-shell Hamiltonian in the "superspace of events" given by the resolving the energy constraint (50), where $\widetilde{T}_{\rm d} = T_{\rm d}[\varphi|\widetilde{\psi}]$, and $T_{\rm d}[\varphi|\widetilde{\psi}]$ is given by Eqs. (84), (87) and (88) where $[\varphi_0|\psi]$ is replaced by $[\varphi|\widetilde{\psi}]$.

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