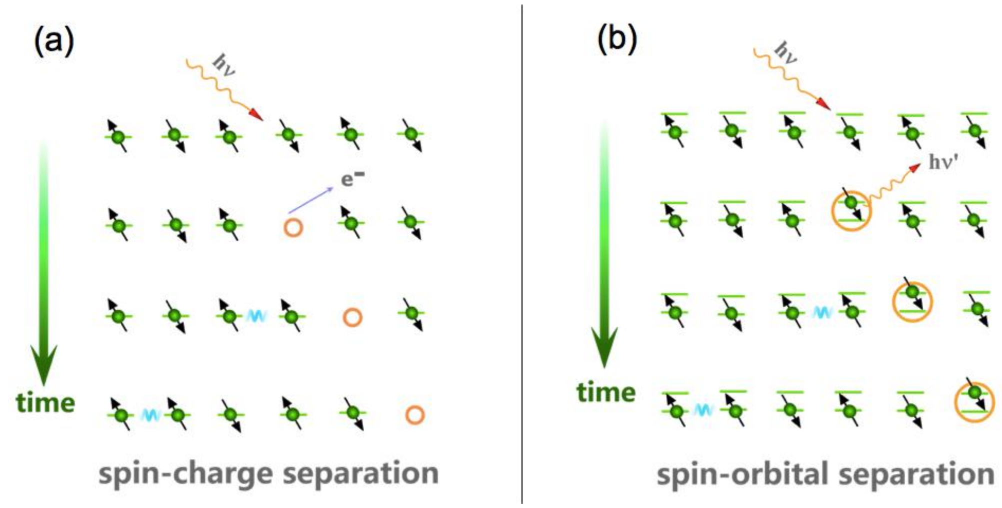

Figure 1: Peculiarities of the quantum nature of quasi-low-dimensionl materials – fractionalization of excitations in the one-dimensional antiferromagnets: (a) a single hole introduced into a one-dimensional antiferromagnet (e.g. in a photoemission experiment) fractionalizes into independent spinon (magnetic domain wall; blue wiggles) and holon (hole without spin; empty orange circle) excitations; (b) a single orbital excitation introduced into a one-dimensional antiferromagnet (e.g. in a resonant inelastic x-ray experiment) fractionalizes into independent spinon (magnetic domain wall; blue wiggles) and orbiton (orbital excitation not carrying spin and charge degree of freedom; empty orange circle) excitations. Figure adopted from J. Schlappa *et al.*, Nature **485**, 82 (2012).

The course is a brief introduction to the basic notions of the quantum theory of magnetism. During the course a particular emphasis will be devoted to: (i) the differences between quantum and classical effects, and (ii) the connections between the real materials and the here discussed effective spin models and its solutions. It is desired that after the lectures, the student should be able to understand the quantum nature of magnetism present in a number of solids – with a particular emphasis on the 'very quantum' behavior of magnetism in the quasi-low-dimensional crystals, cf. Fig. 1.

The plan of the *lectures* is as follows:

Lectures 1-2: Magnetic properties of matter

- Magnetic properties of matter versus spin and orbital quantum numbers of electrons.

- Magnetism in atoms, band magnetism, and magnetism with localized moments (similarities and differences).

- Overview of the topics to be presented during the lectures.

Lectures 3-4: Magnetism with localized magnetic moment – modelling La_2CuO_4

- Overview of the important physics present in La_2CuO_4 (a compound from the high temperature superconductor family).

- Modelling the physics of La_2CuO_4 with the two dimensional (2D) Hubbard Hamiltonian.

- Downfolding the Hubbard Hamiltonian onto an effective 2D spin Heisenberg Hamiltonian.

Lectures 3-8: Quantum magnetism – ground state and excitations of 2D Heisenberg model

(a) Classical (Ising) limit of the 2D Heisenberg model:

- 'Ideal' antiferromagnetic (AF) Neel state as the ground state of the model.

- Excited states: local excitations (dispersionless magnons).

(b) Ground state and excitations:

- Solving the model using linear spin wave theory.

- Neel AF dressed with magnons as the ground state of the model

- Collective excitations (magnons) calculated using linear spin wave approximation.

- Shortcomings and limitations of the linear spin wave approximation.

- Comparison with other approximations and numerical results.

(c) Spontaneous symmetry breaking:

- Symmetries of the model vs. the symmetries of the ground state.
- Order parameter.
- Mermin-Wagner theorem.
- Reduction of order parameter by quantum fluctuations and quantum disorder.

(d) What is quantum and what is classical?

- Definiton of classical and quantum models.
- Intermezzo: introducing ferromagnetic Heisenberg model.
- Not so quantum and not so classical: comparing AF Ising and Heisenberg models with the ferromagnetic model (ground state, excitations, order parameter).

Lectures 9-10: Revisiting La_2CuO_4: comparing the above discussed theoretical results for the 2D Heisenberg model with experiments

- Magnetic moment and other ground state properties: comparison between theory and experiment.
- Introducing spin dynamical structure factor.
- Basics of elastic and inelastic neutron scattering and its relation to spin dynamical structure factor.
- Basics of resonant elastic and inelastic x-ray scattering and its relation to spin dynamical structure factor.
- Overview of experimental results and conclusions on the nature of magnetism in La_2CuO_4.

Lectures 11-14: Properties of other spin models – ground state and collective excitations with a short discussion on its origin, application to materials, and experiments:

(a) One-dimensional Heisenberg model

- Disordered ground state with strong AF correlations.
- 'Domain'–like excitations (spinons).
- Overview of various solutions of the model: Failure of the spin-wave theory, success of particular mean-field approaches and exact Bethe-Ansatz solution.
- Even 'more' quantum nature of the model (than the 2D case).
- Materials and experimental verification of the above theory.

(b) Frustrated spin models

- Introducing frustrated interactions and spin liquids.
- Majumdar-Ghosh model and the valence bond state.
- Bilinear-biquadratic models for S=1.
- Possible experimental realizations of the models.

(c) Adding orbital degrees of freedom (Kugel-Khomskii models)

- Spin exchange interaction in the presence of orbitals. Onset of orbital exchange.
- Overview of various spin-orbital models: SU(4) symmetric spin-orbital model, spin-orbital model for the e_g orbital degrees of freedom, spin-orbital model for the t_{2g} orbital degrees of freedom.
- Ground state and excitations: Goodenough-Kanamori rules and anistropic magnetic order, spin-orbital entanglement.
- Materials and experimental verification of the above theory.

Lecture 15: Summary of open problems in the field

- Existence of the spin liquid phase.
- Nature of spin exciations in the doped magnets (t-J and double exchange models).

- Origin of the $\propto q^2$ ($\propto q$) magnon dispersion relations in the 'more' classical (quantum) models. Relation to the existence of the ordered states in nature.

Literature:

1. "Interacting Electrons and Quantum Magnetism", Assa Auerbach, Springer-Verlag (New York 1994).

2. "Lecture Notes on Electron Correlation and Magnetism", Patrik Fazekas, World Scientific (Singapore 1999).

3. "Magnetism in Condensed Matter", Stephen Blundell, Oxford University Press (Oxford 2001).

4. "Introduction to Frustrated Magnetism: Materials, Experiments, Theory", Editors: C. Lacroix, P. Mendels, and F. Mila, Springer-Verlag (Heidelberg 2011).