

# Proton charge radius and the perturbative quantum electrodynamics

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We argue that the proton charge radius conundrum can be resolved by weakening the assumption of perturbative formulation of quantum electrodynamics within the proton.

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The proton radius conundrum refers to the disagreement between the proton mean square charge radius determined from muonic hydrogen and from electron-proton (e-p) systems: atomic hydrogen and e-p elastic scattering. The muonic hydrogen result [1, 2] of  $r_p = 0.84087 \pm 0.00039$  fm is about 13 times more precise and much different than the CODATA 2010 [3] result of  $r_p = 0.8775 \pm 0.0051$  fm. The CODATA analysis includes atomic hydrogen and the precise cross section measurements of Bernauer et al. [4], which give  $r_p = 0.879 \pm 0.008$  fm. For a recent review, see [5].

It seems [6], that the most likely explanations are related to: (i) novel physics beyond the Standard Model that differentiates  $\mu$ -p and e-p interactions, (ii) novel two-photon exchange effects that differentiates  $\mu$ -p and e-p interactions, (iii) errors in the e-p experiments. Regarding the Standard Model, various extensions have been proposed to explain this discrepancy, by introducing scalar, pseudoscalar or vector particles. The range of available parameters is very limited, if not ruled out, mostly due to various precision tests of the Standard Model, such as  $g - 2$  of the muon.

Here, we assume that the Standard Model is valid, all the above experiments are correct, the only incorrect assumption is an implicit one of the validity of the perturbative QED within the proton. The elastic scattering off the proton by electrons or muons within the Born approximation is described by the electric and magnetic formfactors. Electron interacts with the proton by a single photon exchange, and what matters is the electromagnetic current of the proton which can be decomposed into two independent formfactors. Certainly, the one photon exchange approximation can be supplemented by the exchange of two-photon, soft photon radiation, photon loops, etc. However, this requires justification of all assumptions to build quantum electrodynamics in the presence of strongly interacting particles.

There is no precise formulation in the literature of Quantum Electrodynamics in the presence of hadrons. Let us therefore briefly describe what we think is generally assumed. It is assumed that it makes sense to speak about (chargeless) bare proton and bare neutron, i.e. hadrons with neither electromagnetic nor weak interactions. Such objects would have slightly different masses and different electromagnetic formfactors than the real ones. Now, let us build QED assuming that we have just one proton. The basic assumption, although usually not spelled explicitly, is that one can use perturbation theory to account for electromagnetic interactions. We now want to describe, for example, an electron scattering off

a proton using standard perturbation theory. The single photon exchange between the electron and this bare proton is supplemented by all the proton dressing diagrams, which change the bare mass and bare formfactors into physical mass and physical formfactors. The two-photon exchange diagram will include in addition inelastic contributions. In such a perturbative picture the electron interacts with the proton by the photon exchange and this interaction can be fully described by the electromagnetic current  $J_p^\mu$  of the proton. However, this has never been verified. If the perturbative picture of quantum electrodynamics within the proton fails, one may expect in addition, a different form of an effective electron-proton interaction. A naive example of such interaction is in the elastic positron scattering off an atomic helium in the ground state. Besides the photon exchange, there is the annihilation type of interaction. In analogy one cannot in principle exclude that the effective electron proton interaction may contain nonlocal terms beyond the photon exchange, like for example

$$\bar{\psi}_e(x) e^{-i e \int_x^y A_\mu dz^\mu} \psi_e(y) \bar{\psi}_p(Z) \psi_p(Z) F(x - Z, y - Z) \quad (1)$$

with implicit insertions of Dirac gamma matrices and with  $F$  being some proton formfactor. Another example can be

$$J_p^\mu(x) j_\mu(x) \quad (2)$$

the product of vector currents with the coupling constant, that is different for the electron and the muon. Existence of such interactions is not against the Standard Model and would explain different charge radius seen by the proton and by the muon, because in the nonrelativistic limit they give a local interaction of the type  $\delta^3(r)$ , as the charge radius. The two (and more) hard photon exchange diagrams lead to additional local type of interactions but they are much too small to explain the proton charge radius discrepancy between  $e - p$  and  $\mu - p$  type of experiments. Can the existence of such additional interactions be ruled out on the basis of current experiments? It seems that not at all. The ratio  $G_M/G_E$  of proton formfactors measured by unpolarized electron-proton scattering is different from the ratio obtained by polarization transfer electron-proton scattering, and this difference grows with  $Q^2$ , square of the momentum exchange. The difference has been attributed to the two-photon exchange which contribute more significantly in Rosenbluth separation technique, used to extract formfactors from unpolarized scattering [7]. The evaluation of two-photon exchange diagrams in e-p scattering is problematic, due to insufficient knowledge of inelastic

structure functions or, what we would prefer, lack of correct approach to QED on the proton background. So the proton charge radius difference can be attributed to the existence of additional forms of the effective lepton-proton interaction. An agreement for the hyperfine splitting in H and  $\mu$ H between theoretical predictions and experimental values, suggests that these extra forces are spin independent at low  $Q^2$ , as the mentioned difference for the  $G_M/G_E$  formfactors. We are not aware of any further experimental verifications of the validity of perturbative QED within the proton but we see such possibility by comparison of the electron to positron elastic scattering off the proton. If there are nonperturbative terms beyond the proton formfactors the proton charge radius, as seen by positron, can be different from that seen by the electron. Observation of this difference would be a clear demonstration that current phenomenological treatment of lepton-hadron interactions is incomplete.

In summary, if the usual perturbative assumption does not hold, the effective interactions of leptons with protons, or more generally hadrons, may include additional non-standard

corrections which apparently may violate universality of electromagnetic interactions and explain the observed discrepancies.

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