Quantum electrodynamics and all that

Krzysztof Pachucki

1 Basics

It is the purpose of this small contribution to present an overview of the current status of quantum electrodynamics (QED), open issues and unsolved problems to the large audience not necessarily familiar with sophisticated calculations and experiments performed during the last decades. The complexity in the formulation of bound state problems and the lack of modern textbooks form a difficult to pass barrier. However, the beauty of bound state QED is acknowledged once you perform calculations by yourself.

Quantum electrodynamics as a quantum field theory, has not yet been formulated in a strict mathematical manner. It only exists as a compound of prescriptions for the calculations of physical quantities. These prescriptions are however self consistent. The apparent divergences which arise in the calculations are eliminated through the well established regularization and renormalization procedure. It is the Poincare and gauge symmetry, which uniquely determine QED: no physical quantity may depend on the renormalization scheme. However, what is not well recognized, is the fact that the renormalization procedure works only when asymptotic states are involved, for example for the case of cross sections, bound state energies, or g-factors. Other classes of problems, which involve time dependence, or for example, the shape of natural lines are not yet well formulated within quantum electrodynamics. The renormalization procedure for calculations of processes starting and finishing at some definite time does not exist. There are still many issues to be considered such as infrared divergences or the notion of the wave function in QED. So called infrared divergences are related to the fact that the state of a single electron is not physically distinguishable from the state of a single electron plus long wavelength photons. It affects the meaning of the electron wave function. Moreover the wave function loses the probabilistic interpretation, when relativity is involved. For example, for two particles it would have to depend on two time variables. These problems do not prevent giving precise predictions for static properties of bound states. I will concentrate here on simple systems, such as hydrogen, positronium and anything formed with two particles of the opposite charge.

2 Hydrogen

The spectrum of hydrogen played a crucial role in the development of quantum electrodynamic theory. As it is the simplest of stable atoms, theoretical calculations are the most accurate here. Precise theory has stimulated improved measurements and vice versa. Apart from the well known $2S_{1/2}-2P_{1/2}$ splitting, two photon transitions between S-levels, especially the $1S-2S$ transition, are of main interest. This is because the 2S state is metastable, and the transition to the 1S state has a natural width of 1.3 Hz, as compared to the 2P state, where it is of order 10^8 Hz. The measurement of the 1S-2S transition by Hänsch's group in Garching [1] is the most precise of any optical measurement performed so far. Their result:

$$\nu(1S-2S) = 2\,466\,061\,413\,187\,103(46)\,\mathrm{Hz} \tag{1}$$

with precision of 46 Hz has almost reached the natural width of this transition. There is no hope that theory might be equally accurate, since any theoretical result must involve the electron mass and the fine structure constant. Both of them are known with much less accuracy. Currently this measurement is interpreted as a determination of the Rydberg constant. This is the constant which gives the nonrelativistic transition frequencies in hydrogen:

$$\nu(n \to m) = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{2}$$

A further measurement of a different transition, 2S–6S by Paris group in [2], or 2S–4S by Garching group in [3] leads to the result for such a specific combination, example below

$$\Delta = E(4S-2S) - \frac{1}{4}\,E(2S-1S) \tag{3}$$

which cancels out the nonrelativistic energy. It could be regarded as a test of QED, beside the well known $E(2S-2P)$. The high precision of these measurements has stimulated the intensive development of theoretical calculations. The most interesting were so called recoil corrections, corrections which account for the finite mass of the nucleus. In the Dirac equation one assumes that the nucleus is a static source of the Coulomb field. Finite mass means that the nucleus can move. There does not exist an extension of the Dirac equation which incorporates finite nucleus mass. The use of a reduced mass, instead of the rest mass of the electron, in the Dirac equation is incorrect. For hydrogen like systems it is convenient to perform an expansion in the electron–nucleus mass ratio, and consider each term of this expansion separately. Beautiful formulas were discovered by Shabayev [4] for the first term in the mass ratio, which are valid for any charge of the nucleus, even for hydrogen-like uranium. They have been applied for both analytical and numerical calculations of recoil corrections. Other type of interesting recoil corrections are related to the proton, or in general nucleus, self–energy. As the electron self–energy gives most of the of the Lamb shift, the nuclear self-energy is a tiny correction, diminished by the square of the electron–nucleus mass ratio. Although small, it is nevertheless important in view of high precision hydrogen and deuterium spectroscopy [5]. Moreover it overlaps with the finite nuclear size effect, and a clear distinction between both effects is not possible. Currently, attention is directed toward two-loop corrections, as they give the largest uncertainty in the theoretical predictions. These corrections have very peculiar features, not encountered anywhere else. The power series in $Z\,\alpha$ has very slow convergence [6]. The presence of many logarithmic terms $\ln(Z\,\alpha)$, and the fact the higher order coefficients in this series could be orders of magnitude larger then the leading ones, makes it difficult, if not impossible, to estimate the as yet uncalculated terms in the power series expansion. For that reason attempts to calculate the two-loop correction directly numerically are being made. Very recently, Yerokhin and Shabayev in [7] have been able to complete the two-loop calculations started by Mallampali and Sapirstein few years ago in [8]. However their result is valid only for heavy hydrogen-like ions, and no result exists for hydrogen, except for a few terms in the asymptotic expansion in powers of $Z\,\alpha$. Let us summarize this consideration by a comparison of the current theoretical predictions for the 1S Lamb shift in hydrogen ($Z = 1$) from [6]

