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# Proton structure effects in muonic hydrogen

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## Abstract

Proton structure effects, including finite size, polarizability, and self–energy are considered and their influence on energy levels of muonic hydrogen is recalculated. A new theoretical prediction for the Lamb shift is presented together with improved values of all known QED contributions for muonic hydrogen. This would allow for the substantial improvement of the proton charge radius determination.

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Precision tests of quantum electrodynamics (QED) in atomic systems have reached the level where nuclear structure effects become significant. However, the lack of accurate data on the low energy structure functions of the nucleus strongly limits theoretical predictions. A well known example is the hydrogen hyperfine structure splitting, where the measurement is 6 orders of magnitude more precise than the current theoretical predictions. Another example is the Lamb shift in hydrogen, where inaccuracy in the proton charge radius dominates other theoretical uncertainties. For many years this has been a motivation to study pure QED systems like muonium and positronium, where strong interaction effects are negligible or well estimated at the level of interest. A new possibility for the improvement of QED tests on bound systems has been opened by the proposed measurement of the Lamb shift in muonic hydrogen [1]. Since the muon mass is 200 times larger than the electron mass, it is far more sensitive to the distribution of charge and magnetic moment, or the polarizability of the proton. The main goal of the measurement of the Lamb shift in muonic hydrogen is a precise determination of the mean square proton charge radius. It will verify older measurements of the proton charge radius based on elastic electron–proton scattering. The results of these experiments had to be extrapolated to very low momentum transfer, and therefore might be not very reliable. In contrast, the muonic Lamb shift measurement aims to improve the precision of the proton charge radius by at least 10 times. However, it would require the calculation of all other contributions with comparable or better precision. A few years ago, in a summary of all QED effects known at that time on muonic hydrogen energy levels [2], we concluded that the uncalculated three–loop vacuum polarization limited the precision of theoretical predictions by 0.01 meV, and pointed attention to the proton polarizability effect, which was also not calculated at that time. Progress on preparations for the measurement of the muonic hydrogen Lamb shift has stimulated further theoretical work. The rather difficult three–loop vacuum polarization contribution has recently been calculated by Kinoshita and Nio [3], with the result

$$\Delta E(2P - 2S) = 0.007\,6 \text{ meV}\,. \tag{1}$$

A new contribution coming from hadronic vacuum polarization has been studied by Friar and collaborators [4], who obtained the following result

$$\Delta E(2P - 2S) = 0.011\,3(3) \text{ meV}\,. \tag{2}$$

The proton polarizability correction has been analyzed by Rosenfelder in [5]. His estimate for this effect is

$$\Delta E(2P - 2S) = 0.017(4) \text{ meV}\,. \tag{3}$$

Another recent work [6] gives a similar result of $\sim 0.018$ meV. The calculation of proton polarizability effect is affected by the lack of precise data on low–energy proton structure functions. In this paper we present another estimate of this effect together with a complete review of all other proton structure contributions. We would like to emphasize the importance of the proton self–energy effect and the related problem with the meaning of mean square charge radius. In other words, we present the relation of charge radius as obtained from atomic spectroscopy measurements with that based on electron scattering data.

As is well known, the shift of atomic energy levels due to the finite charge distribution of the atomic nucleus is given by the formula

$$E_{FS} = \frac{2}{3n^3}\,\alpha^4\,\mu^3\,\langle r^2\rangle\,\delta_{l0}\,, \tag{4}$$

which gives $-3.862(108)$ meV contribution to the 2P–2S splitting, were we used $r = 0.862(12)$ fm from [7]. From this one concludes that a measurement of muonic hydrogen Lamb shift with a precision of 0.01 meV will lead to a tenfold improvement in the proton charge radius. This formula in (4) accounts for most of the proton structure effects. Any correction beyond that is much smaller and is usually neglected for light (electronic) atoms. It is our aim to review them in the context of muonic hydrogen Lamb shift, where they are much more significant.

