1. MOTIVATION

Several information theoretic approaches to foundations of quantum theory:

* * Fuchs, Caves, Schack, ...
* * D'Ariano, Chiribella, Perinotti
* * Bub, Clifton, Halvorson
* * Abramsky, Coecke, Vicary, ...
* * Barnum, Wilce, ...
* * Goyal
* * Hardy
* * ...

aim to derive quantum theory from another theory.

BUT:

- What is the range of structures of "quantum theory" to be derived?
  (how about QFT? renormalisation? QG?
  how to construct hamiltonians directly from experimental data? which non-unitary reduction method is correct?)
- Why choose this particular "another theory" as foundations?
  (why Kolmogorov's measure-theoretic probability?
  why symmetric monoidal †-categories?)

NOTE:

- Different foundational approaches have different limits of applicability and leave many problems of quantum theory unsolved
- Their own foundations should be more appealing than the quantum theory itself.

II. AIM & PLAN

- Having these issues in mind, we'll develop new approach to foundations of QT.
- <u>Our aim</u>: to cover most general forms and uses of quantum theory, <u>including quantum field theory</u> (and aiming at quantum gravity)
- <u>Note</u>: at such level, quantum theory still lacks (and asks for) mathematically strict form
- We are less concerned with axiomatisation (at this stage), without strict mathematics, axiomatisation is premature

PLAN OF THE TALK:

1. Review of foundations of probability theory
2. New foundations for probability theory
3. Review of mathematical foundations of quantum theory
4. New mathematical framework: quantum information geometry
5. New conceptual framework: intersubjective coherence interpretation
6. Recovery of traditional quantum theory and probability theory as special cases

III. PROBABILITY THEORY: MATHEMATICAL FRAMEWORKS

Borel – Kolmogorov:

* measure spaces $(\mathcal{X}, \mathfrak{V}(\mathcal{X}), \mu)$, probabilistic models $\mathcal{M}(\mathcal{X}, \mathfrak{V}(\mathcal{X}), \mu) \subseteq L_1(\mathcal{X}, \mathfrak{V}(\mathcal{X}), \mu)^+$
* no *generic* notion of conditional updating of prob.'s
* *note*: there are many measure spaces leading to L_1 spaces isomorphic to $L_1(\mathfrak{V})$ space, where $\mathfrak{V}$ is a caDc boolean algebra, constructed by $\mathfrak{V} := \mathfrak{V}(\mathcal{X}) / \{A \in \mathfrak{V}(\mathcal{X}) \mid \mu(A) = 0\}$. Thus, only $\mathfrak{V}$ is important for probabilistic models.
* *note*: the description in terms of measures μ on $(\mathcal{X}, \mathfrak{V}(\mathcal{X}))$ can be completely replaced by the description in terms of integrals on vector lattice $A(\mathfrak{V})$, canonically associated with $\mathfrak{V}$. Thus, one can deal exclusively with integrals on $\mathfrak{V}$ instead of measures on $\mathfrak{V}(\mathcal{X})$.

Bayes – Laplace:

* finitely additive boolean algebras and cond. prob.'s $p(A|B)$
* Bayes' rule $p(A|I) \longmapsto p_{new}(A|I) := p(A|I) \dfrac{p(B|A \wedge I)}{p(B|I)}$
* no *generic* notion of probabilistic expectation over continuous domain
* *note*: Bayes' rule is a special case of constrained relative entropic updating:

$$p(x|\theta) \longmapsto p_{new}(x|\theta) := \arg\inf_{q \in \mathcal{M}} \left\{ \int E(p,p') D(q,p') + F(q) \right\}$$

for constraints $F(q) = \lambda_1 \left(\int dx \int d\theta\, q(x|\theta) - 1 \right) + \lambda_2 \cdot \left(\int d\theta\, q(x|\theta) - \delta(x-b) \right)$, prior $E(p,p') = \delta(p-p')$ and D given by the Kullback-Leibler divergence.

IV. PROBABILITY THEORY: INTERPRETATIONS

Frequentist: [von Mises, Wald, Church, Martin-Löf]

* still widely believed, but none mathematically strict and logically sound formulation exists
* note: the normalisation of probabilities is required only due to this interpretation

'Subjective' bayesian: [de Finetti, Savage]

* conceptually consistent, but by definition it lacks any intersubjectively valid rules relating the probability assignments (theoretical model construction) with numbers, words, and other knowledge about experiments (experimental setup construction, 'experimental data')
* note: accusations of arbitrariness are unjustified, because any theoretical statement is arbitrary
* note: accusations of arbitrariness are justified, because scientific inquiry seems to be something more than individual personal coherence of bets.

