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LOOP QUANTUM COSMOLOGY

→ doing quantum cosmology using techniques of LQG

Aim: To ask physically relevant questions.

1. Whether one recovers GR in IR limit? (If spacetime curvature is very small)
2. Are there departures from GR when spacetime curvature is large? (UV limit)
3. Singularity resolution.
4. Is it generic?
5. New physics at the Planck scale
6. Phenomenological applications

→ How probable is inflation?

→ Entropy bounds

→ . . . . .

7. Isotropic models $(k=0,\ k=\pm 1)$

→ Bojowald (2000)

→ Ashtekar, Bojowald, Lewandowski (2003)

→ Ashtekar, Pawlowski, PS (2006)

→ Exactly solvable models (2008)

→ Bianchi models

→ Mathematical Issues

→ Ma, French group ⟶ perturbations

(*) Spatially flat (k=0), isotropic, homogenous spacetime

Plan.

* Symmetry reduced variable (c, p)
* Classical theory (c, p), classical dynamics
* Quantum kinematics
* Physical Hilbert space
* Wheeler-DeWitt theory
* LQC ⟶ constraint, physical Hilbert space, physics

CAVEATS : Homogenous spacetime

Phase Space :

* Symmetry reduction:

Metric:

$$ds^2 = -N^2 dt^2 + a^2(t)\, \mathring{q}_{ab}\, dx^a dx^b$$

$N = 1$

$$\mathring{q}_{ab} = \mathring{\omega}_a^i \mathring{\omega}_b^j \delta_{ij}$$

Under the assumption of spatial homogeneity & isotropy

$$A_a^i = \tilde{c}(t)\, \mathring{\omega}_a^i \quad , \quad E_i^a = \sqrt{\mathring{q}}\, \tilde{p}(t)\, \mathring{e}_i^a$$

Relation between $\tilde{c}$, $\tilde{p}$ and scale factor

$$E_i^a E^{bi} = |q|\, q_{ab} \qquad q_{ab} = a^2 \mathring{q}_{ab}$$

$$\longrightarrow \quad \boxed{|\tilde{p}| = a^2}$$

* $$A_a^i = \Gamma_a^i + \gamma K_a^i$$

$$\boxed{\tilde{c} = \mathrm{sgn}(\tilde{p})\, \gamma\, \dot{a}}$$

Classical Hamiltonian in terms of $\tilde{c}$ and $\tilde{p}$

$$C_L = -\frac{1}{\sqrt{\det E}}(1+\gamma^2) E^a_i E^b_j K^i_{[a} K^j_{b]}$$

$$-C_E = \frac{1}{2\sqrt{\det E}} E^a_i E^b_j \epsilon^{ij}{}_k F^k_{ab}$$

In the homogeneous isotropic case:

$$C_L = -3\sqrt{\mathring{q}}\,(1+\gamma^{-2})\sqrt{|\tilde{p}|}\,\tilde{c}^2$$

$$-C_E = 3\sqrt{\mathring{q}}\sqrt{|\tilde{p}|}\,\tilde{c}^2$$

$$\boxed{-C_E + C_L = -3\sqrt{\mathring{q}}\sqrt{|\tilde{p}|}\,\frac{\tilde{c}^2}{\gamma^2}} = -3\sqrt{|p|}\,\frac{c^2}{\gamma^2}$$

* Introduce a fiducial cell $\mathcal{V}$ with fiducial metric $\mathring{V}$.

- Freedom to rescale coordinates ✓
- Freedom of the choice of the cell

$\rightarrow$ $x \longrightarrow x' = l x$; $a \longrightarrow a' = l^{-1} a$

Implies that

$$|\tilde{p}| \rightarrow |\tilde{p}'| = l^{-2}|\tilde{p}|$$

$$\tilde{c} \rightarrow \tilde{c}' = l^{-1}\tilde{c}$$

$$\mathring{V} \longrightarrow \mathring{V}' = l^3 \mathring{V}$$

Natural variables : $c = V_0^{1/3}\,\tilde{c}$, $p = V_0^{2/3}\,\tilde{p}$

$$\boxed{\{c, p\} = \frac{8\pi G\gamma}{3}}$$

Classical Dynamics (with c and p)

$$\mathcal{H}_{classical} = \frac{-3}{8\pi G}\frac{c^2}{\gamma^2}\sqrt{p} + \mathcal{H}_M$$

Hamilton's Eqns

$$\dot{p} = 2\frac{c}{\gamma}p^{1/2} \longrightarrow c = \gamma\dot{a}$$

in term of scale factor for physical solution

$$\boxed{\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\frac{\mathcal{H}_M}{a^3} = \frac{8\pi G}{3}\rho}$$

Friedmann Eq

Hamilton's Eq. for c

$$\dot{c} = \frac{c^2}{2\gamma}p^{-1/2} + \frac{\partial \mathcal{H}_M}{\partial p}$$

$$P = \text{Pressure} = -\frac{\partial \mathcal{H}_M}{\partial V} \;;\; V = p^{3/2}$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)}$$

$\hookrightarrow$ Raychaudhri Eqn.

