The LQG black hole

Hanno Sahlmann

APCTP

1. Introduction

Black hole thermodynamics:

- Mass *is* energy
- Hawking temperature *is* temperature
- Area is entropy ?

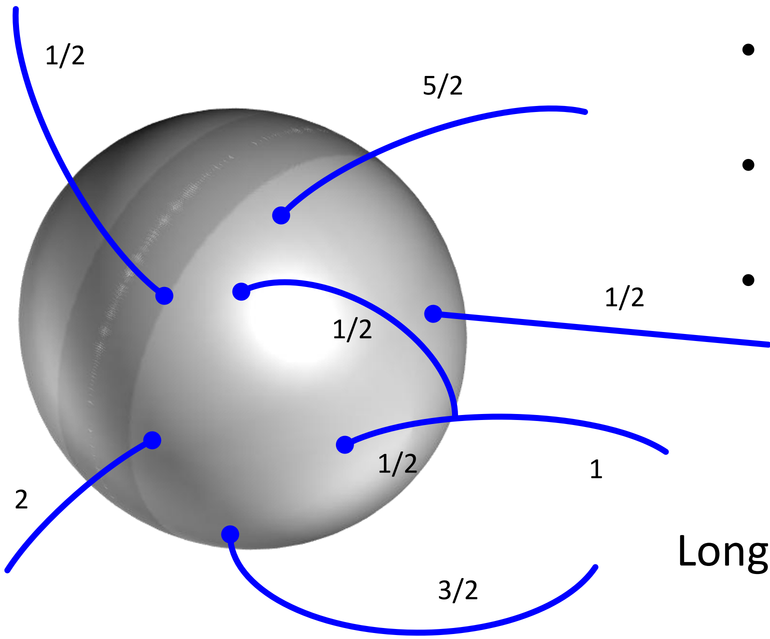
Entropy of what? What are the microstates?

In loop quantum gravity (LQG):

- Area *is* entropy of quantum isolated horizon
- microstates = states of SU(2) Chern-Simons theory with punctures
  ≈ quantum shapes of horizon

But: Treatment not entirely from first principles.

The picture in LQG:



- Polymeric excitations endow horizon with area
- Bulk theory and CS theory coupled at punctures
- number of surface & CS states for fixed area ~ exp A

Long story: Rovelli , Krasnov (‘96)
Ashtekar + Baez + Corichi + Krasnov (‘98)
Domagala + Lewandowski, Meissner (’04)
Engle + Noui + Perez (‘10)

and many more.

2. Isolated horizons

Local notion of black hole horizon? Isolated horizon (Ashtekar Beetle Fairhurst ('98), specialization of *trapping horizon* (Hayward ('94))

Something like a *local Killing horizon*:

- Null surface $H = S^2 \times \mathbb{R}$
- Foliation of H with corresponding null normal non-expanding.
- Pull-back of connection time independent on H.

⇒ Quasi-local expression for horizon mass (angular momentum, charge...)
⇒ Laws of BH mechanics

- KH is IH,
- IH inside/coincides with EH,
- IH + assumptions ⇒ ∃ EH.

For rest of talk: spherically symmetric IH ("Type I")

Ashtekar-Barbero variables (Engle, Noui, Perez ('10))

IH boundary condition: $$\underset{\Leftarrow}{F}^I(A^\gamma) = \frac{(1-\beta^2)\pi}{a_H}\underset{\Leftarrow}{\Sigma}^I$$

with

$$\Sigma^I := \epsilon^I{}_{JK}\, e^J \wedge e^K = \delta^{IJ}\epsilon_{abc}E^a_J\, dx^b \wedge dx^c$$

Presymplectic structure:

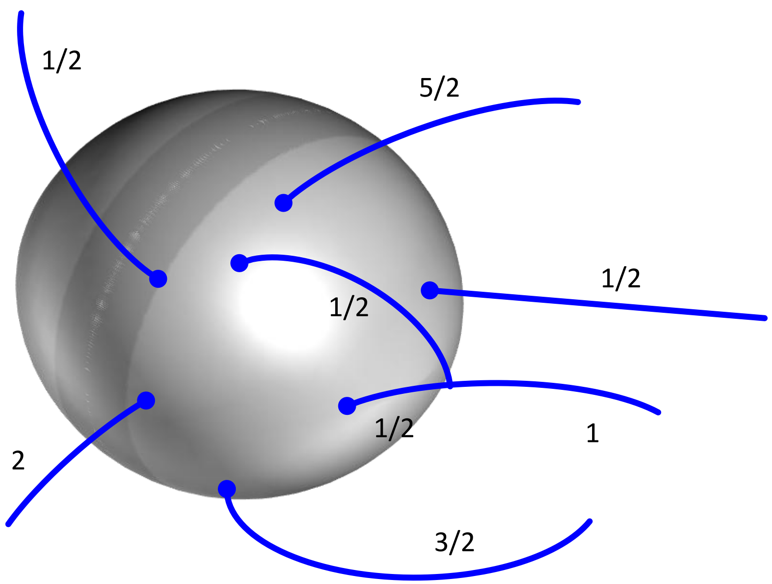
$$\kappa\gamma\Omega(\delta_1,\delta_2) = \int_\Sigma 2\delta_{[1}\Sigma^I \wedge \delta_{2]}A_I - \frac{a_H}{(1-\beta^2)\pi}\int_\Delta \delta_1 A^I \wedge \delta_2 A_I$$

