Homework problems # 3

1. Construct the Noether current in scalar electrodynamics described by the $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\varphi)^*(D^\mu\varphi) - m^2\varphi^*\varphi$ with $D_\mu\varphi \equiv (\partial_\mu - ieA_\mu)\varphi$. Show that the current is conserved for fields which satisfy appropriate equations of motion.

2. Find the Noether current corresponding to space-time translations and the symmetric energy-momentum tensor in scalar electrodynamics.

3. Derive equations of motion for the Yang-Mills theory with the gauge group SU(2) interacting with SU(2) doublet of scalar fields.

4. For the same theory as above, find the symmetric energy-momentum tensor and show that it is conserved if equations of motion are satisfied.