Combine Friedmann Eq & Raychaudhri Eq :

$$\boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0} \leftarrow T^{\mu\nu}{}_{;\mu} = 0$$

Singularity

Massless scalar field.

$$\rho = \frac{\dot{\varphi}^2}{2}, \quad P = \frac{\dot{\varphi}^2}{2} \quad ; \; w = \frac{P}{\rho} = 1$$

$$\rightarrow \rho \propto a^{-3(1+w)}$$ [matter with fixed equation of state]

Ricci Scalar : $R = 6\left(H^2 + \frac{\ddot{a}}{a}\right)$

$$\boxed{R = 8\pi G \rho (1 - 3w)}$$

For massless scalar, $R \propto a^{-6}$

As $a \rightarrow 0$; $H \rightarrow \infty$, $R \rightarrow \infty$

(Big Bang / Big Crunch)

(Big Bang / Big Crunch)

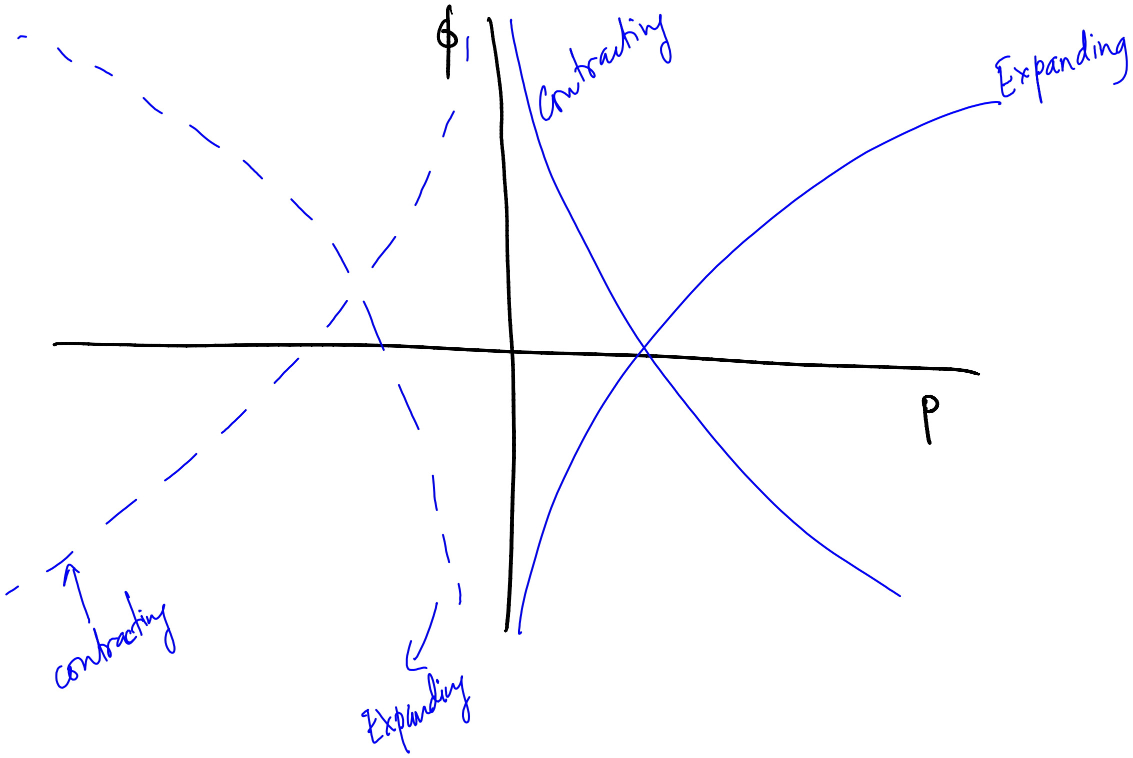
Solutions: $\mathcal{H}_M = \frac{p_\phi^2}{2\, p^{3/2}}$ (massless scalar).