Since the leading contribution to the 2P-2S splitting comes from the electron vacuum polarization (e.v.p.), the second order corrections from combined finite size and e.v.p. effects are nonnegligible. They have been calculated in [2]; here we present a slightly more accurate result

$$\Delta E(2P-2S) = -r^2\,0.0282 = -0.0209(6)\ \mathrm{meV}\,. \tag{5}$$

Further corrections are due to pure photon exchange terms only. The $O(m\,\alpha^5)$ correction is given by the two–photon scattering amplitude with external momenta on mass shell; see Eq. (7). It is the main contribution, which we analyze in this work. However, let us consider first the small corrections beyond this two–photon exchange approximation, namely those of order $m\,\alpha^6$. Since they are small, it is sufficient to calculate them in the external field approximation, or in other words in the limit of infinite nucleus mass. The energy levels are obtained then from the Dirac equation in the potential of the finite size nucleus. They have been considered in detail by Friar in [9]. In the logarithmic approximation they are given by two terms

$$E = E_{FS}\,\alpha^2\,\ln\alpha\left(\frac{2}{3}\,\mu^2\,\langle r^2\rangle - 1\right) \tag{6}$$

and contribute $-0.000\,9$ meV to the 2P-2S splitting and thus are almost negligible.

The correction given by two–photon exchange is the principal concern of this paper. It is given by the following expression

$$\begin{aligned} E &= -\frac{e^4}{2}\,\phi^2(0)\int\frac{d^4q}{(2\,\pi)^4\,i}\,\frac{1}{q^4}\left[T^{\mu\nu}-t^{\mu\nu}(M)\right]t_{\mu\nu}(m) \\ &= -2\,e^4\,\phi^2(0)\,\frac{m}{M}\int\frac{d^4q}{(2\,\pi)^4\,i}\,\frac{(T_2-t_2)(q^2-\nu^2)-(T_1-t_1)\,(q^2+2\,\nu^2)}{q^4\,(q^4-4\,m^2\nu^2)}\,, \end{aligned} \tag{7}$$

where

$$\begin{aligned} T^{\mu\nu} &= -i\int d^4q\,e^{i\,q\,(x-x')}\,\langle P|T\,j^\mu(x)\,j^\nu(x')|P\rangle \\ &= -\left(g^{\mu\nu}-\frac{q^\mu\,q^\nu}{q^2}\right)\frac{T_1}{M}+\left(t^\mu-\frac{\nu}{q^2}\,q^\mu\right)\left(t^\nu-\frac{\nu}{q^2}\,q^\nu\right)\frac{T_2}{M}\,, \end{aligned} \tag{8}$$

$t=(1,0,0,0)$, $P=Mt$ is a proton momentum at rest and $\nu=q^0$. For a point–like proton $T^{\mu\nu}\equiv t^{\mu\nu}(M)$ and

$$t^{\mu\nu}(M) = \mathrm{Tr}\Big[\gamma^\mu \frac{1}{\not p - M}\,\gamma^\nu\,\frac{\gamma^0 + I}{4}\Big] + (q \to -q)\,. \tag{9}$$

From this one obtains

$$t_1 = -\frac{4\,M^2\,\nu^2}{q^4 - 4\,M^2\,\nu^2}\,, \tag{10}$$

$$t_2 = \frac{4\,M^2\,q^2}{q^4 - 4\,M^2\,\nu^2}\,. \tag{11}$$

Since the amplitude $T^{\mu\nu}$ does not have any singularities at small $q^2$ the following holds:

$$T_2 = O(q^2)\,, \tag{12}$$

$$T_1 + \frac{\nu^2}{q^2}\,T_2 = O(q^2)\,. \tag{13}$$

The off–shell (spin averaged) forward Compton amplitude $T^{\mu\nu}$ of the proton is not directly measured. However, it can be expressed in terms of its imaginary part through dispersion relations. Before using them, one notices that the integral in Eq. (7) is infrared divergent. This requires subtraction of the leading finite size term, which has already been accounted for. It also requires an additional subtraction of the proton self–energy term. Since it changes the analytic behavior of $T_1$ and $T_2$ at small $p^2 - M^2$, (see Eq. (20)), this proton self–energy could be only partially accounted for in the proton formfactors, as a contribution to the anomalous magnetic moment or the charge radius. In our opinion this issue is not generally known, we repeat here the analysis from our former paper on radiative recoil corrections and correct some minor misprints.

