Introduction to Intensity Interferometry in Particle Physics

P. A. Kurashvili

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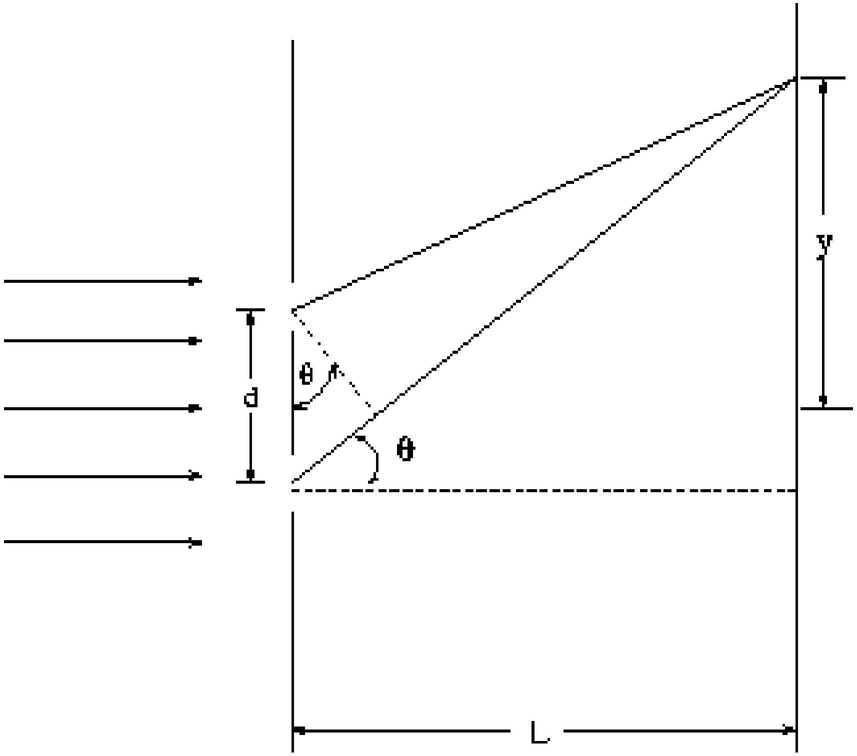
Optics:

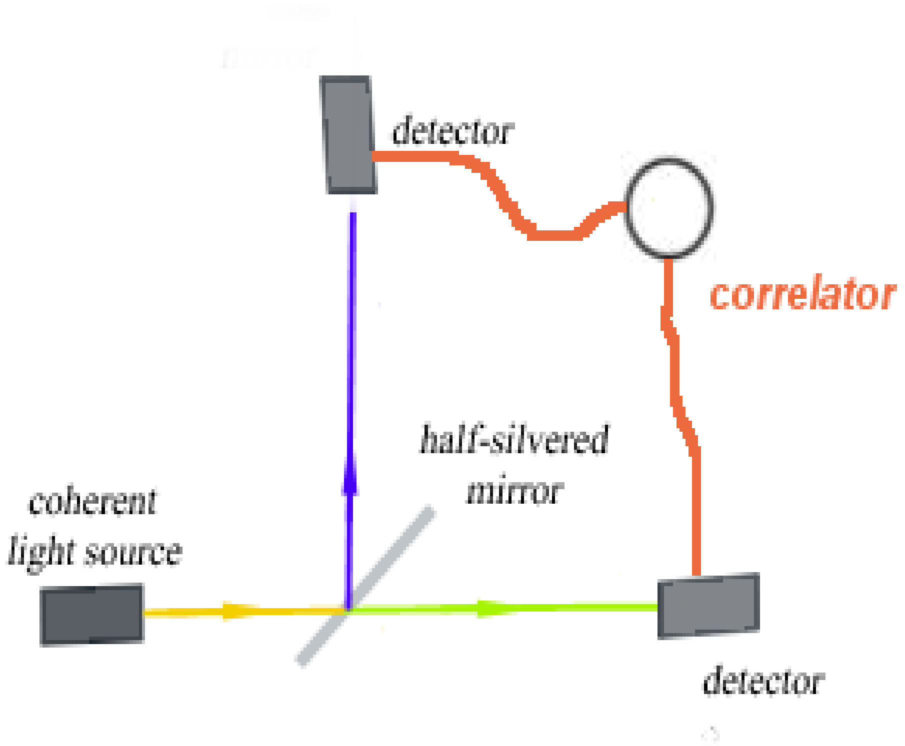
Amplitude interferometry: One measures square of the sum of the amplitudes:

