Homework problems #4

1. Derive equation of motion for a massive photon, so use the following Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$$

and show the for $m \neq 0$ the equation implies $A_{\mu}{}^{,\mu} = 0$.

2. For polarization tensors $\varepsilon^{(a)}_{\mu\nu}(k)$ of a massive graviton, write down the most general form for $\sum_a \varepsilon^{(a)}_{\mu\nu}(k) \varepsilon^{(a)}_{\lambda\sigma}(k)$ using symmetry repeatedly. For example, it must be invariant under the exchange $\{\mu\nu \leftrightarrow \lambda\sigma\}$. You might end up with something like

$$\begin{aligned} &A G_{\mu\nu} G_{\lambda\sigma} + B(G_{\mu\lambda} G_{\nu\sigma} + G_{\mu\sigma} G_{\nu\lambda}) + C(G_{\mu\nu} k_\lambda k_\sigma + k_\mu k_\nu G_{\lambda\sigma}) \\ &+ D(k_\mu k_\lambda G_{\nu\sigma} + k_\mu k_\sigma G_{\lambda\nu} + k_\nu k_\sigma G_{\mu\lambda} + k_\nu k_\lambda G_{\mu\sigma}) + E k_\mu k_\nu k_\lambda k_\sigma \end{aligned}$$

where

$$G_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}$$

with various unknown A,$\cdots$,E . Apply $k^\mu \sum_a \varepsilon^{(a)}_{\mu\nu}(k) \varepsilon^{(a)}_{\lambda\sigma}(k) = 0$ and find out what that implies for the constants. Proceeding this way, derive

$$\sum_a \varepsilon^{(a)}_{\mu\nu}(k) \varepsilon^{(a)}_{\lambda\sigma}(k) = (G_{\mu\lambda} G_{\nu\sigma} + G_{\mu\sigma} G_{\nu\lambda}) - \frac{2}{3} G_{\mu\nu} G_{\lambda\sigma} \,.$$