If we assume a point-like proton the contribution of the proton self–energy to the Lamb shift of S-states is [10]

$$E = \frac{\alpha^5\,\mu^3}{\pi\,n^3\,M^2}\,\left[\left(\frac{10}{9} + \frac{4}{3}\,\ln\frac{M}{\mu\,\alpha^2}\right) - \frac{4}{3}\,\ln k_0(n)\right]\,. \tag{14}$$

For a *true* proton there is a finite size correction as given by Eq. (4). The problem is that the proton self-energy is modified by, and modifies as well, the finite size effect. Therefore some corrections might be counted twice. To incorporate the correction (14) unambiguously we must precisely specify what is the nuclear mean square charge radius. It is usually defined through the Sachs formfactor

$$\frac{\langle r^2\rangle}{6} = \frac{\partial G_E(q^2)}{\partial(q^2)}\bigg|_{q^2=0} \tag{15}$$

which is not correct at our precision level, because the radiative correction to $G_E$ is infrared divergent, which can be regulated with a spurious photon mass. Following [8] we therefore propose a different definition using the forward scattering amplitude described by $T^{\mu\nu}$, or more precisely by its longitudinal component $T_L$ defined by

$$T_L \equiv \Big(1 - \frac{\nu^2}{q^2}\Big)T_2 - T_1\,. \tag{16}$$

For our purpose we consider a nonrelativistic limit $\nu \sim q^2$ and $p^2 - M^2 = (P+q)^2 - M^2 \approx 0$ of $T_L$. For a point-like particle without radiative corrections $T_L$ is

$$T_L \approx M\,\mathrm{Tr}\left[\gamma^0 \frac{1}{\not p - M}\gamma^0 \frac{(\gamma^0 + I)}{4}\right] + (q \to -q) \approx \frac{2\,M^2}{p^2 - M^2} + (q \to -q)\,, \tag{17}$$

where $p = P + q$. With a finite size particle

$$\gamma^\mu \to \Gamma^\mu = \gamma^\mu\, F_1 + i\frac{\sigma^{\mu\nu}}{2M}\, q_\nu\, F_2\,, \tag{18}$$

$T_L$ acquires a correction

$$\Delta T_L \approx \frac{2\,M^2}{p^2 - M^2}[G_E^2(q^2) - 1] + (q \to -q) \approx \frac{2\,M^2}{p^2 - M^2}\, q^2\, \frac{\langle r^2\rangle_{bare}}{3} + (q \to -q)\,, \tag{19}$$

where $G_E = F_1 + \frac{q^2}{4\,M^2}\, F_2$ is an electric formfactor. The radiative corrections for a point-like particle [8] in the nonrelativistic limit are

$$\Delta T_L = \frac{\alpha}{\pi}\, \frac{q^2}{p^2 - M^2}\, \left(\frac{10}{9} + \frac{4}{3}\, \ln \frac{M^2}{M^2 - p^2}\right) + (q \to -q)\,. \tag{20}$$

We define $\langle r^2\rangle$ by the following equation that describes the low-energy behavior of the forward scattering amplitude

$$T_L - t_L \approx \frac{q^2}{p^2 - M^2}\left(\frac{4\,\alpha}{3\,\pi}\, \ln \frac{M^2}{M^2 - p^2} + \frac{2}{3}\, M^2\, \langle r^2\rangle\right) + (q \to -q)\,. \tag{21}$$