$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + \left(A_1^* A_2 + A_2^* A_1\right)$$

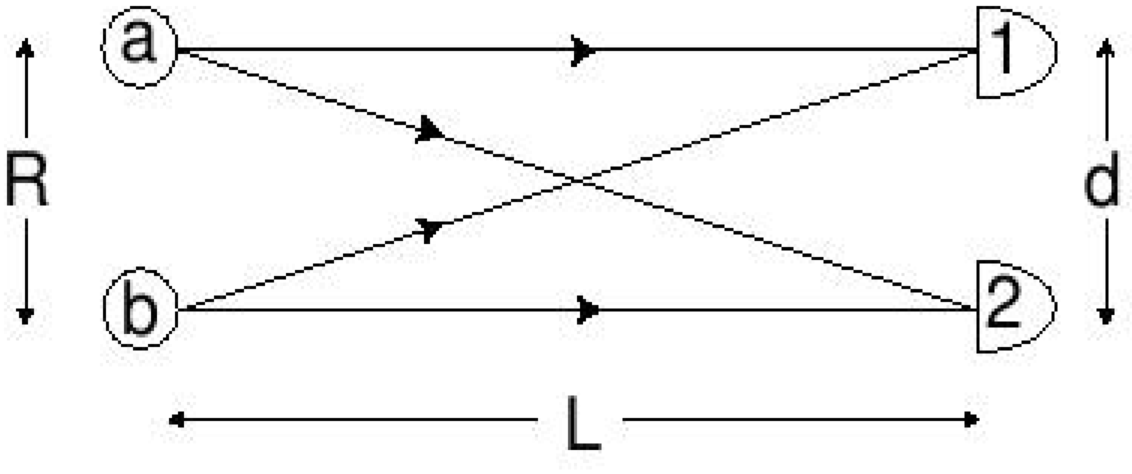


Intesity interferometry: The fringe visibility

$$\langle V^2\rangle = 2\langle|A_1^2|\rangle\langle|A_2^2|\rangle + \langle A_1^{*2} A_2^2\rangle + \langle A_1^2 A_2^{*2}\rangle$$



is measured



The amplitude of the light wave at the detectors:

$$A_1 = \frac{1}{L}\left(\alpha e^{ikr_{1a}+i\varphi_a} + \beta e^{ikr_{2a}+i\varphi_b}\right)$$

Intensity:

$$I_1 = \frac{1}{L^2}\left(|\alpha|^2 + |\beta|^2 + \alpha^* \beta e^{i\left(k\left(r_{1b}-r_{1a}\right)+\varphi_b-\varphi_a\right)} + \alpha^* \beta e^{-i\left(k\left(r_{1b}-r_{1a}\right)+\varphi_b-\varphi_a\right)}\right)$$