$$E_L(1S)_{\rm th} = 8\,172\,816\,(10)(32)\,\mathrm{kHz} \tag{4}$$

with the experimental result from [2] and [3]

$$E_L(1S)_{\rm exp} = 8\,172\,837\,(22)\,\mathrm{kHz} \tag{5}$$

The close agreement indicates that so far QED is a correct theory. The only doubt is with regard to the proton charge radius. It is not yet known with the sufficient accuracy.

In the same way other so called exotic atoms are being studied: muonium, positronium, muonic hydrogen or antiprotonic helium. Their theory does not differ significantly from that in hydrogen, except for positronium. The positronium system is the most beautiful and also the most difficult application of quantum electrodynamic theory. Beautiful, because there is no parameter other than the fine structure constant. It is difficult, because the mass ratio of electron and positron is equal to 1 as long as the CPT theorem is valid, and therefore an analog of Dirac equation for positronium system does not exist. Relativistic corrections could only be calculated by considering the complete QED theory. We will not present details, because they are too difficult. Instead it is more safe to refer to recent works on this subject in [9] and [10].

3 Beyond QED

As the world does not finish with QED, it is natural to investigate effects, usually very small at the atomic scale, coming from weak, strong and gravitational interactions. Weak interactions are clearly detectable because they break parity P symmetry, for a very good review see the book of Khriplovich [11]. The left and right handed chemical molecules may have different vibrational frequencies. States of an atom do not have definite parity, and for example the electric dipole transition between $6S_{1/2}$ and $7S_{1/2}$ in cesium was measured with high accuracy by the Colorado group in [12]. In hydrogen, or better in deuterium, the close lying states of opposite parity $2S_{1/2}$ and $2P_{1/2}$ are being mixed, so one hoped to observe E1 transition $2S_{1/2} \to 1S_{1/2}$. Nevertheless, this effect in hydrogen is too small to be observed. Parity breaking effects are strongly enhanced with Z, the charge of nucleus. Another form of symmetry violation, the combined charge and parity CP symmetry, is the source of asymmetry in the amount of matter and antimatter in our world. On the atomic scale, CP violation leads to the appearance of electric dipole moments for elementary particles or nuclei. These electric dipole moments, or so called Schiff moments of nuclei may induce dipole moments of the whole atom. It is hoped to observe such effects only for heaviest atomic or molecular systems. Current predictions of the standard model, however do not support this hope. Finally, the breaking of CPT symmetry, the combined charge, parity and time reversal symmetry may lead to different transition frequencies for hydrogen and antihydrogen, the atom formed of a positron and antiproton. And once again, the 1S–2S transition, due to its very small width, is the natural candidate for the verification of CPT symmetry.

personal remarks

Author has this chance to work as a theoretician within an experimental group of Ted Hänsch in Garching for three years after finishing his Ph.D. on Lamb shift in hydrogen. Discussions with Martin Weitz, Dietrich Leibfried, Thomas Udem, Jochen Walz, Antoine Weiss and Ted Hänsch have created an atmosphere for intensive scientific work. Let me thank for this nice cooperation.

References

1. M. Niering *et al*, Phys. Rev. Lett. **84**, 5496 (2000)
2. S. Bourzeix, *et al*, Phys. Rev. Lett. **76**, 384 (1996)
3. M. Weitz, A. Huber, F. Schmidt-Kaler, D. Leibfried, and T.W. Hänsch, Phys. Rev. Lett.**72** 328 (1994)
4. V.M. Shabaev, Teor. Mat. Fiz. **63**, 394 (1985) [JETP **37**, 211 (1973)]
5. A. Huber *et al.*, Phys. Rev. Lett. 80, 468 (1998)
6. K. Pachucki, Phys. Rev. A 63, 042503 (2001)
7. V.A. Yerokhin, V.M. Shabaev, physics/0107036
8. S. Mallampali and J, Sapirstein, Phys. Rev. A **57**, 1548 (1998)
9. K. Pachucki, Phys. Rev. A 56, 297 (1997)
10. A. Czarnecki, K. Mielnikov, and A. Yelkhovsky, Phys. Rev. Lett. **82** 311 (1998)

11. I.B. Khriplovich, *Parity Nonconservationin Atomic Phenomena*, Gordon and Breach, Philadelphia (1991)
12. M.C. Noecker, B.P. Masterson, and C.E. Wieman, Phys. Rev. Lett. **61**, 310 (1988); C.S. Wood, *et al.*, Science **275**, 1759 (1997)