We expect that for any nucleus the logarithmic term above will be the same, since it is only related to the fact that nucleus has a charge, and does not depend on other details like the spin. The associated correction to the energy for S-states has the form

$$\Delta E = \frac{2}{3\,n^3}\, \alpha^4\, \mu^3 \langle r^2\rangle + \frac{4\,\alpha^5}{3\,\pi\,n^3}\, \frac{\mu^3}{M^2}\, \left[\ln\left(\frac{M}{\mu\,\alpha^2}\right) - \ln k_0(n)\right]\,. \tag{22}$$

The small second term in the above equation gives $-0.0099$ meV for the 2S state in $\mu$H. For P-states the proton self–energy contributes only through the Bethe log. The correction to the anomalous magnetic moment is already taken into account in the calculation of relativistic effects as given by the Breit Hamiltonian.

We now return to the forward scattering amplitude and its associated correction to the energy of $\mu$H. We can neglect here QED effects on the structure functions, since they are $\alpha$ times smaller. Otherwise dispersion relations for $T_1$ and $T_2$ as in Eq. (29,30) will not be correct. After neglect of QED effects one can expect a separated pole at $2M\nu = q^2$ that is due to the elastic contribution. It is obtained with the help of the proton elastic formfactors $F_1$ and $F_2$; see Eq. (18). One derives for $T_1$ and $T_2$ the following expressions

$$T_1^B = -\frac{1}{(q^4 - 4\,M^2\,\nu^2)}\,(4\,M^2 \nu^2 F_1^2 + 2\,q^4\,F_1\,F_2 + q^4\,F_2^2) \tag{23}$$

$$T_2^B = \frac{1}{(q^4 - 4\,M^2\,\nu^2)}\,(4\,M^2\,q^2\,F_1^2 - q^4\,F_2^2)\,. \tag{24}$$

Any contribution in the two photon exchange beyond this second order Born term is considered to be due to the proton polarizability. The correction to the energy as given by Eq. (7) shows an infrared singularity, which is due to the finite size effect, already accounted for at the lower order in $\alpha$. One first subtracts this singularity and then integrates over $q$. With the commonly used dipole parametrization of proton formfactors and $\Lambda^2 = 0.71$ GeV$^2$ one obtains $\Delta E = 0.018$ meV. However, as pointed out in [1], this result is not correct, because this dipole parametrization corresponds to the proton radius $r = 0.81$ fm, which differs significantly from the value $r = 0.862$ fm. Instead, we use the parametrization by Simon *et al.* [7], with the result

$$\Delta E = 0.0232(10)\,\mathrm{meV} = r^3\, 0.0363\,. \tag{25}$$

The parametrization in [7] is not correct starting from $Q^2 \equiv -q^2 \approx 1$ GeV since for large $Q^2$ it behaves like $Q^{-2}$ instead of $Q^{-4}$; however this does not influence the result at the level of interest. This $r^3$ dependence in Eq. (25) is only an approximate dependence. This dependence become exact in the large nucleus mass limit with the dipole parametrization of proton formfactors. The result, 0.0232(10), was obtained assuming $r = 0.862(12)$ fm. If the proton radius $r$ from $\mu$H measurement turns out to be significantly different, this result should be adjusted with the improved parametrization of proton formfactors.

Let us assume now that the second order Born contribution is subtracted from $T_1$ and $T_2$. The proton polarizability correction $E_{POL}$, as given by Eq. (7), is split into two parts

$$E_{POL} = E_1 + E_2\,, \tag{26}$$

$$E_1 = -2\, e^4\, \phi^2(0)\, \frac{m}{M} \int \frac{d^4 q}{(2\,\pi)^4\, i}\, \frac{q^4 + 2\,\nu^4}{q^6 (q^4 - 4\, m^2\, \nu^2)}\, T_2(\nu, q^2)\,, \tag{27}$$

$$E_2 = 2\, e^4\, \phi^2(0)\, \frac{m}{M} \int \frac{d^4 q}{(2\,\pi)^4\, i}\, \frac{q^2 + 2\,\nu^2}{q^4\, (q^4 - 4\, m^2\, \nu^2)} \left( T_1(\nu, q^2) + \frac{\nu^2}{q^2}\, T_2(\nu, q^2) \right). \tag{28}$$