On averaging over the random phases (incoherent sources),

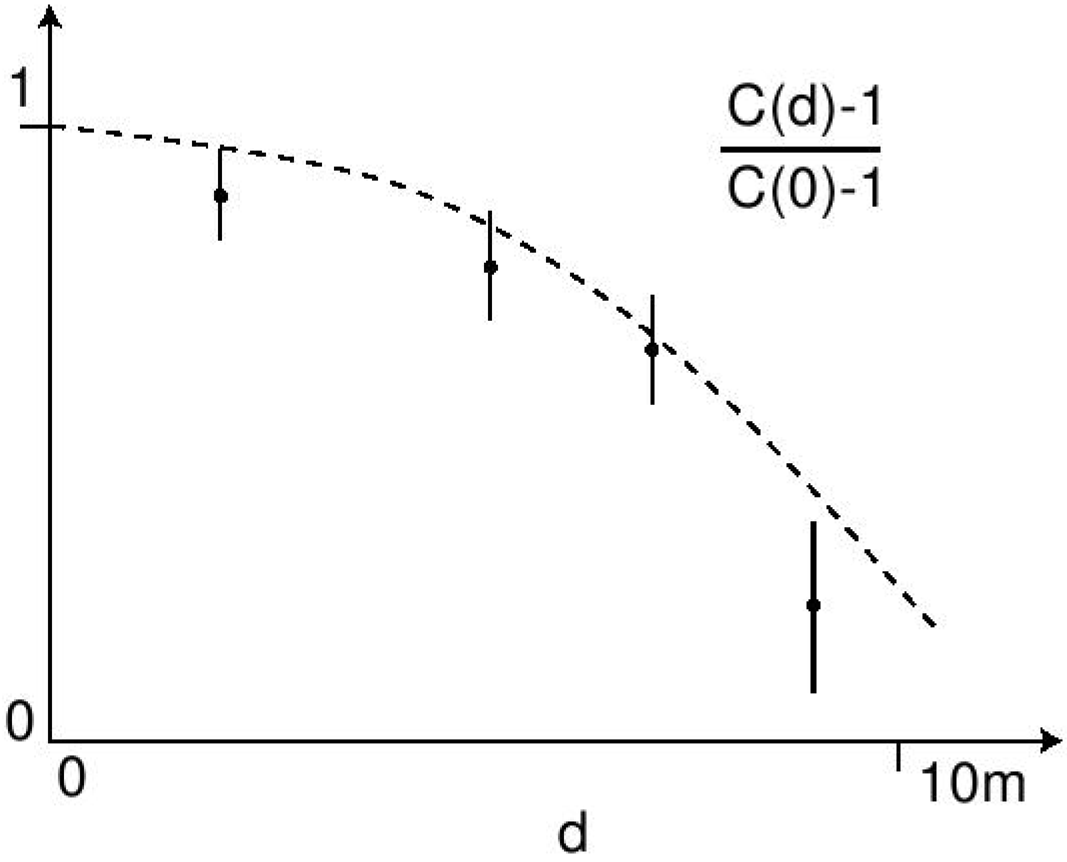
$$\langle I_1 \rangle = \langle I_2 \rangle = \frac{1}{L^2}\left(\langle |\alpha|^2 \rangle + \langle |\beta|^2 \rangle \right)$$

Averaged intensities are independent on the separation of the sources

But this is not the case when we average the product of the intensities!