$\hookrightarrow \rho = \frac{\dot{\phi}^2}{2}$ (using Hamilton's Eqns)

$$\boxed{\phi = \pm \sqrt{\frac{3}{16\pi G}} \ln \left| \frac{p}{p'} \right| + \phi'}$$

$$\frac{d\phi}{dt} = \frac{16\pi G\, p_\phi'}{|p'|^{1/2}} \exp\left(\pm \sqrt{12\pi G}\,(\phi - \phi')\right)$$

p_ϕ = constant



Quantization: Kinematics

$$h_i(A) = \cos\frac{\mu c}{2} + 2\sin\frac{\mu c}{2}\,\tau_i$$

where edge length $\mu \overset{o}{V}^{1/3}$; $\mu \in (-\infty, \infty)$

$$E \propto p\, V_0^{-2/3}$$

algebra of almost periodic functions

$$\mathcal{H}_{Kin} = L^2(R_{Bohr}, d\mu_{Bohr})$$

Elements:

Almost periodic functions form an orthonormal basis

$$\langle e^{i\mu_1 c/2} \,|\, e^{i\mu_2 c/2}\rangle = \delta_{\mu_1,\mu_2}$$

In the triad representation:

$$\hat{p}\,|\mu\rangle = \frac{\mu\gamma l_p^2}{6}\,|\mu\rangle$$

$$\hat{V}\,|\mu\rangle = \frac{|\mu|^{3/2}\gamma^{3/2}\, l_p^3}{6^{3/2}}\,|\mu\rangle$$

$$\langle\mu_1|\mu_2\rangle = \delta_{\mu_1,\mu_2}$$

$$e^{i\mu' c/2}\,|\mu\rangle = |\mu+\mu'\rangle$$

$$\boxed{\hat{h}_k^{(\mu')}|\mu\rangle = \frac{1}{2}\left(|\mu+\mu'\rangle + |\mu-\mu'\rangle\right)\mathbb{I} + \frac{1}{i}\left(|\mu+\mu'\rangle - |\mu-\mu'\rangle\right)\tau_k}$$

$$2i\,\tau_i = \sigma_i$$

<u>Hamiltonian Constraint:</u>

$$C_{grav} = -\frac{1}{\gamma^2}\int_{\mathcal{V}} d^3x\,\epsilon_{ijk}\,e^{-1}E^{ai}E^{bj}F^{i}_{ab}$$

$*$ $\varepsilon_{ijk}\,e^{-1}E^{aj}E^{bk} = \sum_k \frac{(\mathrm{sgn}\,p)}{2\pi G\gamma \mathring{V}^{1/3}\tilde{\lambda}}\,\mathring{\varepsilon}^{abc}\,\mathring{\omega}_c^{\;k}\,\mathrm{Tr}\left(\left(h_k^{(\tilde{\lambda})}\{h_k^{(\tilde{\lambda})-1}, V\}\right)\tau_i\right)$

$*$ Consider a square $\square_{ij}$ in i-j plane spanned by the face of the elementary cell with side $\tilde{\lambda}\,\mathring{V}^{1/3}$

$$F_{ab}^{\;k} = -2\lim_{Ar_\square\to 0}\mathrm{Tr}\left(\frac{h_{\square_{ij}}^{(\tilde{\lambda})}-1}{\tilde{\lambda}^2 V_0^{2/3}}\right)\tau^k\,\mathring{\omega}_a^{\;i}\,\mathring{\omega}_b^{\;j}$$

$$h_{\square_{ij}}^{(\tilde{\lambda})} = h_i^{(\tilde{\lambda})}\,h_j^{(\tilde{\lambda})}\left(h_i^{(\tilde{\lambda})}\right)^{-1}\left(h_j^{(\tilde{\lambda})}\right)^{-1}$$

$$C_{grav} = \lim_{Ar_\square\to 0} C_{grav}^{(\tilde{\lambda})}$$

$$\lim_{Ar_\square \to 0}$$

$$= \frac{-\mathrm{sgn}(p)}{2\pi G \gamma^3 \bar{\lambda}^3} \lim_{Ar_\square \to 0} \sum_{ijk} \mathrm{Tr}\left(h_i^{(\bar\lambda)} h_j^{(\bar\lambda)} \left(h_i^{(\bar\lambda)}\right)^{-1} \left(h_j^{(\bar\lambda)}\right)^{-1} h_k^{(\bar\lambda)} \left\{ h_k^{(\bar\lambda)-1}, V \right\} \right)$$

$$= \lim_{Ar_\square \to 0} \left(\sin(\bar\lambda c) \left(\frac{-1}{2\pi G \gamma^3} \frac{\mathrm{sgn}(p)}{\bar\lambda^3} \sum_k \mathrm{Tr}\, \tau_k h_k^{(\bar\lambda)} \left\{ h_k^{(\bar\lambda)-1}, V \right\} \right) \times \sin(\bar\lambda c) \right)$$