The first, dominant, part can be reliably calculated, while the calculation of the smaller second part will require additional assumptions. This second part was usually neglected in studies of the polarizability effect in composite nuclei; see for example [11]. The imaginary part of $T_i$ as a function of $\nu$ at fixed $q^2$ is obtained from the inclusive cross section $\gamma^* + p \rightarrow X$. The real part could be restored from the imaginary one using the following dispersion relations:

$$T_2(\nu, q^2) = -\int_{\nu_{th}^2}^{\infty} d\nu'^2\, \frac{W_2(\nu', q^2)}{\nu'^2 - \nu^2}\,, \tag{29}$$

$$T_1(\nu, q^2) = T_1(0, q^2) - \nu^2 \int_{\nu_{th}^2}^{\infty} \frac{d\nu'^2}{\nu'^2}\, \frac{W_1(\nu', q^2)}{\nu'^2 - \nu^2}\,, \tag{30}$$

where $\nu_{th}$ is the threshold value of $\nu$ for the production of $\pi$ mesons. These formulas are obtained from [12], by multiplying the right hand side by $-1/2$, due to the different definition of $T_i$ in our work. While $W_2$ for the proton behaves like $1/\nu$ at large $\nu$, $W_1$ goes like $\nu$, and thus the dispersion relation in (30) contains a subtraction at $\nu = 0$. We have assumed here that $T_2$ has an asymptotic behavior similar to $W_2$, i.e. vanishes in the large $\nu$ limit. However, there is not too much information on $T_1(0, q^2)$. One recognizes that in the limit of small $q^2$ it is given by the magnetic polarizability $\beta_M$

$$\lim_{q^2\to 0}\frac{T_1(0,q^2)}{q^2}=\frac{M}{\alpha}\,\beta_M\,, \tag{31}$$

which amounts to $\beta_M = 1.56(57)\cdot 10^{-4}$ fm$^3$, see [13]. Since there is no experimental data, we assume here that the $q$-dependence is governed by the square of the elastic formfactor, namely

$$\beta_M(Q^2)=\beta_M\,\frac{\Lambda^8}{(\Lambda^2+Q^2)^4} \tag{32}$$

with $\Lambda^2 = 0.71$ GeV$^2$. Using equations (27-31) one derives the following expressions for $E_1$ and $E_2$.

$$E_1 = -\alpha^2\,\frac{\phi^2(0)}{m\,M}\int_{\nu_{th}^2}^{\infty}d\nu^2\int_0^{\infty}\frac{dt}{t^2}\,g(\nu,t)\,W_2(\nu,-t)\,, \tag{33}$$

$$\begin{aligned}E_2 = &-\alpha^2\,\frac{\phi^2(0)}{m\,M}\int_{\nu_{th}^2}^{\infty}d\nu^2\int_0^{\infty}\frac{dt}{t^2}\,f(\nu,t)\,W_2(\nu,-t)\,\frac{1}{1+R}\left(1-R\,\frac{\nu^2}{t}\right)\\ &+\alpha^2\,\frac{\phi^2(0)}{m\,M}\int_0^{\infty}dt\,h(t)\,\frac{M}{\alpha}\,\beta_M(t)\,,\end{aligned} \tag{34}$$

where

$$g(\nu,t)=\frac{4}{\pi}\int_0^1 dx\,\sqrt{1-x^2}\;\frac{(1+2\,x^4)}{2\,(x^2+\frac{t}{4\,m^2})(x^2+\frac{\nu^2}{t})}\,, \tag{35}$$

$$f(\nu,t)=\frac{t}{2\,\nu^2}\,\frac{4}{\pi}\int_0^1 dx\,\sqrt{1-x^2}\;\frac{x^2(1+2\,x^2)}{(x^2+\frac{t}{4\,m^2})(x^2+\frac{\nu^2}{t})}\,, \tag{36}$$

