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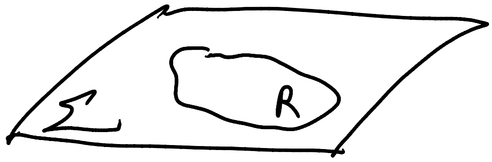
0. MOTIVATION WHY $\widehat{\sqrt{\det q}}$



$$V_R = \int_R d^3x \sqrt{\det q}(x)$$

crucial for LQG

- volume frequently occurs in G
- matter coupling to LQG
- de-parametrized models

LONG STORY

Here: only LQG - Volume
R/S '95-97 A/L '97
T '96

$\exists$ alternative approaches
- SFM
- KOSLOWSKI '07
- BIANCHI '11

(KOSLOWSKI '07, BIANCHI '11: } Volume by surface areas.)

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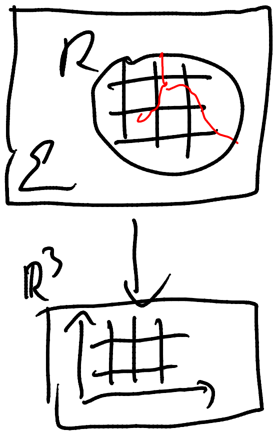
1. CONSTRUCTION / REGULARIZATION

$\sqrt{\det q}$? $\quad E^a_i = \sqrt{\det q}\, e^a_i \Longleftrightarrow \sqrt{|\det E|} = \sqrt{\det q}$

$$V_R = \int_R d^3x \sqrt{\frac{1}{3!}\left|\varepsilon_{abc}\,\varepsilon^{ijk} E^a_i(x) E^b_j(x) E^c_k(x)\right|}$$

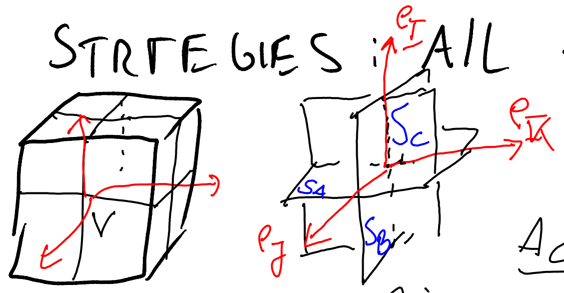
What is $\hat{V}_R$? $\rightarrow$ REG-PROCEDURE

- introduce background R
- int. cubic cell decomp
  $\rightarrow$ boxes with length L



- $\int_R d^3x \sqrt{|\det E|} = \sum_{\square} \int_{\square} d^3x \sqrt{|\det E|}$

$\exists$ STRATEGIES: AL — RS



$$E^i_S = \int_S d^2z\, n^a_S\, E^i_a(z)$$

ACTION ON f_I

$$\hat{E}^i_S f_I = \epsilon(\dot{e}_I, S)\, R^i_I\, f_I$$

$$\det q(v) \propto |\epsilon^{ABC} \epsilon_{ijk} E^i_{S_A} E^j_{S_B} E^k_{S_C}|$$

Evaluate on triple $e_I\ e_J\ e_K$

$$\hat{q}_{IJK}(v)\, T_J = \left(\epsilon_{ijk}\, \epsilon^{ABC}\, \epsilon(\dot{e}_I, S_A)\, \epsilon(\dot{e}_J, S_B)\, \epsilon(\dot{e}_K, S_C)\, R^i_I R^j_J R^k_K\right) T_J$$

$$= \left(\underbrace{\epsilon(IJK)}_{\equiv \mathrm{sgn}(\det(\dot{e}_I\, \dot{e}_J\, \dot{e}_K))}\ \underbrace{\epsilon_{ijk} R^i_I R^j_J R^k_K}_{\hat{q}_{IJK}}\right) T_J$$

Finally (Leibniz rule) at 1 vertex:

$$\hat{q}_v = Z \sum_{e_I \cap e_J \cap e_K \subset v} \epsilon(IJK)\, \hat{q}_{IJK}$$

$$\hat{V}_R T_\gamma = \left[\sum_{\substack{v \in V(\gamma) \\ v \subset R}} \sqrt{|\hat{q}_v|}\right] T_\gamma$$

ACTION DECAYS INTO SINGLE VERTEX ACTION

$$Z = i\, C_{REG}\, \ell_p^6\, \beta^3 \qquad C_{reg} = \frac{1}{48}$$ (GIESEL THIEMANN '05)

Here $Z = 1$

- involves averaging
- Limit $L \to 0$ can be taken

RIS - VOLUME

$$\sqrt{|\hat{q}_v|}\, T_\gamma = \sum_{e_I \cap e_J \cap e_K = v} \sqrt{|\hat{q}_{IJK}|}$$

AL:

$$\sqrt{\left|\sum_{IJK} \epsilon(IJK)\, \hat{q}_{IJK}\right|}$$

GESEL / THIEMANN '05

IDEA: $E^a_i = \sqrt{\det q}\; e^a_i = \det e^i_b \; e^a_i$ (inverse)

$$= \frac{1}{2} \epsilon_{ijk}\, \epsilon^{abc}\, e^j_b\, e^k_c\, \mathrm{sgn}(E)$$

$$\to \quad e^i_b \propto \{A^i_a, V_R\}$$

Assume in QT, that $\hat{V}_e$ is fund. not the flux.