'Objective' bayesian: [Cox, Jaynes]

* claims to provide general rules of assignment of probabilities (= model construction), but fails to provide sound justification for these rules which would be neither epistemic-personalist ('subjective' bayesian) nor ontological-idealist (frequentist)
* note: the intersubjective coherence of relationship between theory and experiment is a crucial idea!

V. PROBABILITY THEORY: NEW APPROACH

<u>Mathematical framework</u>:

* take the best insights from B-K and B-L approaches, and unify kinematic (probabilistic, evaluational) and dynamic (statistic inferential, relational) components
* replace measure spaces $(\mathcal{X}, \mho(\mathcal{X}), \mu)$ by (caDa) boolean algebras $\mho$
* <u>information kinematics</u>: models $\mathcal{M}(\mho) \subseteq L_1(\mho)^+$ defined as spaces of <u>finite positive</u> integrals on $\mho$
* <u>information dynamics</u>: updating by constrained relative entropy maximisation on $\mathcal{M}(\mho)$ with divergence D, prior E, and constraints F.

<u>Interpretation</u>:

* take the best insights from 'subjective' and 'objective' bayesianism & take lessons from Fleck's analysis of the structure of scientific theory
* require that the particular rules of probability assignment should correspond to the particular methods of relating experimental setup construction with theoretic model construction:

> Only some specific constraints are used to check whether some particular (very complicated) apparatus is an intersubjectively acceptable instance of the "ideal experimental setup of a given type".

* beyond community of intersubjective methods of experimentation, the particular model construction rules are <u>irrelevant</u> (arbitrary, personalistic, 'subjective'), but within this range they are <u>indispensable</u> (necessary, scientific, 'objective')

VI. QUANTUM THEORY: HILBERT SPACE FRAMEWORK

Formalism:

* kinematics: Hilbert spaces, algebras of operators, density operators
* dynamics: two different forms of temporal evolution (unitary Schrödinger, non-unitary von Neumann-Lüders)
* quantification: spectral representation in terms of L_2 space based on Kolmogorov's probability measures

Problems:

* it lacks any definite methods of model construction that would correspond to the particular experimental situation (⟹ 'quantisation', 'perturbation', 'renormalisation', and other *ad hoc* techniques)
* it lacks any definite relationship between two temporal evolutions
* it fails to describe relativistic quantum field models and continuous statistical mechanic models

Semi-spectral (POVM) perspective:

* requires more strict relationship between theoretic model construction and experimental setups construction
  [Busch Grabowski Lahti '95, de Muynck '02]
* shows the breakdown of bijection between the notion of 'measurable quantity' and self-adjoint operators
* but shares all problems of von Neumann's approach.

VII. QUANTUM THEORY: ALGEBRAIC FRAMEWORK

Formalism:

$\omega: \mathcal{A} \to \mathbb{C}$

* kinematics: C^*-algebras of operators, states on these algebras
* dynamics: $*$-automorphisms of C^*-algebras and corresponding unitary operator
* quantification: spectral representations again

Virtues:

* improves over Hilbert space approach allowing mathematically strict relativistic quantum field models and continuous quantum statistical models

Problems:

* it lacks any definite methods of model construction that would correspond to the particular experimental situation ($\Rightarrow$ applied QFT's use ad hoc methods)
* it ignores the non-unitary 'reduction', leaving the problem unsolved.

VIII. INSIGHTS

Positive:

* the theoretic model construction should be strongly tided with the experimental setup construction,
* one can resign from interpreting the self-adjoint elements of algebra as 'measurable quantities'
* the relativistic quantum field theory and continuum quantum statistical mechanics can be dealt with, if one works with abstract C^*-algebras (or W^*-) and states on these algebras

Negative:

* two conflicting temporal evolutions in a single time
* no general method of theoretical model construction associated with a given experimental situation
* appealing to Kolmogorov's framework of probabilistic measures for quantitative analysis of quantum theoretic models:
  - Kolmogorov's framework also does not possess any general method of theoretical model construction (⇒ ad hoc methods of statistical inference: estimation, testing, ...)
  - Kolmogorov's framework lacks any sound conceptual justification
  - awkward contextuality related with ~~[illegible]~~ necessity of selection of commutative subalgebra for the purpose of probabilistic interpretation

IX. NEW MATHEMATICAL FOUNDATIONS FOR Q.T.

In face of above insights and new foundations for probability, we deny the following principles:

* quantisation
* spectral representation in terms of measure spaces and Borel-Kolmogorov probability theory
* two evolutions (Schr., vN.-Lüd.) in a single time
* Hilbert spaces

Instead of this, we propose:

* replace Hilbert spaces (too rigid) and C^*-algebras (too general) by W^*-algebras $\mathcal{N}$, because only these non-commutative algebras allow integration theory
* information kinematics: models $\mathcal{M}(\mathcal{N}) \subseteq L_1(\mathcal{N})^+ \cong \mathcal{N}_*^+$, defined as spaces of positive finite integrals on $\mathcal{N}$
* information dynamics: updating by constrained quantum relative entropy maximisation on $\mathcal{M}(\mathcal{N})$ with divergence D, prior E, and constraints F.
* quantitative model construction: based on quantum information geometry on $\mathcal{M}(\mathcal{N})$ and its representations in n-c $L_p(\mathcal{N})$ spaces. $\mathcal{M}(\mathcal{N}) \ni \phi \longmapsto p\phi^{1/p} \in L_p(\mathcal{N})$
* two different times: external-dynamical t and internal-kinematic s.
* external time evolution: non-unitary, uniquely associated with given constraints $F(\omega) = f(t)$, and generated by relative entropic updating on $\mathcal{M}(\mathcal{N})$
* internal time evolution: unitary, uniquely associated with every faithful ~~event~~ information state $\omega \in \mathcal{M}(\mathcal{N})$, and generated by the Tomita-Takesaki modular hamiltonian.

X. RECONSTRUCTION OF STANDARD FRAMEWORKS

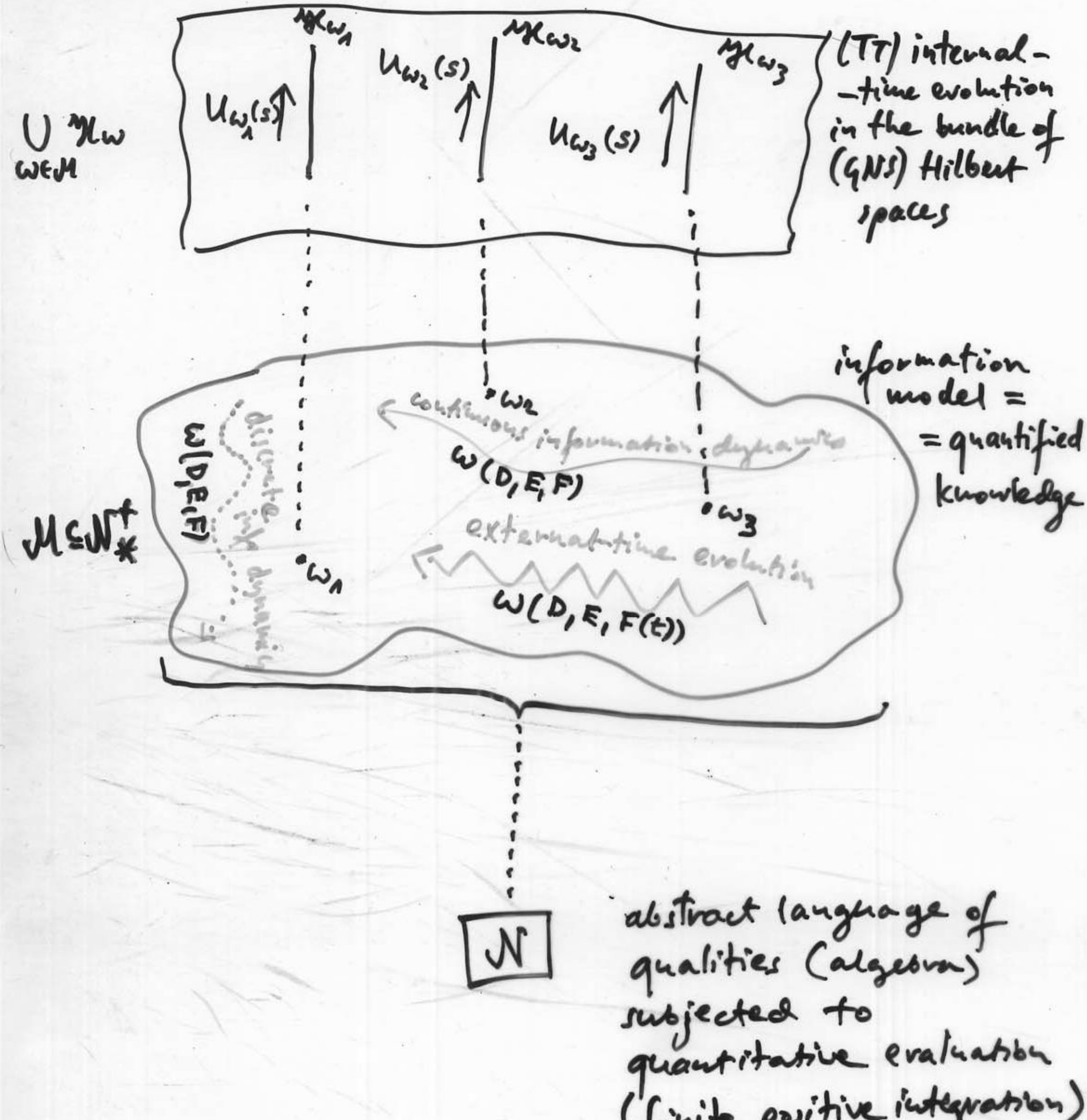
Probability theory:

* if W^*-algebra is commutative, then it naturally generates caDc boolean algebra $\mho$
* all corresponding structures reduce to their commutative counterparts in our new framework for probability theory, except the internal time evolution, which disappears
* our generalisation of probability theory, in appropriate limits, reduces to Borel-Kolmogorov and Bayes-Laplace frameworks

Hilbert space based quantum theory:

* by Gel'fand-Naimark-Segal theorem, each $\omega \in \mathcal{M}(\mathcal{N})$ generates a unique associated Hilbert space $\mathcal{H}_\omega$ equipped with a representation $\pi_\omega(\mathcal{N}) \subseteq \mathcal{B}(\mathcal{H}_\omega)$ of $\mathcal{N}$.
* by Tomita-Takesaki theorem, ~~and~~ if $\omega \in \mathcal{M}(\mathcal{N})$ it faithful, this representation is uniquely equipped with a unitary evolution $U_\omega(s) = e^{-iK_\omega s}$.
* ~~if~~ all representations $\{\pi_\omega(\mathcal{N}) \mid \omega \in \mathcal{M}(\mathcal{N})\}$ are unitary isomorphic (what is the case, by Stone-von Neumann theorem, in orthodox QM), then all fibers of $\bigcup_{\omega \in \mathcal{M}(\mathcal{N})} \mathcal{H}_\omega$ can be identified with a single Hilbert space $\mathcal{H}$
* in such case, the unitary operators generating these isomorphisms become joined with the Tomita-Takesaki evolutions, and recover the ~~orthodox~~ unitary evolution
* the perturbations of this evolution by (already 'renormalised') 'interaction' and the vN-Lüders rule come directly from constrained maximum entropy updating.

XI. PICTURE


$\mathcal{H}_{\omega_1}$
$\mathcal{H}_{\omega_2}$
$\mathcal{H}_{\omega_3}$
$U_{\omega_1}(s)$
$U_{\omega_2}(s)$
$U_{\omega_3}(s)$
$\bigcup_{\omega\in\mathcal{M}} \mathcal{H}_\omega$
(TT) internal-time evolution in the bundle of (GNS) Hilbert spaces
information model = = quantified knowledge
ω_2
continuous information dynamics
$\omega(D, E, F)$
$\omega(D, E, F)$
discrete inf. dynamics
$\mathcal{M} \subseteq \mathcal{N}^+_*$
ω_3
external-time evolution
ω_1
$\omega(D, E, F(t))$
$\mathcal{N}$
abstract language of qualities (algebra) subjected to quantitative evaluation (finite positive integration)

XII. INFORMATION SEMANTICS & INTERSUBJECTIVE INTERPRETATION

Semantics:

* W^*-algebras $\mathcal{N}$ and caDc boolean algebras $\mathcal{V}$ represent the abstract qualitative language used as a reference for intersubjective communication
* information models $\mathcal{M}(\mathcal{N})$ and $\mathcal{M}(\mathcal{V})$ represent the quantified intersubjectively shared knowledge
* information geometry serves for description (in terms of: metrics, connections, divergences, convex sets, ...) and quantification (in terms of L_p space representations) of kinematics and dynamics of information models
* external time is a 'time of becoming' = = an epistemically definable and controllable parameter
* internal time is a 'time of being' = = a mathematical parameter of symmetry arising from representation of qualitative language in context of quantitative knowledge

Interpretation:

* 'Quantum theory' does not govern anything!
* 'Quantum theory' is just a particular mathematical formalism for statistical inference plus particular methods of intersubjective experimentation plus particular methods of relating these two, that are relative to the community of users, which set the range of 'intersubjectivity'
* No paradoxes due to strict denial of ontological idealism.