In quantum theory, limit does not exist. Shrink the loop to the minimum eigenvalue of the area operator

$$\boxed{\bar\mu^2 |p| = \Delta \ell_p^2} \quad \text{with } \Delta = 4\pi\gamma\sqrt{3}$$

$$\bar\mu^2 a^2 = \Delta l_p^2$$

Relevant operators are $\widehat{e^{i\bar\mu(p)c/2}}$ and not $\widehat{e^{i\mu c/2}}$

It is not true that

$$\widehat{e^{i\bar\mu(p)c/2}}\, \tilde\psi(\mu) \neq \tilde\psi(\mu + \bar\mu)$$

$$\widehat{e^{i\bar\mu c/2}}\, \tilde\psi(\mu) = e^{\bar\mu \frac{d}{d\mu}}\, \tilde\psi(\mu)$$

$$v = K\, \mathrm{sgn}(\mu)\, |\mu|^{3/2} \quad ; \quad K = \frac{2}{\sqrt{3\sqrt{3\sqrt{3}}}}$$

$\bar\mu$ forces us that the natural representation is the

$\bar{\mu}$ forces us that the natural representation is the volume representation (and not triad representation)

$$\boxed{\widehat{e^{i\bar{\mu}c/2}}\,\Psi(v) = \Psi(v+1)}$$

$$\widehat{\cos\frac{\bar{\mu}c}{2}}\,|v\rangle = \frac{1}{2}\left(|v+1\rangle + |v-1\rangle\right)$$

Inverse triads.

$$\epsilon_{ijk}\, e^{-1} E^{aj} E^{bk} \longrightarrow \frac{p_\phi^2}{|p|^{3/2}}$$

One can show that

$$\frac{\mathrm{sgn}(p)}{|p|^{1/2}} = \frac{1}{2\pi G\gamma\bar{\mu}}\,\mathrm{Tr}\left(\sum_i \tau^i h_i \{h_i^{-1}, V^{1/3}\}\right)$$

$$h_i = e^{\bar{\mu}c\tau_i} \qquad \tau_i = -\frac{i\sigma_i}{2}$$

$$\mathrm{Tr}\sum_i \tau^i h_i \{h_i^{-1}, V^{1/3}\} = \mathrm{Tr}\sum_i \tau^i e^{\bar{\mu}c\tau_i}(-\bar{\mu}\tau^i)\frac{e^{-\bar{\mu}c\tau_i}}{2}$$

$$|p|^{-1/2}\,\mathrm{sgn}(p)\left(\frac{8\pi G\gamma}{3}\right)$$

$$\widehat{\frac{\mathrm{sgn}(p)}{|p|^{1/2}}}|v\rangle = \frac{-i}{2\pi G\gamma\bar{\mu}}\,\mathrm{Tr}\left(\tau^i \hat{h}_i\left[\hat{h}_i^{-1}, \hat{V}^{1/3}\right]\right)|v\rangle$$

$$= \frac{-12i}{8\pi G\gamma\bar{\mu}}\left[\widehat{\sin\frac{\bar{\mu}c}{2}}\,\hat{V}^{1/3}\,\widehat{\cos\frac{\bar{\mu}c}{2}} - \widehat{\cos\frac{\bar{\mu}c}{2}}\,\hat{V}^{1/3}\,\widehat{\sin\frac{\bar{\mu}c}{2}}\right]|v\rangle$$

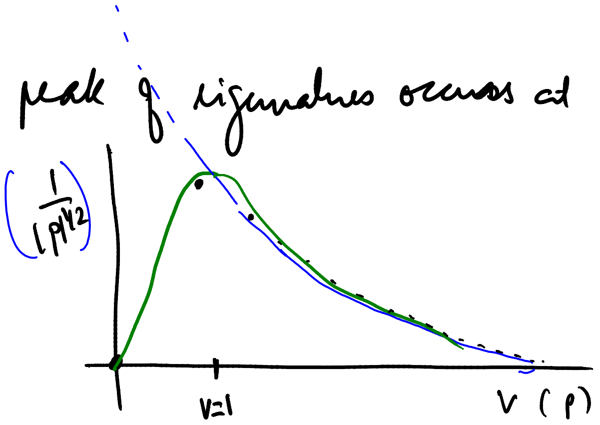
$$= \frac{-12i}{8\pi G \gamma \bar{\mu}} \left[\widehat{\sin\frac{\bar{\mu}c}{2}}\, \hat{V}^{1/3}\, \widehat{\cos\frac{\bar{\mu}c}{2}} - \widehat{\cos\frac{\bar{\mu}c}{2}}\, \hat{V}^{1/3}\, \widehat{\sin\frac{\bar{\mu}c}{2}} \right] |v\rangle$$

$$\boxed{\widehat{\frac{\mathrm{Sgn}(p)}{|p|^{1/2}}}|v\rangle = \frac{3\alpha}{4(8\pi G)\gamma} K^{2/3} v^{1/3} \left[|v+1|^{1/3} - |v-1|^{1/3} \right] |v\rangle}$$

$$\text{where } \alpha = \left(\frac{8\pi\gamma}{6}\right)^{1/2} \frac{l_p}{K^{1/3}}$$

For $v \gg 1$, eigenvalues approximate to $\frac{1}{|p|^{1/2}}$

The peak of eigenvalues occurs at $v = 1$



Thiemann Trick is very useful, but one has to be very careful while using it.