Can I get the flux of LQG back?

$$E^i_S f_e = \epsilon(e,S) R^i_e f_e$$

✓ for A/L

X for R/S

$\longrightarrow$ use AL-version

2. Evaluation of Matrix Elements

$$\hat{q}_{IJK} = \epsilon_{ijk} R^i_I R^j_J R^k_K$$

$$\sqrt{2j+1}\,[\pi_j(h)]_{mn} \Leftrightarrow \langle h | j\, m; n \rangle$$

$$R^i_I \Leftrightarrow J^i_I$$

Rewrite SNF as abstract spin system

$$\epsilon_{ijk} J^i_I J^j_J J^k_K = \frac{i}{4}\left[(J_{IJ})^2, (J_{JK})^2\right]$$

J_{IJ} is casimir of

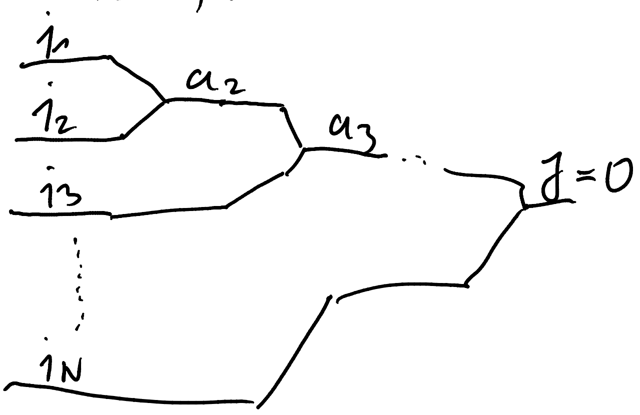
$$\underline{\pi_{j_I} \otimes \pi_{j_J}} = \bigoplus_{a=|j_I-j_J|}^{j_I+j_J} \pi_a$$

N-valent vertex: $_N$

$$T_{v\vec{j}\vec{m}\vec{n}} \doteq \bigotimes_{K=1} |j_K m_K; n_K\rangle$$

RECOUPLING SCHEMES AS BASIS

$\mathcal{H}_{kin,v}$



$$|\vec{a}\, J M; \vec{j}, \vec{n}\rangle$$

$$M = \sum_K m_K$$

GAUGE INVARIANCE AT V

|||

$$J=0 \qquad J_N = -\sum_{I=1}^{N-1} J_I$$

$$\hat{V}_R T_\gamma = \sum_v \sqrt{|\hat{q}_v|}$$

$$\hat{q}_v = \sum_{I,J,K<N} \sigma(IJK)\, \hat{q}_{IJK}$$

$$\sigma(IJK) = \epsilon(IJK) - \epsilon(JKN) + \epsilon(IKN) - \epsilon(IJN)$$

$$\sigma(IJK) = -4, \dots, 4$$

3. ANALYSIS OF RESULTING $\hat{V}_R$

4-vertex: tridigonal Matrix

use

$\sigma(123)$ only scaling of Spec $\hat{V}$

techniques from
Jacobi - matrices

ANALYTICALLY:

- $\ker(\hat{V})$
- $\mathrm{spec}(\hat{V})$ is non degenerate
- $\exists\ \lambda_{min} > \ell_p^3 \sqrt{|Z|\ \sigma(123)\ j_{max}}$
- upper bound on $\lambda_{max} \propto (j_{max})^{3/2}$

NUMERICALLY.

Now: GENERAL PICTURE N-valent v

Now $\sigma(IJK)$

<u>NEED:</u> permutation equiv. classes of non-diffeomorphic embeddings of N-valent vertex.

ORIENTED MATROID FRAMEWORK ALLOWS TO HANDLE SIGNS OF ARBRITRARY VALENT VERTICES.

SIGN-DATA FROM OM-COMPUTATION

$$\hat{q}_v \propto \sum_{IJK \le N} \sigma(IJK)\ \hat{q}_{IJK}$$

$\Downarrow$ (Numerical analysis)

$\Rightarrow$ Numerical analysis.

SUMMARY & OUTLOOK

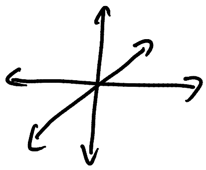
- $\hat{V}_R$ technically involved.

  <u>but</u>: it can now be analyzed numerically

  $\rightarrow$ Spectral properties (VOLUME GAP) <u>depend</u> on the <u>local</u> vertex geometry

<u>TO DO</u>: • Fully examin the OM-framework $\rightarrow$ can compute <u>all</u> spectra (also the one with degenerated triples .

$\Rightarrow$ Compare semiclass. analysis. (Thiemann, Flori '08)

 Cubic singled out or not?

$\Rightarrow$ do computations with $\hat{C}_E$ $\rightarrow$ better understand graph changing op's

- have seen that OM cover both:
  vertices and di-graphs
  → does this have some deeper meaning
