Lecture 1
1 marca 2011
07:10

QUANTUM GRAVITY

WHAT DO WE KNOW?

- QM
- S.M.
- GR

I. STATES OPERATORS -

MATH. $SU(2) \ni h$ $\quad J_i = -\frac{i}{2}\sigma_i$ $\quad i=1,2,3$

- Haar measure
- Represent. th. $\quad j$ spin $\quad j = 0, \frac{1}{2}, 1 \ldots \quad d_j = 2j+1$

$$\mathcal{H}_j \ni v^m \qquad D^j(h)^m{}_n$$

$$\int dh\, \overline{D^j(h)^m{}_n}\, D^{j'}(h)^{m'}{}_{n'} = \delta^{jj'}\delta^{mm'}\delta_{nn'}\frac{1}{2j+1}$$

$$L_2[SU(2)] \ni \psi(h) \qquad \langle h|jmn\rangle$$

$$\langle jmn|j'm'n'\rangle = \delta\delta\delta\frac{1}{d_j}$$

$$L_2[SU(2)] = \oplus_j \left(\mathcal{H}_j^* \otimes \mathcal{H}_j\right)$$

$$\vec{L} = \{L_i\} \qquad L_i = \frac{d}{dt}\psi\left(h e^{tJ_i}\right)\Big|_{t=0} \qquad R_i = \frac{d}{dt}\psi\left(e^{tJ_i}h\right)\Big|_{t=0}$$

$$L^2 = L_i L_i \qquad L^2 D^j = j(j+1) D^j$$

$$L_i D^j{}^m{}_n(h) = D^j{}^m{}_{n'}(h)\, \overset{(j)}{J}_i{}^{n'}{}_n$$

$$\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3} \cdots \otimes \mathcal{H}_{j_n} \ni v^{m_1 \ldots m_n}$$

$$\mathcal{H}_{j_1}\otimes\mathcal{H}_{j_2}\otimes\mathcal{H}_{j_3}\cdots\otimes\mathcal{H}_{j_n} \ni \underline{\underline{V^{m_1\cdots m_n}}}$$

$$D^{j_1}(h)^{m_1}{}_{\tilde m_1}\cdots D^{j_n}(h)^{m_n}{}_{\tilde m_n}\, V^{\tilde m_1\cdots \tilde m_n} = V^{m_1\cdots m_n}$$

$$\mathcal{H}_1\otimes\mathcal{H}_1\otimes\mathcal{H}_1 : v^{ijk} \qquad \epsilon^{ijk}$$

$$\underbrace{\mathcal{H}_1\cdots\mathcal{H}_1}_{4T.} \ni v^{ijkl} \qquad \underline{\delta^{ij}\delta^{kl}} \qquad \underline{\delta^{il}\delta^{jk}} \qquad \underline{\delta^{ik}\delta^{jl}}$$

$$K_{j_1\cdots j_n} = \text{Inv}\left[\mathcal{H}_{j_1}\otimes\cdots\otimes\mathcal{H}_{j_n}\right]$$ intertwiner space

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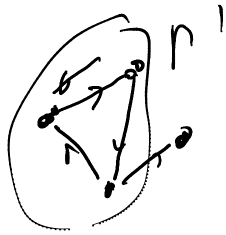
GRAPHS $\quad \Gamma = (\mathcal{L}, \mathcal{N}, \partial)$ $\quad$ L "links" l

N "nodes" n

$\partial : l \to (s(l), t(l))$



Γ'

- l^{-1} $\quad s(l) = t(l^{-1})$

  $t(l) = s(l^{-1})$

- AUTOMORPH.
- SUBGRAPH $\quad \Gamma' < \Gamma$

$$f = \lim_{\Gamma\to\infty} f_\Gamma$$

STATES $\qquad$ **TRUNCATION of d.o.f.**

QED $\quad \mathcal{H}_1 \quad L_2[M] \quad \mathcal{H}_n = L_2[M^n]/n$

$$\mathcal{H}_N = \bigoplus_n^N \mathcal{H}_n \qquad \mathcal{F} = \lim_{N\to\infty} \mathcal{H}_N$$

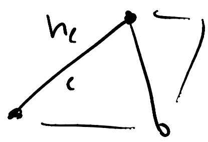
QCD

$$\mathcal{H}_{\text{latt.}} = L_2[G^L] \qquad L : \#\text{ of links}$$

$$\psi(h_e) = \psi(\Lambda_{s(e)}\, h_e\, \Lambda^{-1}_{t(e)})$$

$$\mathcal{H}_{\text{latt}} = L_2[G^L/G^N]$$

STATES of LOOP GRAVITY



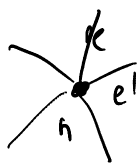
(i) $\mathcal{H}_\Gamma = L_2[SU(2)^L/SU(2)^N] \ni \psi(h_e)$

(ii) $\tilde{\mathcal{H}}_\Gamma = \mathcal{H}_\Gamma/\sim$

(iii) $\mathcal{H} = \lim_{\Gamma\to\infty} \tilde{\mathcal{H}}_\Gamma$

Separable.