Quantum Constraint:

$$\hat{C}_{grav} = \sin(\bar{\mu}c)\, \hat{A}\, \sin(\bar{\mu}c)$$

$$\hat{A} = \frac{24i\, \mathrm{sgn}(\mu)}{8\pi\gamma^3 \bar{\mu}^3 l_p^2} \left(\sin\left(\frac{\bar{\mu}c}{2}\right) \hat{V} \cos\left(\frac{\bar{\mu}c}{2}\right) - \cos\left(\frac{\bar{\mu}c}{2}\right) \hat{V} \sin\left(\frac{\bar{\mu}c}{2}\right) \right)$$

$$\hat{C}_{grav}\Psi(v) = f_+(v)\Psi(v+4) + f_0(v)\Psi(v) + f_-(v)\Psi(v-4)$$

$$f_+(v) = \frac{27}{16}\sqrt{\frac{8\pi}{6}}\, K \frac{l_p}{\gamma^{3/2}} |v+2| \Big| |v+1| - |v+3| \Big|$$

$$K = \frac{2}{3\sqrt{3\sqrt{3}}}$$

$$\hat{V}|v\rangle = \left(\frac{8\pi\gamma}{6}\right)^{3/2} \frac{|v|}{K} l_p^3 |v\rangle$$

$$f_- = f_+(v-4) \;, \quad f_0(v) = -f_+(v) - f_-(v)$$

* The constraint leads to a difference equation which couples Ψ in uniform steps of v.

* Total constraint: $(\hat{C}_{grav} + \hat{C}_m)\Psi = 0$

* Massless scalar field:

$$C_m = 8\pi G |p|^{-3/2} p_\phi^2$$

$$\widehat{|p|^{-3/2}}\Psi(v) = \left(\frac{6}{8\pi\gamma l_p^2}\right)^{3/2} B(v)\Psi(v)$$

$$B(v) = \left(\frac{3}{2}\right)^3 K |v| \Big| |v+1|^{1/3} - |v-1|^{1/3} \Big|^3$$

The total constraint can be written as

$$\boxed{\partial_\phi^2 \Psi(v,\phi) = -\Theta_{(v)}\Psi(v,\phi)}$$

where $\Theta\Psi := B^{-1}_{(v)}\Big(C^+_{(v)}\Psi(v+4,\phi) + C^\circ_{(v)}\Psi(v,\phi) + C^-_{(v)}\Psi(v-4,\phi)\Big)$

$$C^+_{(v)} = \frac{3\pi K G}{8} |v+2| \Big| |v+1| - |v+3| \Big|$$

$$C^-_{(v)} = C^+_{(v-4)}, \quad C^\circ_{(v)} = -C^+_{(v)} - C^-_{(v)}$$

* The constraint takes the form of a Klein-Gordon Eq.
  - $\phi \longrightarrow$ emergent time
  - $\Theta \longrightarrow$ spatial Laplacian

Wheeler-De Witt Quantization

$$\partial_\phi^2 \Psi(v,\phi) = -\Theta_W \Psi(v,\phi)$$

where $\Theta_W \Psi = 12\pi G\, v\, \partial_v \big(v\, \partial_v \Psi(v,\phi)\big)$

In the limit $v \gg 1$, Θ_{LQC} yields Θ_W

Eigenfunctions of Θ_W:

$$\Theta_W e_K = \omega^2 e_K \qquad ; \quad \omega^2 \geq 0$$

$$e_k(v) = \frac{1}{\sqrt{2\pi}} e^{ik \ln|v|} \quad ; \quad (e_k, e_{k'}) = \delta(k,k')$$

A general solution of the constraint:

$$\Psi(v,\phi) = \int_{-\infty}^{\infty} dk \left(\tilde{\Psi}_+(k)\, e_k(v)\, e^{i\omega\phi} + \tilde{\Psi}_-(k)\, \bar{e}_k(v)\, e^{-i\omega\phi} \right)$$

In WDW theory:

$k > 0$, Ψ ~~is~~ incoming or contracting

$k < 0$, Ψ is outgoing or expanding

Ψ is of positive frequency if $\tilde{\Psi}_-$ vanishes.

