12 marca 2011
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EPRL map for Spin4 and SL(2, ℂ) and integrability of Lorentzian spin foams.

Spin foam for Spin4

$V_{j^+j^-}$ for faces
ι - intertwiners on edges

Vertex amplitude is the evaluation of this spin-network ~ contraction of the intertwiners

$\in \mathrm{Inv}\, V_{j_f^+ j_f^-} \otimes \ldots$ (faces that meet in the edge)

Choice of intertwiners:

$$Y_\gamma : V_k \longrightarrow V_{j^+j^-} \qquad j^\pm = \frac{1 \pm \gamma}{2} k$$

Immirzi parameter

$$\mathrm{Res}_{SU(2)} V_{j^+ j^-} = \bigoplus_{j=|j^+-j^-|}^{j^+ + j^-} V_j$$

$$P \circ V^{\otimes}_{\gamma} : \mathrm{Inv}_{SU(2)} \bigotimes V_{e_f} \to \mathrm{Inv}_{Spin4} \bigotimes V_{j^\pm}$$

projection onto Invariants spin4.

Problems for $SL(2,\mathbb{C})$

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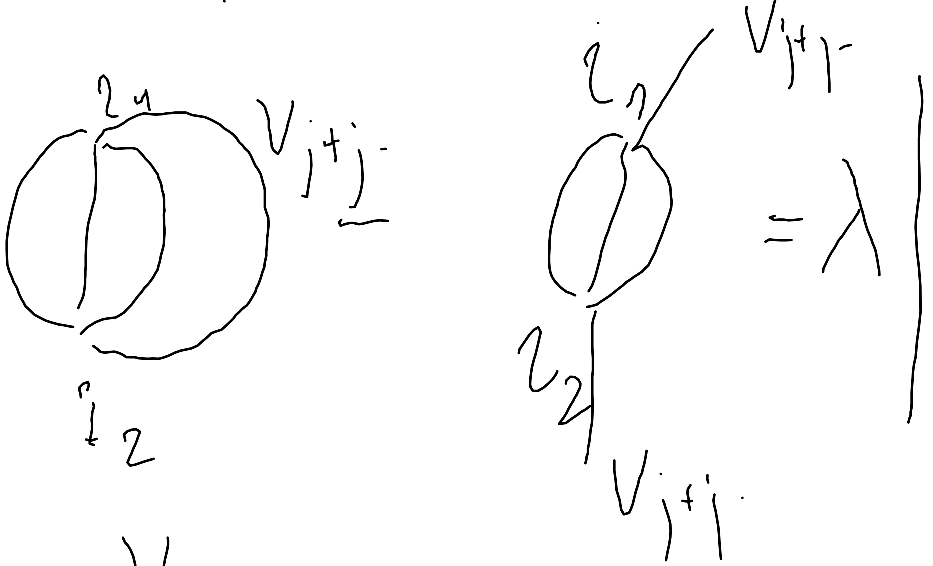


$\lambda \dim V_{j^+ j^-}$

For $SL(2,\mathbb{C})$ contraction of intertwiners ill-defined

$-\ \mathrm{Inv}_{SL(2,\mathbb{C})} \bigotimes V_{n,\rho} \not\subset \bigotimes V_{n,\rho}$

Representations of $SL(2,\mathbb{C})$

$f(z_1, z_2)$ – $\qquad z_1, z_2 \in \mathbb{C}$

$$f(\lambda z_1, \lambda z_2) = \lambda^k \bar{\lambda}^{-k} |\lambda|^{2i\rho-2} f(z_1, z_2)$$

$V_{k,\rho}$

On the sphere $|z_1|^2 + |z_2|^2 = 1$

$$f(e^{i\varphi} z_1, e^{i\varphi} z_2) =$$
$$= e^{2ik\varphi} f(z_1, z_2)$$

Basis $\bigoplus_{j-k \in \mathbb{Z}} V_j$ as $SU(2)$ representation

Group averaging

Way of defining $\mathrm{Inv}_{SL(2,\mathbb{C})}$

$\bigotimes V_{k_i,\rho_i} \supset D \qquad [\psi] \in D^*$

$[\varphi] \quad : [\varphi]\varphi' =$

$\varphi \in \mathcal{D} \qquad = \int dg \langle \varphi | U(g) \varphi' \rangle$

it gives us a natural candidate for scalar product on the space of invariants

Example $L^2(\mathbb{R}) \quad e^{ikx}$

$\mathcal{D} = C_0^\infty(\mathbb{R})$.

$$\int dk \langle \varphi \, e^{ikx} \varphi' \rangle = \bar{\varphi}(0) \varphi'(0)$$

if we take $\mathcal{D} = C_0^\infty(\mathbb{R})$

$\varphi(0) = 0$

In our case

$$\mathcal{D} = \sum_{\{j_i\}} V_{j_1} \otimes \ldots \otimes V_{j_n}$$

$V \qquad = \oplus V$

$v_{n_i, g_i} \quad \underbrace{\quad}_{ji} \quad v_{ji}$

$\int dg \, \langle \psi | U(g) \psi' \rangle$

is absolutely convergent provided that $n \geq 3$

- Contraction:

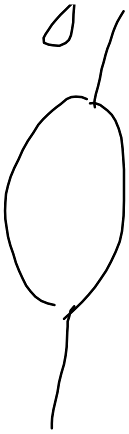
$[\varphi_i]$ for every node of the graph.

$$\int \prod dg_i \; \varphi_1 \cdots \varphi_n \, D(g_1^{-1} g_n) \cdots$$

Gauge symmetry

$g_i \to h g_i$

Gauge fixing $g_0 = e$



$\simeq \lambda \; \Big| \; \lambda$

Spin 4

$$S(g) = \sum_{j^+ j^-} (2j^+ + 1)(2j^- + 1)\, \Theta_{j^+ j^-}(g)$$

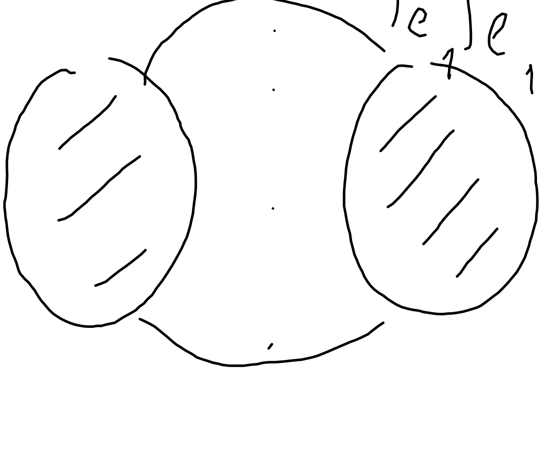
$SL(2,\mathbb{C})$ Harish-Chandra

$$S(g) = \frac{1}{2}\sum_{k} \int d\rho \, (\rho^2 + k^2)\, \Theta_{k,\rho}(g)$$

$$\Theta_{-k,-\rho}(g) = \Theta_{k,\rho}(g)$$

$$\text{"}\dim\text{"}\, V_{k,\rho} = (k^2 + \rho^2)$$

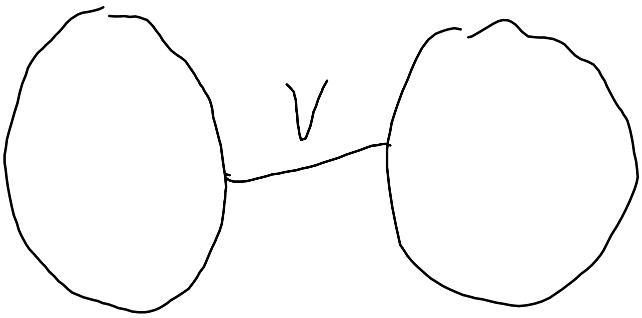
First prescription $= \lambda \dim V_{j_e^+ j_e^-}$



Spin4.

$$V_{\,)e_2}^{+} \quad {}_{)e_2}^{-}$$

$\rightsquigarrow SL(2,\mathbb{C})$ 2-edge reducible graph should not be absolutely conv.



What with 3-edge connected graphs

- Every such integral (evaluation) is absolutely convergent.

Proof:

$$\int \prod_{i \in E_0 \setminus \{0\}} dg_i \prod_i \varphi_i^{\mu_{e_1} \cdots \mu_{e_n}}$$

$$\prod_{e \in E^1_{i(\Gamma_e)}} D(g_i^{-1} g_j)$$

$$\langle \xi_{ie},\ D(g_i^{-1} g_j) \xi_{je} \rangle$$

$$\varphi = \bigotimes_{e \text{ coming to } i} \xi_{ie} \qquad \xi_{ie} \in V_{j_{ie}}$$

$$\int \prod_i dg_i \prod_e \langle \quad \rangle$$

$$|K \quad \rangle| \leq C(j_{ie}, j_{je}) \frac{1}{(\cosh r)^{1-\tau}}.$$

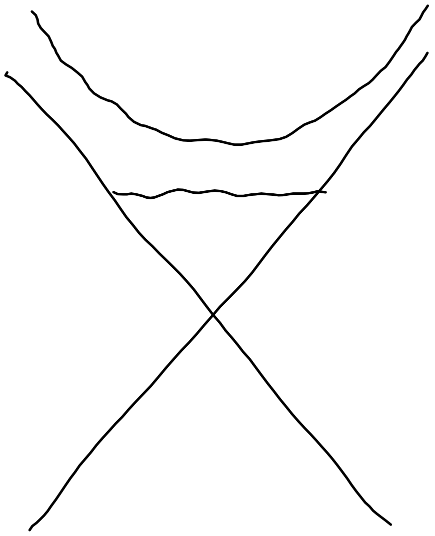
$$\cosh r = \operatorname{tr}(g_i^{-1} g_j)^\dagger (g_i^{-1} g_j)$$

$$dg = dp\,du$$

pu of angles

$$\leq \int \prod dp_i \prod_e \left(\frac{1}{\cosh r_{ij}}\right)^{1-\tau}$$

measure on the unit hyperboloid



$(\eta_0, \bar\eta)$ $\eta_0^2 - \bar\eta^2 = 1$

$(\epsilon, \bar\xi)$ 3 dim vector with norm 1

$$\int \prod_i \frac{d\epsilon_i}{\epsilon_i^3} \prod_i d\xi_i$$

$$\prod \left(\frac{\epsilon_i \epsilon_j}{\theta_{ij}^2 + \epsilon_i^2 + \epsilon_j^2}\right)^{1-\tau}$$