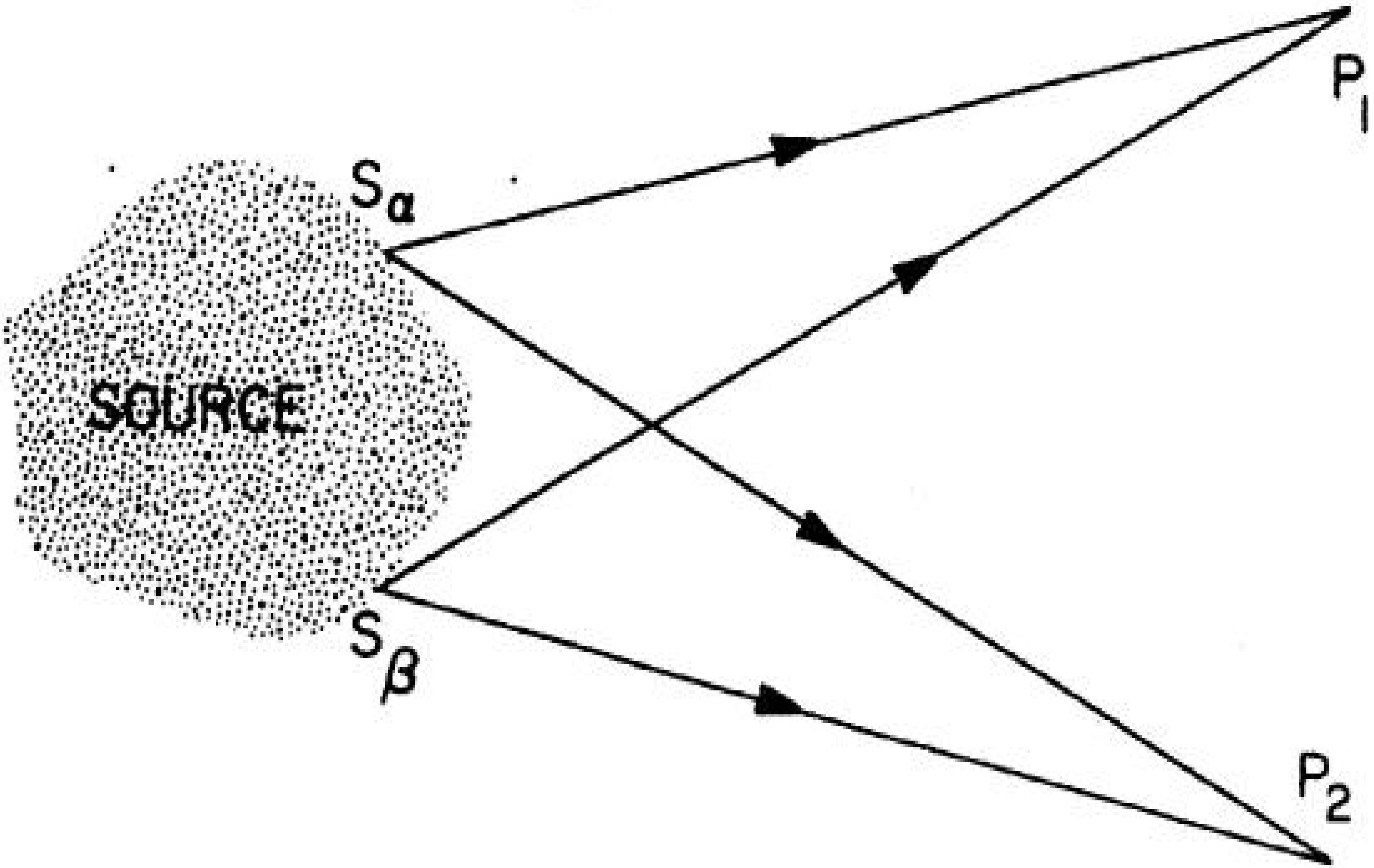
$$\langle I_1 I_2\rangle=\langle I_1\rangle\langle I_2\rangle+\frac{2}{L^4}|\alpha|^2|\beta|^2\cos\left(k\left(r_{1a}-r_{2a}-r_{1b}+r_{2b}\right)\right)$$

The correlated signal varies as a function of the detector separation d on the characteristic length scale $d=\frac{\lambda}{\theta}$, where $\theta=R/L$ is the angular size of the source



Measuring the angular size of Sirius

Particle sources:



Quantum Mechanics

Two regimes in the phase space for a pair of identical particles:

$(x_1 - x_2)(p_1 - p_2) \gg 2\pi\hbar$ - the QS correlation effect is negligible

$(x_1 - x_2)(p_1 - p_2) \geq 2\pi\hbar$ - the QS correlations become significant

Distributions:

$$P_1(\vec{p}) = E\frac{dN}{d^3p} = E\langle a_p^\dagger a_p \rangle$$

$$P_2(\vec{p}_a,\vec{p}_b) = E_a E_b \frac{dN}{d^3p_a d^3p_b} = E_a E_b \langle a_{p_a}^\dagger a_{p_b}^\dagger a_{p_a} a_{p_b} \rangle$$

$$\langle \hat{O} \rangle = tr(\rho \hat{O})$$

Normalization:

$$\int \frac{d^3 p}{E} P_1(\vec{p}) = \langle N \rangle$$

$$\int \frac{d^3 p_a}{E_a} \frac{d^3 p_b}{E_b} P_2(\vec{p}_a, \vec{p}_b) = \langle N(N-1) \rangle$$

Correlation function:

$$C(\vec{p}_a, \vec{p}_b) = \frac{\langle N \rangle^2}{\langle N(N-1) \rangle} \frac{P_2(\vec{p}_a, \vec{p}_b)}{P_1(\vec{p}_a) P_2(\vec{p}_b)}$$

N - average number of particles

The Generalized Wick Theorem

$$P_2(\vec{p}_a, \vec{p}_b) = \frac{\langle N(N-1)\rangle}{\langle N\rangle^2}\left(P_1(\vec{p}_a) P_1(\vec{p}_b) \pm |S(\vec{p}_a, \vec{p}_b)|^2\right)$$

here

$S(\vec{p}_a, \vec{p}_b) = (E_a E_b)^{1/2} \langle a^{\dagger}_{p_a} a_{p_b} \rangle$ - if the particles are emitted independently

$S(\vec{p}_a, \vec{p}_b) = 0$ - for the coherenet case

Correlation functions :

$$C_I(\vec{p}_a \vec{p}_b) = 1 \pm \frac{\left|\langle a^{\ddagger}_{p_a} a_{p_b} \rangle\right|^2}{\langle a^{\ddagger}_{p_a} a_{p_a} \rangle \langle a^{\ddagger}_{p_b} a_{p_b} \rangle}$$ - incoherent sources

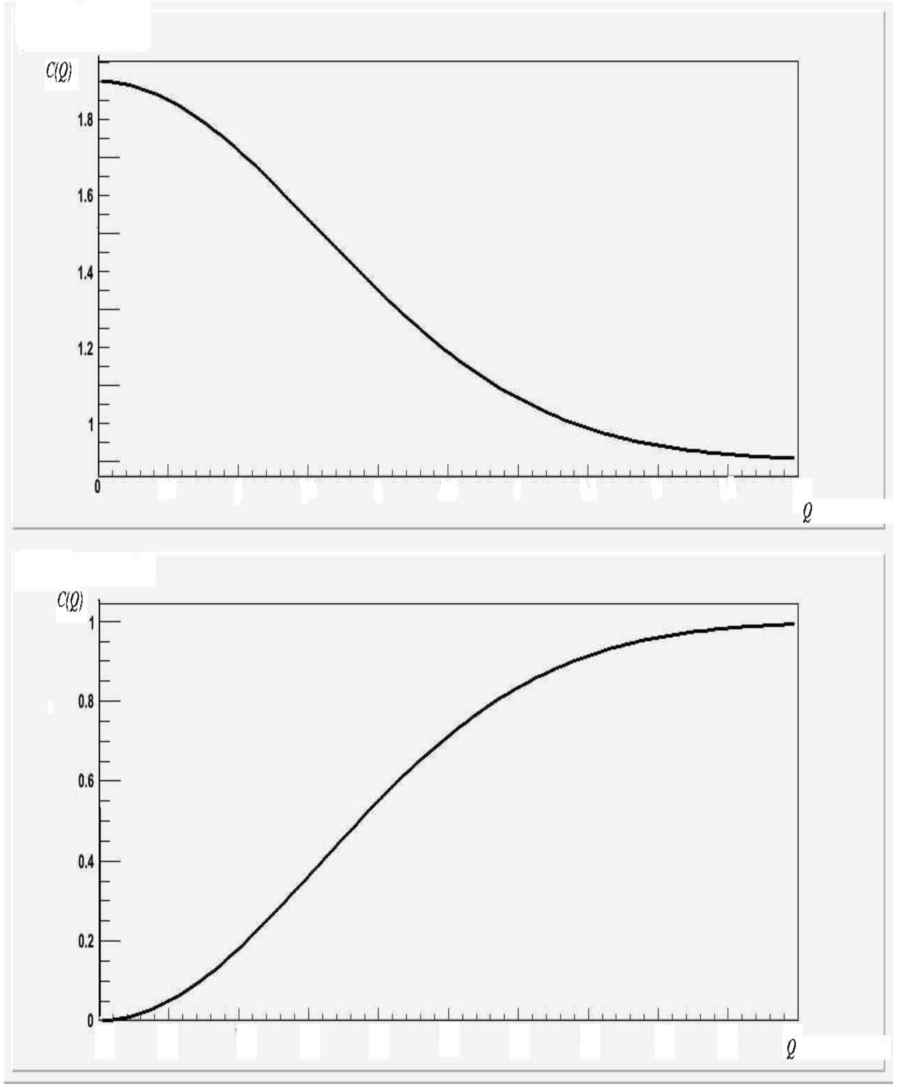
$$C_c(\vec{p}_a, \vec{p}_b) = 1$$ - for coherent emitters