" negative " " $\tilde{\Psi}_+$ " .

* Positive and negative frequency solutions can be obtain

$$\mp i \partial_\phi \Psi(v,\phi) = \sqrt{\Theta}\, \Psi(v,\phi)$$

Evolution : $f(v)$ as initial data at $\phi = \phi_0$

$$\Psi_\pm(v,\phi) = e^{\pm i\sqrt{\Theta}(\phi - \phi_0)} f(v)$$

* $k = \dfrac{-P_\phi}{\sqrt{12\pi G \hbar^2}}$, $\omega = -\sqrt{12\pi G}\, k$

* Dirac observables :

$$\hat{P}_\phi \Psi(v,\phi) = -i\hbar \frac{\partial \Psi}{\partial \phi}$$

$$\hat{v}\big|_{\phi=\phi_o}\Psi(v,\phi) = e^{i\sqrt{\Theta}(\phi-\phi_o)}\,|v|\,\Psi(v,\phi_o)$$

* Inner Product:

$$\langle\Psi_1|\Psi_2\rangle_{phy} = \int_{\phi=\phi_o} dv\ \bar{B}(v)\ \bar{\Psi}_1(v,\phi)\ \Psi_2(v,\phi)$$

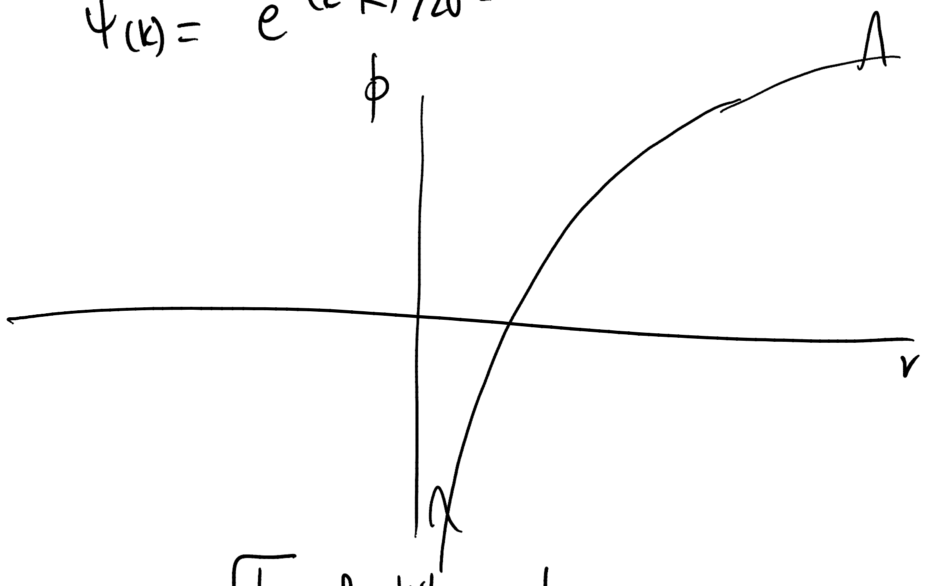
* Physical Implication:

Consider a state peaked at classical trajectory at large volume and small spacetime curvature

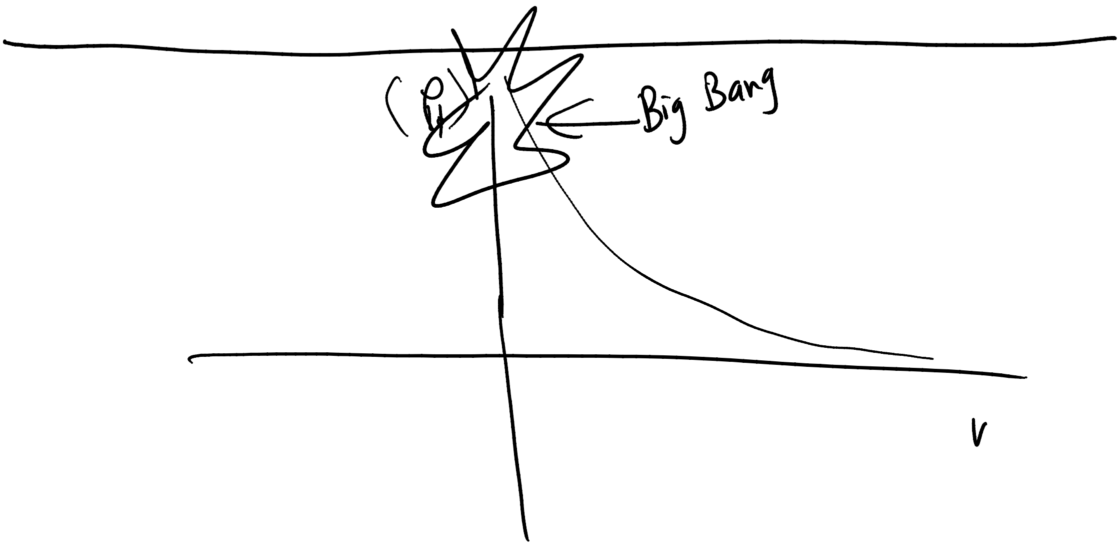
The state is sharply peaked in v^*, P_ϕ^*