Appearance of Chern-Simons term on boundary suggests separate quantization of boundary DOF. More later.

3. Quantum theory

Bulk theory as before, but now spin-net edges can end on the horizon. Fix graph. Standard LQG-results give

$$\mathcal{H} = (\otimes_i j_{p_i}) \otimes \mathcal{H}_{\text{bulk}} \qquad \ni \psi = |\{j_{p_i}, m_{p_i}\}, \ldots\rangle$$



$$[\widehat{J}^I(p), \widehat{J}^J(p)] = \epsilon^{IJ}{}_K \widehat{J}^K(p)$$

$$\underset{\Leftarrow}{\widehat{E}}^I(p) = 16\pi G\beta \sum_{p_i} \delta(p, p_i) \widehat{J}^I(p)$$

$$\widehat{a}_H = 8\pi\beta l_P^2 |\widehat{J}(p)|$$

Boundary theory must satisfy:

$$-\frac{a_H}{\pi(1-\beta^2)}\epsilon^{ab}\widehat{F}_{ab}(p) \stackrel{!}{=} 16\pi G\beta \sum_{p_i} \delta(p,p_i)\widehat{J}^i(p)$$

Again, follow ENP: Take SU(2) CS theory with particles,

$$\begin{aligned} S[A,\Lambda_i] =& \frac{k}{4\pi}\int \mathrm{tr}[A\wedge dA + \frac{2}{3}A\wedge A\wedge A] \\ &+ \sum_i \int_{c_i} \mathrm{tr}[\tau_3({\Lambda_i}^{-1}d\Lambda_i + {\Lambda_i}^{-1}A\Lambda_i)] \end{aligned}$$

Canonical analysis:

$$(\lambda_i, S_i): \qquad \{S_i^I, S_i^J\} = \epsilon^{IJ}{}_K S_i^K, \qquad \{S_i^I, \Lambda_i\} = -\tau^I\Lambda_i$$

plus three constraints per particle. Further constraint:

$$\frac{k}{4\pi}\epsilon^{ab}\widehat{F}_{ab}(p) \stackrel{!}{=} \sum_i \delta(p,p_i)\widehat{S}_i$$

This suggests:

$$k = \frac{a_H}{4\pi l_P^2 \beta(1-\beta^2)}$$

$$\widehat{S}_i = \widehat{J}(p_i)$$

$$\mathcal{H} = \mathcal{H}_{CS}(j_1, j_2 \ldots) \otimes \mathcal{H}_{\text{bulk}}$$

Note furthermore: Because Δ is simply connected

$$\mathcal{H}_{CS}(j_1, j_2, \ldots) \subset \text{Inv}(j_1, j_2, \ldots)$$

In fact, for k large, it can be shown that

$$\mathcal{H}_{CS}(j_1, j_2, \ldots) = \text{Inv}(j_1, j_2, \ldots)$$

for large level k (large BH).

Finally: Constraints don't change the picture.

4. State counting

Count: All sequences $j_1, j_2, \ldots$ such that

$$4\pi\beta l_P^2 \sum_i \sqrt{j_i(j_i+1)} \leq a_H$$

with multiplicity $\dim \mathrm{Inv}(j_1 \otimes j_2 \otimes \ldots)$

Agullo, Barbero, Borja, Diaz-Polo, Villasenor (‘10) find:

$$\begin{aligned} N_{\leq}(a) = & \frac{2}{(2\pi)^2 i} \int_0^{2\pi} \int_{x_0 - i\infty}^{x_0 + i\infty} \frac{\sin^2 \omega}{s} \\ & \times \left(1 - \sum_{k=1}^{\infty} \frac{\sin(k+1)\omega}{\sin \omega} e^{-s\sqrt{k(k+2)}}\right)^{-1} \\ & \times e^{as}\, ds\, d\omega \end{aligned}$$

Analysis of the pole structure of the integrand gives:

$$\ln N_{\leq}(a) = \frac{\widetilde{\beta}}{2\pi\beta}\frac{a_h}{4l_P^2} - \frac{3}{2}\ln(a_H/l_P^2) + O(a_H^0)$$

with $\widetilde{\beta}$ determined by

$$\sum_{k=1}^{\infty}(k+1)e^{-\widetilde{\beta}\sqrt{k(k+2)}/2} = 1$$

This leads to the choice

$$\beta = \widetilde{\beta}/(2\pi) \approx 0.274067$$

which reproduces the Bekenstein-Hawking entropy law.