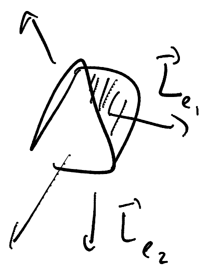
OPERATORS $\psi(h_e)$



$\vec{L}_e \qquad G_{ee'} = \vec{L}_e \cdot \vec{L}_{e'} = L^i_e L^i_{e'}$

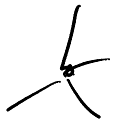
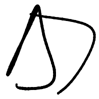
metric operator

$$A_e^2 = G_{ee}$$



$$C_n = \sum_{e \in n} \vec{L}_e = 0$$

$$\delta_{ij} L^i_e L^j_{e'} = G_{ee'}$$

$V_n = \frac{\sqrt{2}}{3}\sqrt{L_1\cdot(L_2\times L_3)}$  

$\mathcal{H}_{1111}$ $\quad$ $\mathcal{H}_{\frac12\frac12\frac12\frac12}$

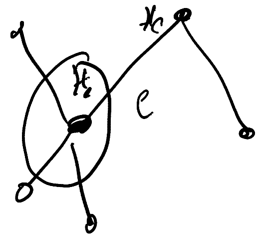
- $G_{ee'}$ do not commute
- A_e V_n → diag. then basis

$\psi_{\Gamma j_e v_n}(h_e)$ $\qquad$ j_e: quant. n. Area

$=$

$L_2[SU(2)] = \oplus_j (\mathcal{H}_j\otimes\mathcal{H}_j)$

$L_2[SU(2)^L] = \bigoplus_{\{j_e\}} \bigotimes_e (\mathcal{H}_{j_e}\otimes\mathcal{H}_{j_e})$

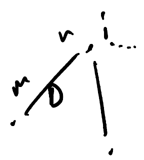


$= \bigoplus_{\{j_e\}}\bigotimes_n \mathcal{H}_n$ $\qquad$ $\mathcal{H}_n = \mathcal{H}_{j_1}\otimes\dots\otimes\mathcal{H}_{j_n}$

$K_n = \mathrm{Inv}\,\mathcal{H}_n$

Volume V_n acts on K_n

$L_2[SU(2)^L/SU(2)^N] = \bigoplus_{\{j_n\}}\bigotimes K_n =$ " $\bigoplus_{\{j_n\},\{v_n\}} |j_e v_n\rangle$ "

$|\Gamma, j_e, v_n\rangle \in \mathcal{H}$ 

$\langle h_e|\Gamma, j_e, v_n\rangle = \bigotimes_e D^j(h_e) \cdot_n \bigotimes_n v_n$

$\mathcal{H}_\Gamma$ can be interpreted as representing a set of N polyhedra with areas & volumes $j_e(j_e+1)$, v_n -

— QUANTUM GEOMETRY

— i) area & vol discrete

— ii) $G_{ee'}$ op do not commute -

— iii) a generic state is a superposition

① The minimal non vanishing area is $\sqrt{j(j+1)} = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)} = \frac{\sqrt{3}}{2}$

minimal area

$$\vec{L}_e \qquad \underline{G_{ee'}} = \cos \vec{L}\,\vec{L}$$

$$\uparrow \ \uparrow\uparrow$$

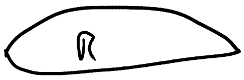
② SCALE

$$a_m = \frac{\sqrt{3}}{2} L^2_{loop} \qquad \text{free parameter -}$$

$$L_{loop} = 8\pi \overbrace{G\hbar}^{L_P^2}\gamma \qquad \gamma\ (\text{Immirzi})$$

$$\simeq 10^{-33}\,\mathrm{m}$$

③ $\mathcal{H}$ state space? $\quad \mathcal{H}_{in} \otimes \mathcal{H}_{out}$



geometry on boundary $\underline{\psi \in \mathcal{H}}$