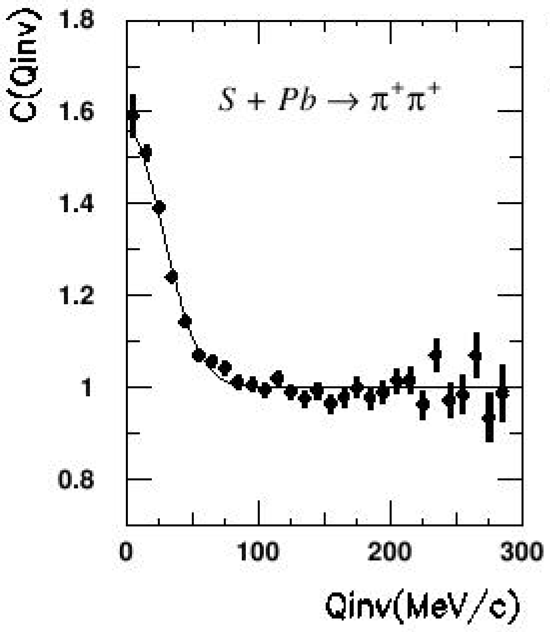
in general, $\rho = \alpha \rho_{chaotic} + (1 - \alpha) \rho_{coherent}$

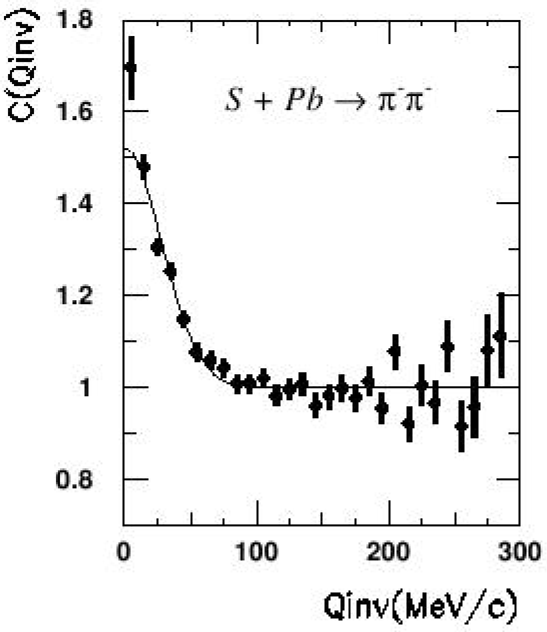
Simplest Invariant parametrization:

$$C(\vec{q},\vec{K})=1\pm\lambda\exp[-Q_{inv}^{2}R_{inv}^{2}] \quad , \quad Q_{inv}^{2}=(\vec{p}_a-\vec{p}_b)^{2}-(E_a-E_b)^{2}$$