5. Remarks

1. History:

- Rovelli , Krasnov ('96): Counting punctures

- Ashtekar + Baez + Corichi + Krasnov ('98):
    - Use of isolated horizon condition
    - Chern-Simons on boundary
    - gauge fixing to U(1) on boundary
    - state counting only approximate

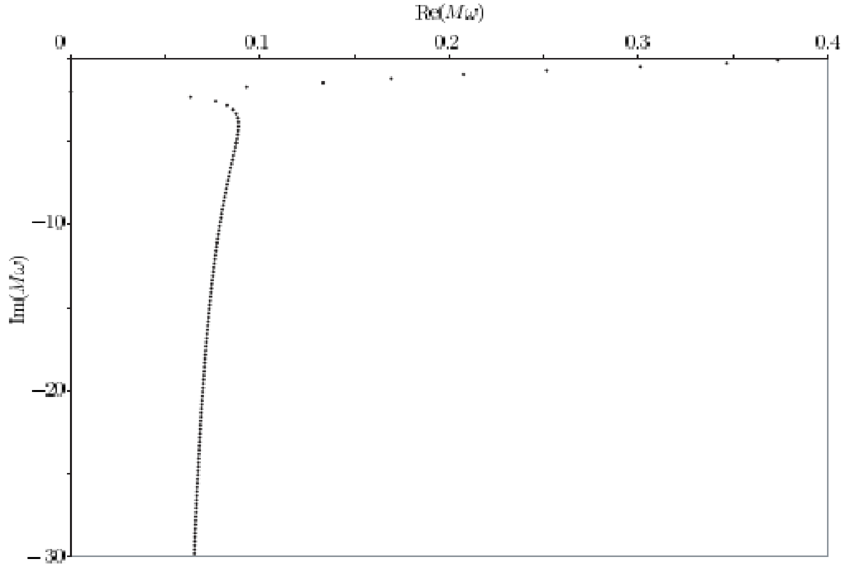
- Domagala + Lewandowski, Meissner ('04): correct combinatorial formulation, counting, statistics.

- Engle + Noui + Perez ('10): Treatment without extraneous gauge fixing

2. Relation to quasinormal modes (QNM):

QNM in this context: ringing modes of fields (metric perturbations) on BH spacetimes

Complex frequencies. For scalar modes on Schwarzschild:



(image from: Dreyer ‘02)

In fact, limit of real part

$$\lim_{n\to\infty} \mathrm{Re}(M\omega_n) \approx 0.04371235$$

Hod's prediction ('98):

$$lim_{n\to\infty}\mathrm{Re}(M\omega_n) = \frac{\ln 3}{8\pi}$$

Proven by Motl ('03). Why is this interesting?

Bekenstein ('73): Area quantum

$$\Delta A = 4\ln(k)\, l_P^2$$

Hod's reasoning: With k=3:

$$\Delta A = 32\pi M \Delta M = 32\pi\hbar\omega_{\mathrm{QNM}}$$

QNM spectrum = emission spectrum of quantum BH??

Situation in LQG:
Area spec much more complicated, but there is minimal nonzero eigenvalue.

Dreyer ('03): Can get $\Delta A = 4\ln(3)\, l_P^2$ for gauge group SO(3) in LQG

Uses (incorrect)

$$\beta^{\mathrm{ABCK}} = \frac{\ln 3}{2\pi\sqrt{2}}$$

Domagala, Meissner, Lewandowski: Does not work for correct counting.

