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Hyperfine structure of muonic helium

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Abstract

The hyperfine structure of ground state muonic helium is calculated numerically using the variational approach. Comparison to former theoretical and experimental results is made and prospects for the high precision determination of related physical constants are drawn. ($r = r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$)

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Muonic helium is an exotic atom consisting of an electron, muon, and an alpha particle nucleus [1]. It is almost a hydrogenic atom, as the muon and alpha particle can be treated as a soft nucleus as a first approximation. However, when comparison to precision experiment [2,3] is performed correlation effects can not be neglected. The spin–spin interaction between the electron and muon leads to the hyperfine structure. The most precise theoretical analysis so far was performed by Lakdawala and Mohr in [4], where the correlation effects were studied in the perturbation approach. Here we calculate the wave function and energy numerically using the variational method. Our calculations resolve a discrepancy between former theoretical results in [4–6] in favor of that in [4]. Our new, more precise result is ($r = r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$)

$$\langle \pi\, \delta^3(r) \rangle = 0.9857078(1). \tag{1}$$

For hydrogen this expectation value is equal to 1, so these system do not differ significantly. The nonrelativistic hamiltonian for muonic helium in atomic units is:

$$H = \frac{\boldsymbol{p}_1^2}{2} + \frac{m_e}{m_\mu}\frac{\boldsymbol{p}_2^2}{2} + \frac{m_e}{m_\alpha}\frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2} - \frac{2}{r_1} - \frac{2}{r_2} + \frac{1}{r} \tag{2}$$

where electron–muon and electron–alpha mass ratios are [8]:

$$\frac{m_e}{m_\mu} = 0.00483633210\,, \tag{3}$$

$$\frac{m_e}{m_\alpha} = 0.00013709335611\,, \tag{4}$$

and uncertainties in above mass ratios are irrelevant.

In the numerical calculations we follow the approach advanced by Korobov in [7]. We use exponential basis set of the form

$$\phi(r_1, r_2, r) = \sum_i v_i\, e^{-\alpha_i\, r_1 - \beta_i\, r_2 - \gamma_i\, r} \tag{5}$$

with real α, β, γ chosen randomly and homogeneously between some minimal and maximal values. It was essential to use at least two such basis sets with the second having much larger minimal and maximal values, due to strong electron-muon correlations at small distances. All these values could be determined by minimization of the ground state energy. Because of numerically instability of the linear independence of this basis set we had to use multi precision package by Bailey [9] with $N = 48$ digits. Results of these calculations versus size of the basis set are presented in Table I. The value of the hyperfine splitting is obtained from the Fermi formula:

$$\delta\nu = \frac{2\,\alpha}{3}\,\frac{g_e\, g_\mu}{m_\mu\, m_e}\,(m_e\,\alpha)^3\,\langle\phi|\pi\,\delta^3(r)|\phi\rangle \tag{6}$$

where the expectation value of the delta function is in atomic units, as in Table I. The electron and muon g-factors g_e and g_μ are $g = 2(1 + \alpha/(2\,\pi))$: higher order QED corrections to g-factors as well as other QED corrections to hyperfine structure will be neglected. Our final result is (values of physical constants from [8])

$$\delta\nu_{\rm th} = 4464.54(60)\ \mathrm{MHz} \tag{7}$$

where the uncertainty is a rough estimate of higher order QED effects. It agrees well with two experimental values obtained: at Swiss Institute for Nuclear Research (now Paul Scherrer Institute)

$$\delta\nu_{\rm exp} = 4464.95(6)\ \mathrm{MHz}\,, \tag{8}$$

and at Los Alamos Meson Physics Facility

$$\delta\nu_{\rm exp} = 4465.004(29)\ \mathrm{MHz}\,. \tag{9}$$

The purpose of this paper is to demonstrate the availability of high precision results for the muonic helium atom. We think that the ultimate theoretical precision could be almost as high as for hydrogenic systems such as muonium. The finite size of alpha particle has almost negligible influence on hyperfine structure in muonic helium as compared to the hydrogen atom. Moreover, the calculation of QED effects in three body systems could be performed in higher orders of perturbation theory as presented in [11] for the case of helium 2^3S_1 Lamb shift. Since nowadays the experimental precision could be improved by factor of 50 or so [10], muonic helium may serve as a complementary to muonium tool, for high precision determination of fine structure constant α, electron–muon mass ratio or even for the verification of the CPT symmetry.

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TABLES

TABLE I. Nonrelativistic ground state energy and the expectation value of $\pi\,\delta^3(r)$ versus length of the exponential basis set, in electron mass units

N	energy	$\langle \pi\,\delta^3(r) \rangle$
100	-402.63726963200300530	0.985756016
300	-402.63726963219002850	0.985706929
600	-402.63726963219007283	0.985707641
900	-402.63726963219007444	0.985707734
1200	-402.63726963219007450	0.985707756
∞	-402.6372696321900745(1)	0.9857078(1)