$$h(t)=\left[1+\left(1-\frac{t}{2\,m^2}\right)\left(\sqrt{\frac{4\,m^2}{t}+1}-1\right)\right]\,, \tag{37}$$

and $R$ is equal to $R = W_L(\nu,q^2)/W_1(\nu,q^2) = \sigma_L/\sigma_T$, the ratio of longitudinal to transverse cross sections, a measurable quantity. A recent work [14] by E143 collaboration presents the most recent parametrization of $R$ in the momentum range $Q^2 \geq 1$ GeV. This parameterization shows a weak dependence of $R$ on $\nu$ for small $Q^2$, of order a few GeV; see for example Figure 3 of this work. Moreover $R$ is between $0.3-0.4$. However, there is no precise data on the most important region for us of $Q^2 < 1$ GeV. One knows from conditions (12,13), that $R\sim Q^2$ at small $Q^2$ and constant $\nu$. The only known paper that comments on R, by Brasse *et al.* [15], shows that R is of order 10% at the resonance region. Since, as will be shown later, $E_2$ is small, and we do not have precise data on $R$ at the resonance region we simply neglect $R$ in the expression for $E_2$ in (34). We expect this approximation will not alter significantly the final result for the polarizability correction. Various parametrizations of the structure function $W_2$ have been presented in the literature. We have chosen a parameterization which matches the $q^2 = 0$ (photoproduction) limit, and is also correct for the region of $\nu$ and $Q^2$ of order a few GeV. We use the parametrization from [15] for the resonance region. It is given by a compact formula and a long table of numbers, see Table 1 of [15]. For $\nu$ close to the threshold for the $\gamma^* + p^+ \to \pi^+ + n$ process, we assume a behavior

$\sqrt{W - W_{th}}$ and match the value at $W = 1.110$ GeV, as given in [15]. The photoproduction of $\pi^0$ can be neglected close to this threshold value. Out of the resonance region, $W > 1.990$ GeV, we use an ALLM97 parametrization [16]. With these parametrizations the integrals in (33,34) are performed numerically with the results

$$E_{POL} = E_1 + E_2 = -\alpha^2 \,\frac{\phi^2(0)}{m\,M}\,(1.82 + 0.12 - 0.26) = -\alpha^2\,\frac{\phi^2(0)}{m\,M} 1.68(33)\,, \tag{38}$$

where the uncertainty of 0.33 forms 20% of the final result and is an estimate due to approximations and assumptions performed during this derivation, namely: unknown $R$, unknown $Q^2$ dependence of $\beta_M$, and inaccuracy of ALLM97 parametrization at low $Q^2$ and $\nu$. The contribution to the (2P-2S) splitting of $n = 2$ states is

$$E_{POL} = 0.012(2)\,\mathrm{meV}\,. \tag{39}$$

It is slightly lower than the result obtained by Rosenfelder in [5]. We think it is due to the approximate treatment of the $Q^2$-dependence in his work. Other contributions to the Lamb shift in muonic hydrogen are presented in Table I. It is an improved version of the former Table I in [2]. All QED corrections are recalculated with a better precision. The final uncertainty for all QED corrections excluding finite size effect is 0.003 meV, what is relevant for the determination of the proton charge radius from the measurement of the 2S-2P transition frequency in muonic hydrogen. The final result for the Lamb shift differs from that in [2] by more than the error estimate there. This is due to a mistake in the sum in Table I in [2] and to the inclusion of new terms: proton polarizability, third order electron v.p. and the hadronic v.p.. It should be noted here that the same effects should be included in the hydrogenic Lamb shift with the new value of the proton charge radius determined from $\mu H$. This has not been necessary so far, since in the electron scattering measurements of the proton radius these effects have not been excluded. The uncertainty in the final result has two main sources: the proton polarizability and the estimate of higher order QED effects. While the first one is rather difficult to improve, the second source of uncertainty, higher order QED corrections, can be calculated. By these higher corrections we mean the diagrams presented in Figure 1. Diagrams in the second row have been calculated in an approximate way in [2], and are named in Table I as a muon self–energy with electron v.p. We note that further theoretical work will also be necessary on the hyperfine splitting of S-levels, since the proton polarizability effect is also present there. In summary, having now precise theoretical predictions, the measurement of the 2P-2S transition frequency in muonic hydrogen will lead to an improved value of the proton charge radius.