Θ_{ij} - distance on 3-dim sphere between $\vec{\xi}_i$ and $\vec{\xi}_j$.

Subdivide region of integration into sub-region of different scales,

$$\Theta_{ij}\ ,\ \epsilon_i$$

$$\mathrm{I}\quad \epsilon_1 \leq \Theta_{12} \leq \dots$$

$$\int_0^{\Theta_{12}} d\epsilon_1 \int \dots \prod \left(\quad \right)^{1-\tau}$$

this allows us to estimate

Example

$$\int^{\Theta_{12}} d\epsilon_1 \left(\frac{\epsilon_1 \epsilon_2}{\Theta_{12}^2 + \epsilon_1^2 + \epsilon_2^2} \right)^{1-\tau} \simeq$$

$$\simeq \Theta_{12}\left(\frac{\Theta_{12}\epsilon_2}{\Theta_{12}^2+\Theta_{12}^2+\epsilon_2^2}\right)^{1-\tau}$$

true if some powercounting gives positive result

$$\simeq \int d\epsilon_1\, \epsilon_1^{1-\tau}\left(\quad\right)^{1-\tau}$$

After each step power of the smallest variable need to be positive.

In each step we replace all variables $\leq$ that the given one by x. Then we put $x =$ variable ω_n

$$\left(\frac{\epsilon_1 \epsilon_2}{\theta_{12}^2+\epsilon_1^2+\epsilon_2^2}\right)^{1-\tau}$$

$$\longrightarrow \underline{1}$$

if we integrate over ϵ_1, ϵ_2 and θ_{12}

$$d\bar{\xi}_i \, d\bar{\xi}_j \simeq \theta_{12} \, d\theta_{12} \, d\phi \, d\hat{\xi}_{ij}$$

After every integration we have additional χ coming $d\epsilon_i$

χ^2 coming from $\theta_{ij} \, d\theta_{ij}$

Power counting $\geq$ external legs of the following graph:

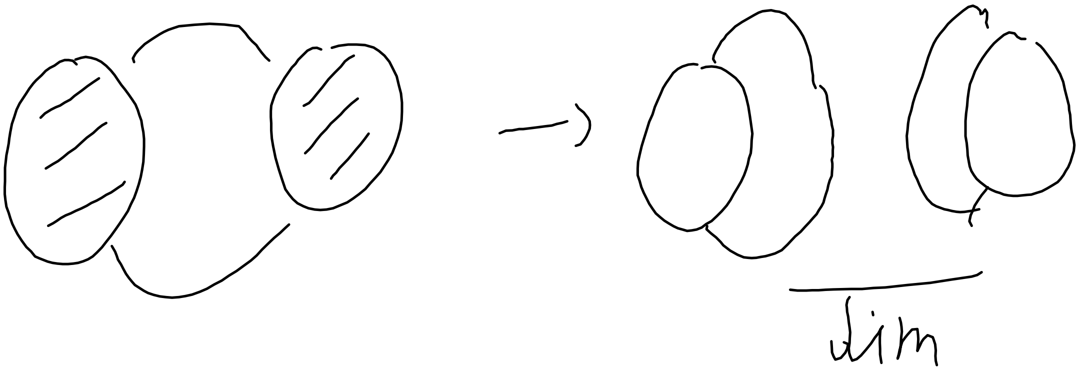
In our spin-network we take all vertices that correspond to ϵ_i integrated out; edges we take all edges with θ_{ij} integrated out ϵ_i, ϵ_j int.

$$\text{power} \geq \underbrace{\widetilde{\sum}}_{\text{conned.}} -2 + E_n (1 - \gamma)$$

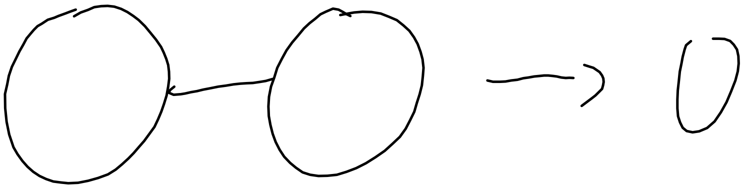
number of external legs

technicalities

2 - edge reducible.



There are ways to obtain reasonable evaluation for every non-edge-reducible graph

Ex 

Remark: There is dependence on the choice of Haar measure