Ideal (no FSI and completely chaotic) case:




C(Qinv)
1.8
1.6
1.4
1.2
1
0.8
0
100
200
300
Qinv(MeV/c)
$S + Pb \rightarrow \pi^+\pi^+$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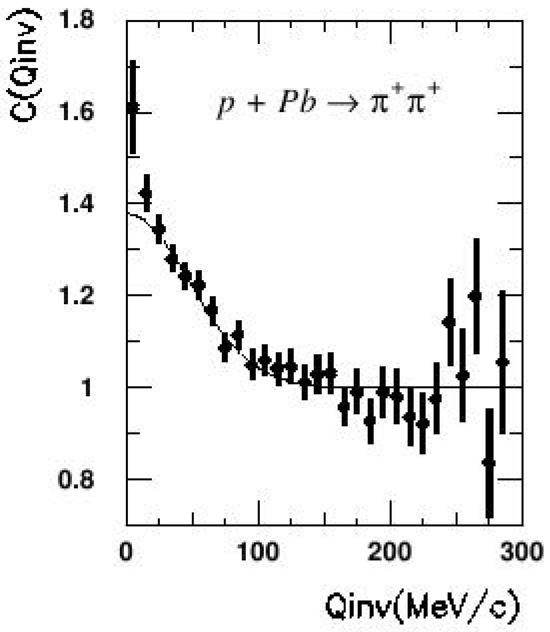
C(Qinv)
1.8
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0.8
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100
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300
Qinv(MeV/c)
$p + Pb \rightarrow \pi^+\pi^+$

Source function:

$$(\Box + m^2)\phi(x) = J(x)$$

Introduce the emission function:

$$S(x,K) = \int \frac{d^4 y}{2(2\pi)^3} e^{(-iKy)} \langle J^*(x+\frac{1}{2}y) J(x-\frac{1}{2}y)\rangle$$

- Wigner function

$$E_K \frac{dN}{d^3 K} = \int d^4 x\, S(x,K)$$

$$C(\vec{q},\vec{K}) = 1 + \frac{|\int d^4 x\, S(x,K) e^{iqx}|^2}{\int d^{4x} S(x,K+\frac{1}{2}q)\ \int d^{4x} S(x,K-\frac{1}{2}q)}$$

$$q = p_a - p_b, \quad K = (p_a + p_b)/2 \quad , \quad qK = 0 \quad , \quad E_K = K^0 = (m^2 + K^2)^{\frac{1}{2}}$$

If the particles are massive one finds small q -s :

$$K^0 = E_K\left(1 + \frac{q^2}{8E_K{}^2} + \ldots\right) \approx E_K$$

Then

$$S\left(x, K - \frac{1}{2}q\right) S\left(y, K + \frac{1}{2}q\right) \approx S(x, K)\, S(y, K)$$

and

$$C(q, K) = 1 \pm \frac{\left|\int d^4x\, S(x, K)\, e^{iqx}\right|^2}{\left|\int d^4x\, S(x, K)\right|^2} = 1 + \left|\left\langle e^{iqx}\right\rangle\right|^2$$

The Full Gaussian Parametrization:

$$S(x,K)=N(K)\,S(\bar{x}(K),K)\exp\left[-\frac{1}{2}\tilde{x}^{\mu}(K)\,B_{\mu\nu}(K)\,\tilde{x}^{\nu}\right]+\delta S(x,K)$$

$$\int d^4x\,\delta S(x,K)=\int d^4x\,x^{\mu}\,\delta S(x,K)=\int d^4x\,x^{\mu}x^{\nu}\,\delta S(x,K)=0$$

$$\tilde{x}^{\mu}(K)=\langle x^{\mu}\rangle$$

$$(B^{-1})_{\mu\nu}=\langle\tilde{x}_{\mu}\tilde{x}_{\nu}\rangle=\langle(x-\bar{x})_{\mu}(x-\bar{x})_{\nu}\rangle$$