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## TABLES

| correction | value in meV | uncertainty |
|---|---|---|
| leading order e.v.p. | 205.0074 | |
| rel. corr. to e.v.p . | 0.0594 | |
| double e.v.p. | 0.1509 | |
| two-loop e.v.p. | 1.5079 | |
| three-loop e.v.p. | 0.0076 | |
| muon self-energy + muon v.p. | $-0.6677$ | |
| muon self-energy with e.v.p. | $-0.006$ | 0.001 |
| recoil of order $\alpha^4$ | 0.0575 | |
| recoil of order $\alpha^5$ | $-0.0450$ | |
| recoil of order $\alpha^6$ | 0.0003 | |
| proton self energy | $-0.0099$ | |
| leading finite size of order $\alpha^4$ | $-r^2\,5.1974 = -3.862$ | $0.108^*$ |
| finite size of order $\alpha^5$ | $r^3\,0.0363 = \quad 0.0232$ | 0.0010 |
| finite size of order $\alpha^6$ | $-0.0009$ | 0.0003 |
| e.v.p. with finite size | $-r^2\,0.0282 = -0.0209$ | $0.0006^*$ |
| hadronic v.p. | 0.0113 | 0.0003 |
| proton polarizability | 0.012 | 0.002 |
| estimate of uncalculated terms | | 0.002 |
| sum of corrections to Lamb shift in $\mu$H with $r = 0.862(12)$ fm | $206.085(3) - r^2\,5.2256 + r^3\,0.0363 = 202.225(108)$ meV | |

TABLE I. Summary of results for corrections to the Lamb shift in muonic hydrogen, e.v.p. denotes the electron vacuum polarization. QED corrections are calculated according to Ref. [17] and [2]. A subscript * denotes that this uncertainty comes only from the uncertainty in the electron scattering determination of the proton charge radius.

## FIGURES

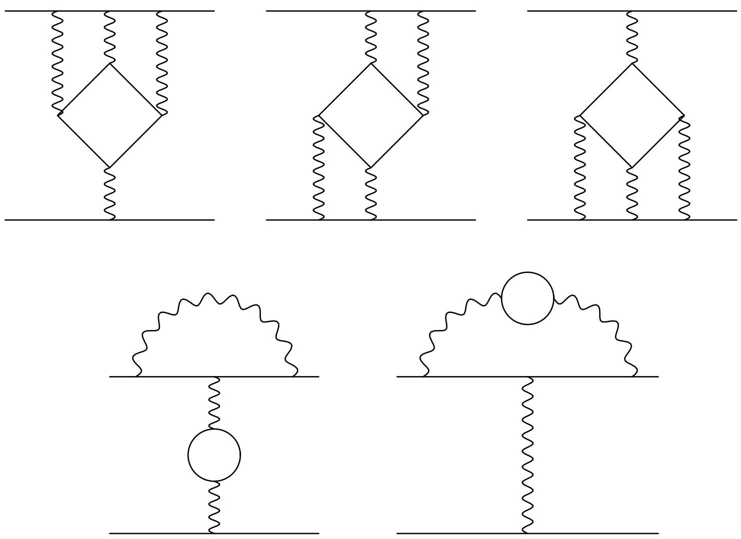

FIG. 1. Higher order diagrams contributing to muonic hydrogen Lamb shift. The upper and lower horizontal lines denote the muon and the proton respectively. A closed line denotes the electron loop.