$v^* \gg 1$ and $P_\phi^* \gg \hbar$

$$\tilde{\Psi}(k) = e^{-(k-k^*)^2/2\sigma^2}$$



$$\phi = \sqrt{\frac{1}{12\pi G}}\ \ln\frac{|v|}{|v^*|} + \phi_o$$



LQC.

$$\partial_\phi^2 \Psi(v,\phi) = -\Theta\, \Psi(v,\phi)$$

Physical Hilbert space : $L^2(\mathbb{R}_{Bohr}, B(v)\, d\mu_{Bohr})$

$$\Theta := -(B(v))^{-1}\left(C^{+}(v)\, \Psi(v+4,\phi) + C^{0}(v)\, \Psi(v,\phi) + C^{-}(v)\, \Psi(v-4,\phi)\right)$$

$$v = \pm|\varepsilon| + 4n$$

Dirac observables: $\hat{p}_\phi$, $\hat{v}|_{\phi_0}$

Inner Product:

$$\langle \Psi_1 | \Psi_2 \rangle_\varepsilon = \sum_{\mathcal{L}_\varepsilon} B(v)\, \bar{\Psi}_1(v,\phi_0)\, \Psi_2(v,\phi_0)$$

Exactly Solvable Model (k=0, massless scalar)

Take $N = a^3$ then the classical constraint

is $$-\frac{3}{4\pi G \gamma^2}\, c^2 p^2 + p_\phi^2 = 0$$

Quantize using LQC:

$$\partial_\phi^2 \tilde{\Psi}(v,\phi) = 3\pi G \, v \frac{\sin \lambda b}{\lambda} v \frac{\sin \lambda b}{\lambda} \tilde{\Psi}(v,\phi)$$

b is the conjugate to v ; $[\hat{b}, \hat{v}] = 2i$

c, p, V, b ; $b = \frac{c}{|p|^{1/2}}$, $\lambda^2 = 2\sqrt{3}\pi\gamma l_p^2$

In classical theory $c = \gamma \dot{a}$ $\Rightarrow$ $b = \frac{\gamma \dot{a}}{a} = \gamma H$

The constraint in the b representation:

$$\partial_\phi^2 \chi(b,\phi) = 12\pi G \left(\frac{\sin \lambda b}{\lambda} \partial_b\right)^2 \chi(b,\phi)$$

$$x = \frac{1}{\sqrt{12\pi G}} \ln\left(\tan \frac{\lambda b}{2}\right)$$

x ranges from $-\infty$ to ∞

$$\boxed{\partial_\phi^2 \chi(x,\phi) = \partial_x^2 \chi(x,\phi)}$$

$\longrightarrow$ Exactly solvable model

$$\hat{V}|_{\phi_0} \chi(x,\phi) = e^{i\sqrt{\Theta}(\phi-\phi_0)} (2\pi\gamma l_p^2 |\hat{v}|) \chi(x,\phi_0)$$

$$\boxed{\langle \hat{V}|_\phi \rangle = V_+ e^{\sqrt{12\pi G}\phi} + V_- e^{-\sqrt{12\pi G}\phi}}$$

where V_+ & V_- are strictly positive

* There exists a minimum vol.

$$V_{min} = \frac{2\,(V_+ V_-)^{1/2}}{||x||^2}$$

It is reached at 'time' $\phi_B = \frac{1}{2\sqrt{12\pi G}} \ln \frac{V_-}{V_+}$

$\langle \hat{\rho}_{lq} \rangle$; $\rho_{sup} = \frac{\sqrt{3}}{32\pi^2 G^2 \hbar \gamma^3} = 0.41\, \rho_{Pl}$

$(\gamma = 0.24)$

Effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{-3}{8\pi G \gamma^2} \frac{\sin^2(\lambda b)}{\lambda^2} V + \mathcal{H}_M$$

where $\{b, V\} = 4\pi G \gamma$

Derive Hamilton's Eqns:

Modified Friedmann Eq:

$$\frac{\dot{V}}{V} = \frac{3}{\gamma} \frac{\sin(\lambda b)\cos(\lambda b)}{\lambda} \longrightarrow (1)$$

$$\mathcal{H}_{eff} \simeq 0 \Rightarrow \rho = \frac{3}{8\pi G} \frac{\sin^2(\lambda b)}{\gamma^2 \lambda^2}$$