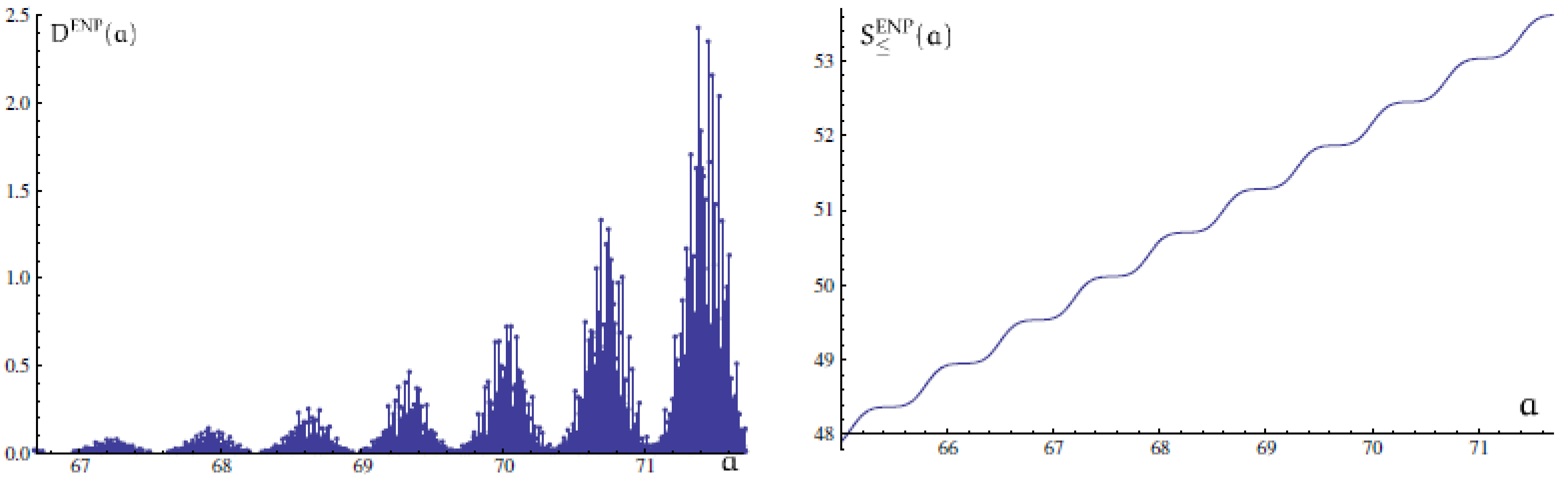
Possible way out: Different ordering in area operator gives equidistant area spectrum in LQG. Dreyer's argument then seems to work again.

But: Extension to charged, rotating case unclear (Perez, Sahlmann, Sudarsky ('04))

3. Entropy quantization

Area spectrum in LQG not equidistant. But…

A. Corichi, J. Diaz-Polo and E. Fernandez-Borja (‘07):



With

$$\Delta a = \beta \chi l_P^2, \quad \text{with} \quad \chi \approx 8.80 \stackrel{?}{=} 8 \ln 3$$

Fully analytic investigation using tools from number theory in progress:
Agullo, Barbero, Borja, Diaz-Polo, Villasenor ('10).

Structure seems to go away for large black holes.

4. Connection to convex polyhedra, polymers

Beautiful recent work by Bianchi, Dona, Speziale ('10):

Convex Polyhedra ↔ Closed polymers ↔ Space of intertwiners

Given uniquely by set of N normalized vectors $\vec{n}_i$ and N numbers A_i, with closure relation

$$\sum_i \vec{n}_i A_i = 0$$

Quantization of symplectic space

- derivation of area-entropy relation from polymer physics
- Horizon DOF as DOF of quantum convex polyhedron?

6. Summary / Outlook

In LQG:

- BH area *is* entropy of quantum isolated horizon
- microstates = states of SU(2) Chern-Simons theory with punctures
  ≈ quantum shapes of horizon

Things fit very nicely together. Seems the LQG picture captures at least part of the truth.
But much more to understand:

- Charged, rotating black holes
- Dynamical situation
- Holography?
- How does the thermodynamics come in?
- …

2. LQG basics

Variables: (Ashtekar, Barbero)

$$S[\omega, e] = \frac{1}{4k}\int_M \left[\epsilon_{ijkl} e^i \wedge e^j \wedge \Omega^{kl} - \frac{2}{\beta} e^i \wedge e^j \wedge \Omega_{ij}\right]$$

$$= \frac{1}{k\gamma}\int dt \int_{\Sigma_t} E^a_I \dot{A}^I_a - \omega^I_0 G_I + N^a C_a + NC$$

$$A^I_a = \Gamma^I_a + \beta K^I_a \qquad E^a_I = \sqrt{|\det q|}\, e^a_I$$

$$i, j, \ldots : \mathrm{SO(3,1)} \qquad I, J, \ldots : \mathrm{SO(3)(or\ SU(2))}$$

$$\{A^I_a(x), E^J_b(y)\} = 8\pi\beta G \delta^b_a \delta^I_J \delta(x, y)$$