From these

$$C(\vec{q},\vec{K})=1+\exp\left[-q^{\mu}q^{\nu}\langle\tilde{x}_{\mu}\tilde{x}_{\nu}\rangle(\vec{K})\right]+\delta C(\vec{q},\vec{K})$$

Coordinate system:

$$\vec{K} = (K_x, K_y, K_z) = (K_\perp, 0, K_L)$$

Axes:

l – longitudinal, coincides with the beam axis
o – outward, orthogonal to l and lying in the K-l plane
s – sideward, orthgonal to the above two

a. Standard Cartesian (Bertch-Pratt) Parametrization

$$q^0 = \beta_\perp q_o + \beta_l q_l$$

$$C(\vec{q}, \vec{K}) = 1 + \exp[-R_s^2(\vec{K}) q_s^2 - R_o^2(\vec{K}) q_o^2 - R_l^2(\vec{K}) q_l^2 - 2 R_{ol}^2(\vec{K}) q_o q_l]$$

with

$$R_s^2(\vec{K}) = \langle \tilde{y}^2 \rangle$$
$$R_o^2(\vec{K}) = \langle (\tilde{x} - \beta_\perp \tilde{t})^2 \rangle$$
$$R_l^2(\vec{K}) = \langle (\tilde{z} - \beta_l \tilde{t})^2 \rangle$$
$$R_{ol}^2(\vec{K}) = \langle (\tilde{x} - \beta_\perp \tilde{t})(\tilde{z} - \beta_l \tilde{t}) \rangle$$

b. Yanoo-Koonin-Podgoretski parametrization

Another choice of basic variables:

$$q_{\perp}=(q_o^2+q_s^2)^{\frac{1}{2}},\ \ q^0,\ \ q_3$$

$$C(\vec{q},\vec{K})=1+\exp[-R_{\perp}^2 q_{\perp}^2-R_{\parallel}^2(q_l^2-(q^0)^2)-(R_0^2+R_{\parallel}^2)(qU)^2]$$

Where

$$U(\vec{K})=\gamma(\vec{K})(1,0,0,v(\vec{K}))$$
$$R_{\perp}^2(\vec{K})=R_s^2(\vec{K})=\langle\tilde{y}^2\rangle$$
$$R_{\parallel}^2(\vec{K})=\langle(\tilde{z}-\beta_l\tilde{x}/\beta_{\perp})^2\rangle-\beta_l^2\langle\tilde{y}^2\rangle/\beta_{\perp}^2\approx\langle\tilde{z}^2\rangle$$
$$R_0^2(\vec{K})=\langle(\tilde{t}-\tilde{x}/\beta_{\perp})^2\rangle-\langle\tilde{y}^2\rangle/\beta_{\perp}^2\approx\langle\tilde{t}^2\rangle$$

Yanoo-Koonin frame: $v(\vec{K})=0$

Longitudinally CoMoving System (LCSM): $\beta_l=0$

Relations between the Cartesian and YKP parameters:

$$R_s^2 = R_\perp^2$$

$$R_{diff}^2 = R_o^2 - R_s^2 = \beta_\perp^2 \gamma^2 \left(R_0^2 + v^2 R_\parallel^2 \right)$$

$$R_l^2 = \left(1 - \beta_l^2 \right) R_\parallel^2 + \gamma^2 \left(\beta_l - v \right)^2 \left(R_0^2 + R_\parallel^2 \right)$$

$$R_{ol}^2 = \beta_\perp \left(-\beta_l R_\parallel^2 + \gamma^2 \left(b_l - v \right)^2 \left(R_0^2 + R_\parallel^2 \right) \right)$$