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{\dot{V}^2}{9V^2} = \frac{1}{\gamma} \frac{\sin^2(\lambda b)}{\lambda^2}\left(1 - \sin^2(\lambda b)\right)$$

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{\dot{V}^2}{9V^2} = \frac{1}{\gamma}\frac{\sin^2(\lambda b)}{\lambda^2}\left(1 - \sin^2(\lambda b)\right)$$

$$= \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

$$\rho_c = \frac{3}{8\pi G\gamma^2\lambda^2} \simeq 0.41\,\rho_{pl}$$

For $\rho \ll \rho_c$, $H^2 = \frac{8\pi G}{3}\rho$

At $\rho = \rho_c$, $H = 0$

Modified Raychaudhuri Eq:

(from Hamilton's Eq for b)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho\left(1 - 4\frac{\rho}{\rho_c}\right) - 4\pi G P\left(1 - 2\frac{\rho}{\rho_c}\right)$$

Combine Friedmann & Raychaudhuri Eqns.

$$\dot{\rho} + 3H(\rho + P) = 0$$

Ricci Scalar:

$$R = 8\pi G\rho\left(1 - 3w + \frac{2\rho}{\rho_c}(1 + 3w)\right); \quad w = \frac{P}{\rho}$$

<u>New Physics:</u>

$$\dot{H} = -4\pi G(\rho+P)\left(1-2\frac{\rho}{\rho_c}\right)$$

Classically $\dot{H}_{cl} = -4\pi G(\rho+P)$

For $\rho_c > \rho > \frac{\rho_c}{2}$, $\dot{H} > 0 \longrightarrow$ Super-Inflation

For matter which satisfies NEC : $\rho + P \geq 0$

$\dot{H}_{cl} > 0$ is not allowed.

In LQC for all matter satisfying NEC,

$\dot{H} > 0$ for $\rho_c > \rho > \frac{\rho_c}{2}$

$$P = -\frac{\partial H_m}{\partial V} \quad ; \quad P = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$\rightarrow$ Repulsiveness of Gravity:

In GR: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$

For massless scalar $\rho = P$; $\frac{\ddot{a}}{a} < 0$

In LQC: $\frac{\ddot{a}}{a} > 0$ for $\rho > \frac{2}{5}\rho_c$

For dust : $\frac{\ddot{a}}{a} > 0$ for $\rho > \frac{\rho_c}{4}$

$$\rightarrow |H|_{max} = \left(\frac{1}{\sqrt{3}\, 16\pi G\hbar\gamma^3}\right)^{1/2} = \frac{\theta_{max}}{2}$$

$\rightarrow$ $$|H|_{max} = \left(\frac{1}{\sqrt{3}\, 16\pi G \hbar \gamma^3}\right)^{1/2} = \frac{\dot{\theta}_{max}}{3}$$

$\rightarrow$ Can be shown that geodesics are complete in effective LQC for $k=0$ model for arbitrary matter. (Assuming Effective Hamiltonian is valid)

$\rightarrow$

<u>Issue of Fiducial Cell.</u>

$$\mathcal{V} \longrightarrow \mathcal{V}' \; ; \quad \mathring{V}' = \alpha^3 \mathring{V}$$

$$c' = \alpha c \,, \quad p' = \alpha^2 p \,, \quad V' = \alpha^3 V \,, \quad b' = b \,, \quad \rho' = \rho$$

$$p_\phi' = \alpha^3 p_\phi$$

<u>Alternate quantizations of LQC.</u>

μ_0 quantization (old quantization)

$$\frac{3}{\gamma^2} \frac{\sin^2(\lambda_c c)}{\lambda_c^2} |p|^{1/2} = 8\pi G H_m$$

$$\rightarrow \quad \rho_c = \sqrt{2} \left[\frac{3}{8\pi G \gamma^2 \lambda_c^2}\right]^{3/2} \frac{1}{p_\phi}$$

<u>Some more ways</u>

New phase space variables:

$$P_g = c\, p^m \,, \quad g = \frac{p^{(1-m)}}{1-m}$$

(c, p)

when $m=0 \quad (\rho_g, g) \longrightarrow (c, p)$

$m=-1/2 \quad (\rho_g, g) \longrightarrow (b, V)$

$$\rho_c = \frac{3}{8\pi G \gamma^2 \lambda_{\rho_g}^2} \left(\frac{8\pi G}{6} \gamma^2 \lambda_{\rho_g}^2 \, p_\phi^2 \right)^{\frac{2m+1}{2m-2}}$$

works only for $m=-\frac{1}{2}$